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Component commonality under no-holdback allocation rules

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ABSTRACT

We study the value of component commonality in assemble-to-order systems under no-holdback allocation rules. We prove that the total product backorder and on-hand component inventory decrease with probability one as the degree of commonality increases; however, the average cost may not decrease unless a certain cost symmetric condition is imposed.

1. Introduction

ATO systems are an important business model for improving supply chain performance. By eliminating expensive finished-product inventory and carrying only component inventory, ATO systems hold the promise of achieving customization, lower inventory cost and fast response to demand simultaneously. In an ATO system, a product may require a subset of components, and a component can be required by different products. The issues of component commonality and component inventory management (replenishment policy and allocation rule) are critical to the success of an ATO system.

Component commonality is a key enabler of ATO systems. Examples can be found in many industries, such as computers, electronics and automobiles [4]. The value of component commonality has been studied in the operations management literature for decades, with a focus primarily on static models without lead times [16,17]. Recent studies have extended this literature to dynamic inventory systems with lead times [15]. These studies focus on practical but sub-optimal allocation rules because the optimal allocation rules are not known (except for a few special cases, e.g., [3,13]) and only simple and suboptimal allocation rules are implemented in practice.

Because a common component is shared by many products, it allows us to explore the effect of risk pooling in assembly systems. Risk pooling is an important concept in supply chain management [4]: by aggregating demands from different products, we may reduce overall demand uncertainty and improve the cost/effectiveness of the system. In dynamic ATO systems with lead times under heuristic allocation rules, the value of commonality (or risk pooling effect) is more delicate. For example, for a two-product ATO system under a first-come first-serve (FCFS) allocation rule, Song [14] provides a couple of numerical examples to show that component commonality does not lower the total backorder and improve inventory performance. Song and Zhao [15] shows that component commonality does not always generate savings on inventory investment or service-level improvements. In fact, the value of component commonality depends strongly on how the component inventory is managed, e.g., the common component allocation rules, as well as various system parameters such as component costs and lead times. Thus, it is of interest to study the impact of commonality on backorders and inventory performance in ATO systems, but under a class of allocation rules different from FCFS, that is, the no-holdback (NHB) rules.

Song and Zhao [15] first defines the NHB rules. To see how it works, let us compare the FCFS rule with the first-ready first-serve (FRFS) rule (a special case of the NHB rules, see [15]). Under the FCFS rule, demand for each component is fulfilled in exactly the same sequence as it occurs. When a demand arrives, if some of its components are available while others are not, the available components are put aside as committed stock. Under the FRFS rule, however, we do not allocate or commit those available components to the order unless doing so leads to the fulfillment of this order. When a replenishment arrives, we satisfy the oldest backorder for which all required components are available. The FRFS rule is widely used in practice, see, e.g., [12] for an example in Dell Computer Corporation.

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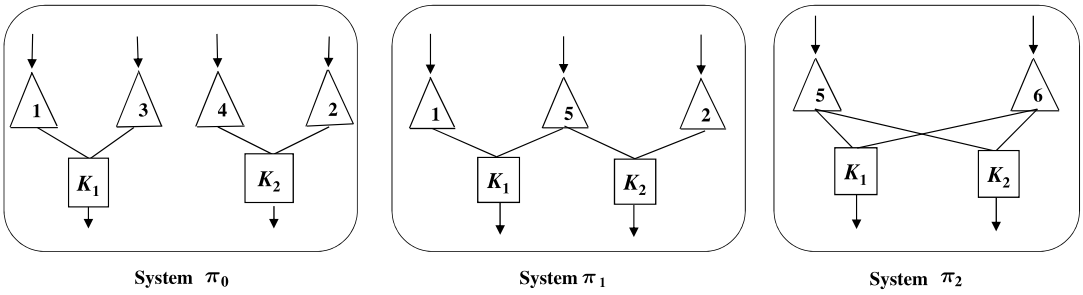

Fig. 1. Two-component ATO systems.

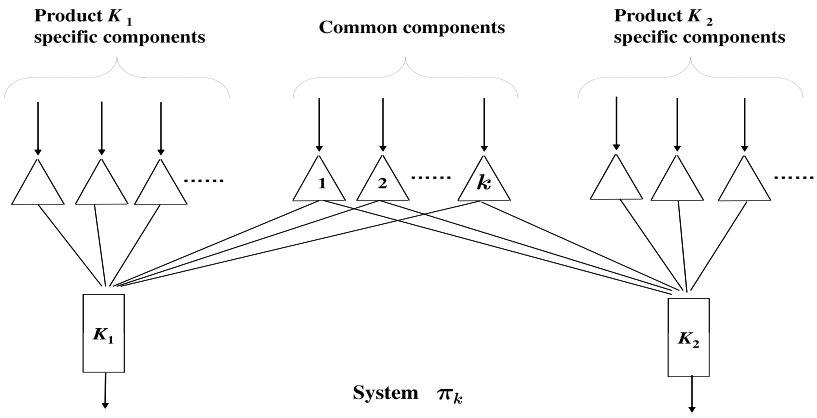

Fig. 2. ATO system with k common components.

Lu et al. [13] studies the NHB rules in continuous-review ATO systems of a general product structure, and identifies conditions on product and cost structure under which the NHB rules outperform all other component allocation rules. Dogru et al. [7] studies the general class of the NHB rules in a two-product system with identical constant lead times. However, both papers do not consider the impact of component commonality. In addition to [15], several other papers study the value of component commonality in dynamic inventory systems with lead times [1,6,14]. They assume continuous-review inventory control and focus on the FCFS rule; although Agrawal and Cohen [1] use a fair-share rule for demand realized in the same period, the paper assumes FCFS for satisfying demand in different time periods.

In the component commonality literature, researchers have employed a representative two-product ATO system in which each product is assembled from two components [2,9] as illustrated in Fig. 1. In system π_0, products are assembled from product-specific components only and there is no common component. In system π_1, the products share the common component 5 that replaces components 3 and 4 in System π_0. In system π_2, the two products share both the common components 5 and 6 where component 6 replaces components 1 and 2 in system π_1.

In this paper, we study the value of component commonality in a generalized version of the aforementioned two-component systems in Fig. 1 with lead times and the NHB rules. Using a sample path analysis, we show that under any NHB rule, both the total product backorder and total on-hand component inventory decrease in any event as the degree of commonality increases. However, the system-wide average cost does not always decrease unless a certain cost symmetric condition is imposed. Finally, we consider systems with general cost structure and conduct a numerical study to quantify the impact of commonality on system-wide average cost and its dependence on various system parameters.

2. The model and preliminary results

We consider a multi-product ATO system with m different components labeled by $i \in \{1, 2, 3, \ldots, m\}$. At most one unit of a component is required for each product. Let $\mathcal{K}$ denote the set of products. Each product $K \in \mathcal{K}$ is assembled by the component set $K \subseteq \{1, 2, 3, \ldots, m\}$. For each product $K \in \mathcal{K}$, it requires a set of common components $\mathcal{M}$ (which is shared by all products) and a set of product-specific components $K \setminus \mathcal{M}$. If $|\mathcal{M}| = k$, we denote the system by π_k. Fig. 2 provides an example with two products.

In the sequel, we use subscripts to indicate components and superscripts to indicate products. Let $\mathcal{K}_i$ be the product set that requires component i. We assume that the demand process $\{D^K(t), t \geq 0\}$ for product $K \in \mathcal{K}$ follows an arbitrary stochastic or deterministic process which could be dependent or independent of the others, such as ARMA [8] (or a vector ARMA), ARIMA [10], quasi-ARMA [11] and MMFE [5].

The component inventory is controlled by an independent continuous-time base-stock policy with base-stock levels $\mathbf{s} = (s_1, s_2, \ldots, s_m)$. Thanks to its simplicity, this class of inventory policies is well adopted in practice and studied in the literature [16]. We note that the base-stock policy is suboptimal in a general assembly system, and refer the reader to [16] for a detailed review of the literature. Without loss of generality, we assume that the initial on-hand inventory of component i equals s_i. For component i, let L_i be the replenishment lead time which is a constant, $I_i(t)$ be the on-hand inventory at time t, and $O_i(t)$ be the outstanding order that is the amount of orders placed but not replenished by t. Because the arrival of each demand requiring component i triggers a replenishment order for this component, we have

$$O_i(t) = \sum_{K \in \mathcal{K}_i} D^K(t - L_i, t), \tag{1}$$

where $D^K(t - L_i, t)$ denotes lead time demand of product K during the time period $(t - L_i, t]$. We assume full backorder for any demand which cannot be satisfied upon arrival. Let $B_i(t)$ be the shortage of component i at time t, and it is expressed as $B_i(t) = [O_i(t) - s_i]^+$, where $[x]^+ := \max\{x, 0\}$. Let $B^K(t)$ be the backorder of product $K \in \mathcal{K}$ at time t. For steady state variables, we omit the parameter t.

The following flow conservation equation holds under any allocation rule for component i [13].

$$I_i(t) = s_i - O_i(t) + \sum\nolimits_{K\in\mathcal{K}_i} B^K(t). \tag{2}$$

Since the on-hand inventory at any time is nonnegative, i.e., $I_i(t) \geq 0$, by Eq. (2), we obtain

$$\sum\nolimits_{K\in\mathcal{K}_i} B^K(t) \geq B_i(t), \quad \forall\, i. \tag{3}$$

An allocation rule is no-holdback (NHB) if the following condition is satisfied for all t [13,15],

$$B^K(t) \times \min\{I_i(t), i \in K\} = 0, \quad \text{for all } K \in \mathcal{K}. \tag{4}$$

In other words, under an NHB allocation rule, a product demand is backordered if and only if at least one of its required components is out of stock. For the total product backorder, Lu, et al. [13] provide the following result: for system π_k with the common component set $\mathcal{M}$ under an NHB allocation rule, on all sample paths, the total product backorder satisfies

$$\sum_{K\in\mathcal{K}} B^K(\pi_k, t) = \max\left\{\max_{i\in\mathcal{M}}\{B_i(\pi_k, t)\}, \sum_{K\in\mathcal{K}} \max_{j\in K\setminus\mathcal{M}}\{B_j(\pi_k, t)\}\right\}. \tag{5}$$

3. Degree of commonality

We define the *degree of commonality* to be the number of common components, $|\mathcal{M}|$. Consider a series of ATO systems $\{\pi_k, k = 0, 1, 2, 3, \ldots\}$ with the corresponding common component sets $\{\mathcal{M}_k, k = 0, 1, 2, 3 \ldots\}$ such that $\mathcal{M}_0 \subset \mathcal{M}_1 \subset \mathcal{M}_2 \subset \mathcal{M}_3 \subset \cdots$, where $k = |\mathcal{M}_k|$. For the multi-component system π_k, where $k = 0, 1, 2, 3, \ldots$, let the total product backorder at time t be

$$B(\pi_k, t) = \sum_{K\in\mathcal{K}} B^K(\pi_k, t).$$

To compare the performance of systems with different degree of commonality, we investigate systems π_k and π_{k+1}. In particular, let n be the index of the *critical component* such that $\mathcal{M}_{k+1} = \mathcal{M}_k \bigcup \{n\}$. Furthermore, let $s_n(\pi_{k+1})$ be the base-stock level of the component n in system π_{k+1}, and $s_n^K(\pi_k)$ be the base-stock level of the product-K specific component in system π_k which is replaced by the common component n in system π_{k+1}. For ease of exposition, we make the following assumption.

Assumption 1. For systems π_k and π_{k+1},

(a) the lead time and holding cost for each component do not change before/after component sharing;
(b) for component n, the base-stock level is accumulated after component sharing, that is

$$s_n(\pi_{k+1}) = \sum_{K\in\mathcal{K}} s_n^K(\pi_k); \tag{6}$$

(c) for other components, the base-stock levels do not change before/after component sharing.

Let $O_n(\pi_{k+1}, t)$ denote the outstanding order of component n in system π_{k+1}, and $O_n^K(\pi_k, t)$ be the outstanding order of the product-K component in system π_k that is replaced by the common component n in system π_{k+1}. Then, we have the following observation.

Observation 1. *Under Assumption* 1, *for the critical component* n, *the following result holds on all sample paths,*

$$O_n(\pi_{k+1}, t) = \sum_{K\in\mathcal{K}} O_n^K(\pi_k, t). \tag{7}$$

Theorem 1. *For the class of general ATO systems under any base-stock levels, Assumption* 1 *and any NHB allocation rule, the total product backorder decreases as the degree of commonality increases on all sample paths, that is,*

$$B(\pi_k, t) \geq B(\pi_{k+1}, t), \quad k = 0, 1, 2, 3, \ldots.$$

Proof. Note that for any $i \in \mathcal{M}_k$, by Assumption 1, we have

$$\begin{aligned} B_i(\pi_k, t) &= [O_i(\pi_k, t) - s_i(\pi_k)]^+ \\ &= [O_i(\pi_{k+1}, t) - s_i(\pi_{k+1})]^+ = B_i(\pi_{k+1}, t), \end{aligned}$$

and for any product-specific component $j \in K \setminus \mathcal{M}_{k+1}$ we have

$$\begin{aligned} B_j(\pi_k, t) &= [O_j(\pi_k, t) - s_j(\pi_k)]^+ \\ &= [O_j(\pi_{k+1}, t) - s_j(\pi_k)]^+ = B_j(\pi_{k+1}, t). \end{aligned}$$

By Eq. (5), we further have

$$\begin{aligned} B(\pi_k, t) &= \max\left\{\max_{i\in\mathcal{M}_k}\{B_i(\pi_k, t)\}, \sum_{K\in\mathcal{K}} \max_{j\in K\setminus\mathcal{M}_k}\{B_j(\pi_k, t)\}\right\} \\ &= \max\left\{Q(t), \sum_{K\in\mathcal{K}} \max\{R^K(t), B_n(\pi_k, t)\}\right\}; \end{aligned} \tag{8}$$

and

$$\begin{aligned} &B(\pi_{k+1}, t) \\ &= \max\left\{\max_{i\in\mathcal{M}_{k+1}}\{B_i(\pi_{k+1}, t)\}, \sum_{K\in\mathcal{K}} \max_{j\in K\setminus\mathcal{M}_{k+1}}\{B_j(\pi_{k+1}, t)\}\right\} \\ &= \max\left\{\max\{Q(t), B_n(\pi_{k+1}, t)\}, \sum_{K\in\mathcal{K}} R^K(t)\right\}, \end{aligned} \tag{9}$$

where $Q(t) = \max_{i\in\mathcal{M}_k}\{B_i(\pi_k, t)\}$ and $R^K(t) = \max_{j\in K\setminus\mathcal{M}_{k+1}}\{B_j(\pi_{k+1}, t)\}$. For any product $K \in \mathcal{K}$, let $B_n^K(\pi_k, t) = [O_n^K(t) - s_n^K]^+$. By Assumption 1, we have the following result for the total shortage of component n before and after sharing,

$$\begin{aligned} B_n(\pi_{k+1}, t) &= [O_n(\pi_{k+1}, t) - s_n(\pi_{k+1})]^+ \\ &= \left[\sum_{K\in\mathcal{K}} (O_n^K(\pi_k, t) - s_n^K(\pi_k))\right]^+ \\ &\leq \sum_{K\in\mathcal{K}} \left[O_n^K(\pi_k, t) - s_n^K(\pi_k)\right]^+ = \sum_{K\in\mathcal{K}} B_n^K(\pi_k, t). \end{aligned} \tag{10}$$

Here the second equality holds by Eqs. (6) and (7), and the inequality holds by the fact that $[\sum a^K]^+ \leq \sum [a^K]^+$ with $a^K = O_n^K(\pi_k, t) - s_n^K(\pi_k)$. To compare Eq. (8) with Eq. (9), we consider the following two cases:

- case 1: $Q(t) \geq B_n(\pi_{k+1}, t)$. By Eq. (9), we have

$$\begin{aligned} B(\pi_{k+1}, t) &= \max\left\{Q(t), \sum_{K\in\mathcal{K}} R^K(t)\right\} \\ &\leq \max\left\{Q(t), \sum_{K\in\mathcal{K}} \max\{R^K(t), B_n^K(\pi_k, t)\}\right\} \\ &= B(\pi_k, t); \end{aligned}$$

- case 2: $Q(t) < B_n(\pi_{k+1}, t)$. By Eq. (9), we have

$$\begin{aligned} B(\pi_{k+1}, t) &= \max\left\{B_n(\pi_{k+1}, t), \sum_{K\in\mathcal{K}} R^K(t)\right\} \\ &\leq \max\left\{\sum_{K\in\mathcal{K}} B_n^K(\pi_k, t), \sum_{K\in\mathcal{K}} R^K(t)\right\} \\ &\leq \sum_{K\in\mathcal{K}} \max\left\{B_n^K(\pi_k, t), R^K(t)\right\} \end{aligned}$$

$$= \max\left\{Q(t), \sum_{K\in\mathcal{K}} \max\{R^K(t), B_n^K(\pi_k, t)\}\right\}$$
$$= B(\pi_k, t).$$

Here, the first inequality holds by Eq. (10), and the second to the last by the fact that

$$Q(t) < B_n(\pi_{k+1}, t) \le \sum_{K\in\mathcal{K}} B_n^K(\pi_k, t)$$
$$\le \sum_{K\in\mathcal{K}} \max\left\{R^K(t), B_n^K(\pi_k, t)\right\}.$$

The proof is finally concluded by cases 1 and 2 above. □

For system π_k, we denote the total on-hand component inventory by

$$I(\pi_k, t) = \sum_{i=1,2,\ldots} I_i(\pi_k, t). \tag{11}$$

Theorem 2. *Consider the class of general ATO systems under any base-stock levels, Assumption 1 and any NHB allocation rule. If all the products $K \in \mathcal{K}$ require the same number of components, then the total on-hand inventory decreases as the degree of commonality increases on all sample paths, that is*

$$I(\pi_k, t) \ge I(\pi_{k+1}, t), \quad k = 0, 1, 2, 3, \ldots$$

Proof. Let v be the number of components required by each product $K \in \mathcal{K}$. By Eqs. (2) and (11), we rewrite the total on-hand inventory of system π_k, as follows,

$$I(\pi_k, t) = \sum_{i=1,2,\ldots} s_i(\pi_k) - \sum_{i=1,2,\ldots} O_i(\pi_k, t) + v \cdot B(\pi_k, t). \tag{12}$$

Next, by Assumption 1(b), systems π_k have the same total base-stock level (corresponding to the first term on the right hand side of Eq. (12)). Furthermore, by Observation 1, systems π_k have the same total outstanding orders (corresponding to the second term on the right hand side of Eq. (12)). Finally, the proof readily follows from Theorem 1. □

4. Average cost analysis

In this section, we consider the long-run time-average cost which is the sum of inventory holding costs of all components and backorder penalty costs of all products per unit of time. We assume a linear holding cost for each component's on-hand inventory and a linear backorder cost for each product backorder. In particular, let h_i denote the inventory holding cost per unit of component i, and b^K denote the backorder cost per unit of product K. For any base-stock policy $\mathbf{s}$, the expected total average cost of the system in its steady state is

$$C(\pi_k, \mathbf{s}) = \sum_{i=1,2,\ldots,m} h_i \mathbf{E}\left[I_i(\pi_k)\right] + \sum_{K\in\mathcal{K}} b^K \mathbf{E}\left[B^K(\pi_k)\right].$$

Substituting Eq. (2) into the above equation yields

$$C(\pi_k, \mathbf{s}) = \sum_{i=1,2,\ldots,m} h_i\left[s_i(\pi_k) - \mathbf{E}[O_i(\pi_k)]\right] + \sum_{K\in\mathcal{K}} \beta^K \mathbf{E}\left[B^K(\pi_k)\right], \tag{13}$$

where

$$\beta^K = \sum\nolimits_{i\in K} h_i + b^K. \tag{14}$$

System π_k is called *cost symmetric* if $\beta^K = \beta$ for each product K, where $\beta > 0$ is a positive constant. Otherwise, it is called *cost asymmetric*.

Table 1
$\mathbf{E}[B^{K_1}]$ and $\mathbf{E}[B^{K_2}]$ of systems π_0 and π_1.

	$\lambda^{K_1} = 1$		$\lambda^{K_1} = 3$	
	$\mathbf{E}[B^{K_1}]$	$\mathbf{E}[B^{K_2}]$	$\mathbf{E}[B^{K_1}]$	$\mathbf{E}[B^{K_2}]$
System π_0	1.34	5.08	9.00	5.03
System π_1	2.00	4.07	8.40	5.61

If system π_k is cost symmetric, then by Eqs. (13) and (5), we have the following expression:

$$C(\pi_k, \mathbf{s}) = \sum_{i=1,\ldots,m} h_i\left[s_i(\pi_k) - \mathbf{E}[O_i(\pi_k)]\right] + \beta \cdot \mathbf{E}\left[B(\pi_k)\right], \tag{15}$$

where the total backorder is given by

$$B(\pi_k) = \max\left\{\max_{i\in\mathcal{M}_k}\{B_i(\pi_k)\}, \sum_{K\in\mathcal{K}} \max_{j\in K\setminus\mathcal{M}_k}\{B_j(\pi_k)\}\right\}.$$

The following theorem presents the value of component commonality for cost-symmetric systems.

Theorem 3. *For* cost-symmetric systems *(i.e., $\beta^K = \beta$ for each product K) with any given base-stock levels, $\mathbf{s}$, under Assumption 1 and any NHB allocation rule, the average cost decreases as the degree of commonality increases, that is*

$$C(\pi_k, \mathbf{s}) \ge C(\pi_{k+1}, \mathbf{s}), \quad k = 0, 1, 2, \ldots$$

Proof. For cost-symmetric systems π_k and π_{k+1}, we compare the average costs given by Eq. (15). By Assumption 1, before and after component sharing, the holding cost for each component does not change, and the base-stock level and outstanding order for each non-critical component (see Section 3 for a definition of the critical component) do not change. For the critical component, its base-stock level and outstanding order after sharing are the sums of their counterparts before sharing. Then, the first and second terms in Eq. (15) do not change after component sharing. Finally, the result readily follows from Theorem 1. □

Note that Theorem 3 may not hold for a cost-asymmetric system. For example, we consider the two-component systems π_0 and π_1 illustrated in Fig. 1 under the FRFS allocation rule, and we set $L_1 = L_2 = L_6 = 1$ and $L_3 = L_4 = L_5 = 4$; $s_1 = 2, s_2 = 2, s_3 = 3, s_4 = 3, s_5 = 6$ and $s_6 = 4$. We assume that the demand process of each product follows an independent Poisson process with rate λ^{K_i}. We set $\lambda^{K_2} = 2$, and study the expected backorder for each product while $\lambda^{K_1} = 1$ or 3.

Table 1 exhibits the expected backorder for each product obtained via simulation (the simulation error is controlled within 0.5%). Because the expected backorders of individual products may not decrease as the degree of commonality increases, the average cost may not decrease for cost-asymmetric systems. For example, when $\lambda^{K_2} = 1$, one can choose a β^{K_1} that is sufficiently larger than β^{K_2} such that $C(\pi_1, \mathbf{s}) > C(\pi_0, \mathbf{s})$ by Eq. (13).

5. Numerical study

In this section, we conduct a numerical study to quantify the value of component commonality and its dependence on various system parameters. To this end, we consider the two-component systems π_0 and π_1 in Fig. 1 under the FRFS allocation rule. For each instance, we compute the minimum average cost by searching the optimal base-stock levels $\mathbf{s}^*$ using simulation. The percentage saving (or the value of commonality) is defined as follows,

Value of commonality or % of saving

$$= \frac{[C(\pi_0, \mathbf{s}^*) - C(\pi_1, \mathbf{s}^*)]}{C(\pi_0, \mathbf{s}^*)} \times 100\%,$$

where $C(\pi_k, \mathbf{s}^*) = \min_{\mathbf{s}} C(\pi_k, \mathbf{s})$, $k = 0, 1$.

Table 2
The impact of lead times on the value of commonality.

L_a	L_b									
	1	2	3	4	5	6	7	8	9	10
1	7.00	18.37	18.76	20.97	20.36	22.81	27.73	35.72	46.68	57.97
2	12.79	4.87	14.81	15.97	17.71	18.21	20.46	24.12	31.33	41.73
3	11.32	13.33	7.26	14.02	16.17	16.77	18.30	18.99	22.45	29.23
4	10.14	12.06	11.63	6.06	13.57	14.32	15.81	16.65	17.74	21.11
5	9.67	12.64	12.45	11.65	5.14	12.99	15.11	15.70	16.38	17.61
6	9.39	11.88	11.94	11.87	12.54	6.46	12.36	14.58	14.94	15.67
7	9.28	11.73	11.73	11.48	12.61	11.61	6.17	11.68	13.39	14.18
8	11.07	11.99	11.56	12.35	12.76	12.71	12.95	5.09	11.49	13.36
9	15.38	13.32	11.67	11.89	12.51	12.43	14.49	17.72	5.53	11.87
10	21.93	16.90	12.95	11.85	12.30	12.25	14.17	18.52	24.39	6.20

Table 3
The impact of inventory holding cost on the value of commonality.

h_b	$\mathbf{s}^*(\pi_0) = (s_1, s_2, s_3, s_4)$	$\mathbf{s}^*(\pi_1) = (s_1, s_2, s_5)$	$C(\pi_0, \mathbf{s}^*)$	$C(\pi_1, \mathbf{s}^*)$	% saving
1	(13, 13, 13, 13)	(13, 13, 26)	40.104	39.938	0.4149
3	(12, 12, 12, 12)	(13, 13, 24)	52.299	51.131	2.2328
5	(12, 12, 12, 12)	(12, 13, 23)	62.727	60.020	4.3153
7	(11, 11, 11, 11)	(12, 12, 22)	70.839	67.032	5.3742
9	(11, 11, 11, 11)	(12, 12, 21)	78.446	73.548	6.2432

Impact of lead time: this study tries to answer the following question: shall we share components with longer lead times or the opposite? To this end, we set the demand arrival rates $\lambda^{K_1} = \lambda^{K_2} = 1$; $h_i = 1$ for all i; and $b^{K_1} = b^{K_2} = 20$. We vary lead times $L_1 = L_2 = L_a$ and $L_3 = L_4 = L_b$, where the lead time of the common component, L_a, and the lead time of product-specific component, L_b, take value from 1 to 10. We calculate the value of commonality for each combination of L_a and L_b. The result is summarized in Table 2. We make the following observations:

- if $L_b \geq L_a$: keeping L_a fixed, a longer L_b tends to increase the value of commonality;keeping L_b fixed, a longer L_a tends to decrease the value of commonality.
- except for 5 cases, any upper diagonal element of the table is always greater than its corresponding element in the lower diagonal of the table, e.g., the value of commonality when $L_a = 3$ and $L_b = 6$ is greater than the value of commonality when $L_a = 6$ and $L_b = 3$.

In summary, sharing components with longer lead time tends to result in higher value of commonality. When $L_b \geq L_a$, the larger the difference in lead time between product-specific component and common component, the higher the value of commonality. However, when $L_b < L_a$, there is no clear trend on how lead time affects the value of commonality.

Impact of component holding cost: this study tries to address the following question: shall we share expensive components or the opposite? To this end, we design a numerical study with Poisson demand arrival rates $\lambda^{K_1} = \lambda^{K_2} = 1$, $L_i = 10$ for all i, $b^{K_1} = b^{K_2} = 20$, and $h_1 = h_2 = 3$. We vary $h_3 = h_4 = h_b$ from 1, 3, 5, 7 and 9. The result is summarized in Table 3 which shows that the value of commonality always increases as the shared components become more expensive to carry in inventory. Thus, sharing more expensive components tends to result in higher value of commonality.

Although our unconstrained model of backorders and inventory cost differs from the fill-rate constrained model of [15], our numerical result on the impact of lead time and inventory holding cost agrees well with those of [15]. Specifically, in both models, sharing components with longer lead times and higher inventory holding costs tends to result in a higher value of commonality.

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References

[1] N. Agrawal, M. Cohen, Optimal material control and performance evaluation in an assembly environment with component commonality, Nav. Res. Logist. 48 (2001) 409–429.
[2] K.R. Baker, M.J. Magazine, H.L.W. Nuttle, The effect of commonality on safety stocks in a simple inventory model, Manag. Sci. 32 (1986) 982–988.
[3] S. Benjaafar, M. ElHafsi, Production and inventory control of a single product assemble-to-order system with multiple customer classes, Manag. Sci. 52 (2006) 1896–1912.
[4] G. Cachon, C. Terwiesch, Matching Supply with Demand, vol. 2, McGraw-Hill, New York, 2006.
[5] L. Chen, H. Lee, Information sharing and order variability control under a generalized demand model, Manag. Sci. 55 (5) (2009) 781–797.
[6] F. Cheng, M. Ettl, G.Y. Lin, D.D. Yao, Inventory-service optimization in configure-to-order system, Manuf. Serv. Oper. Manag. 4 (2002) 114–132.
[7] M.K. Dogru, M.I. Reiman, Q. Wang, A stochastic programming based inventory policy for assemble-to-order systems with application to the W model, Oper. Res. 58 (2010) 849–864.
[8] G. Gaalman, Bullwhip reduction for ARMA demand: the proportional policy versus the full-statefeedback policy, Automatica 42 (2006) 1283–1290.
[9] Y. Gerchak, M.J. Magazine, A.B. Gamble, Component commonality with service level requirements, Manag. Sci. 34 (1988) 753–760.
[10] K. Gilbert, An ARIMA supply chain model, Manag. Sci. 51 (2005) 305–310.
[11] A. Giloni, C. Hurvich, S. Seshadri, Forecasting and information sharing in supply chains under quasi-ARMA demand, IIE Trans. 46 (1) (2014) 35–54.
[12] R. Kapuscinski, R. Zhang, P. Carbonneau, R. Moore, B. Reeves, Inventory decisions in Dell's supply chain, Interfaces 34 (2004) 191–205.
[13] Y. Lu, J.S. Song, Y. Zhao, No-holdback allocation rules for continuous-time assemble-to-order systems, Oper. Res. 58 (2010) 691–705.
[14] J.S. Song, Order-Based backorders and their implication in multi-item inventory systems, Manag. Sci. 48 (2002) 499–516.
[15] J.S. Song, Y. Zhao, The value of component commonality in a dynamic inventory system with lead times, Manuf. Serv. Oper. Manag. 11 (2009) 493–508.
[16] J.S. Song, P. Zipkin, Supply Chain Operations: Assemble-to-Order Systems, in: Handbooks in Operations Research and Management Science, vol. 11, 2003, pp. 561–596. Chapter 11.
[17] J.A. Van Mieghem, Note-commonality strategies: value drivers and equivalence with flexible capacity and inventory substitution, Manag. Sci. 50 (2004) 419–424.