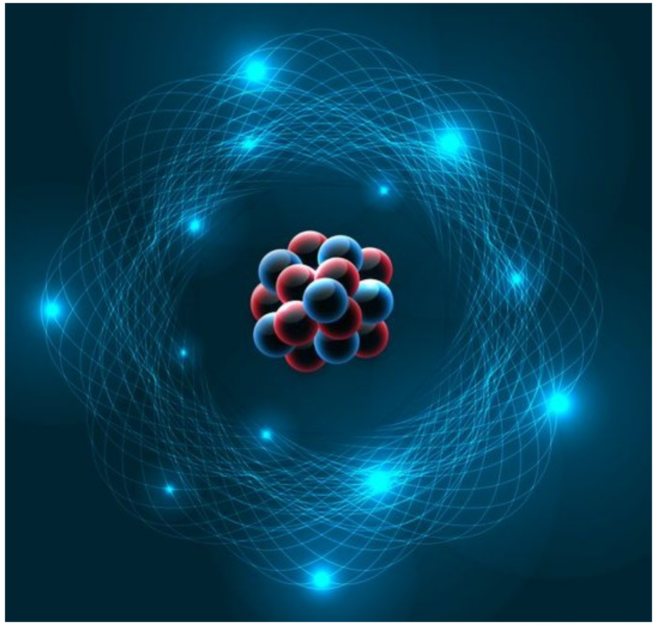
Physics 111: Mechanics
Chapter 1



Junjie Yang
Department of Physics, NJIT

Quiz 1

10^x	Prefix	Symbol
x=-1	deci	d
-2	centi	c
-3	milli	m
-6	micro	μ
-9	nano	n
-12	pico	p
-15	femto	f
-18	atto	a

Quiz 2

1 mile = 1609 m, 1 m = 6.215×10^{-4} mile,

1 hour = 3600 s, 1 s = 2.778×10^{-4} hour

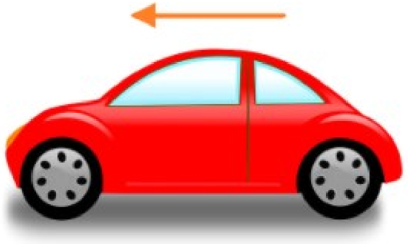
Vectors and Scalars

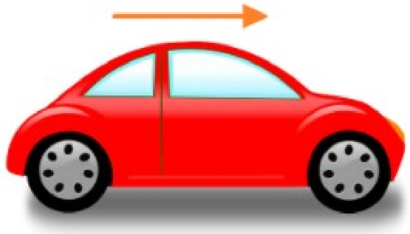
- A car: mass 1000 kg, can have a velocity of 20 m/s towarding to West (-20 m/s) or East (+20 m/s).
- A *scalar quantity* can be described by a *single number,* like mass.
- A *vector quantity* has both a ***magnitude*** and a ***direction*** in space, like velocity.

20 m/s is the magnitude, and +/- is its direction.

➢ Scalar examples: mass, time, kinetic energy

➢ Vector examples: displacement, velocity and force

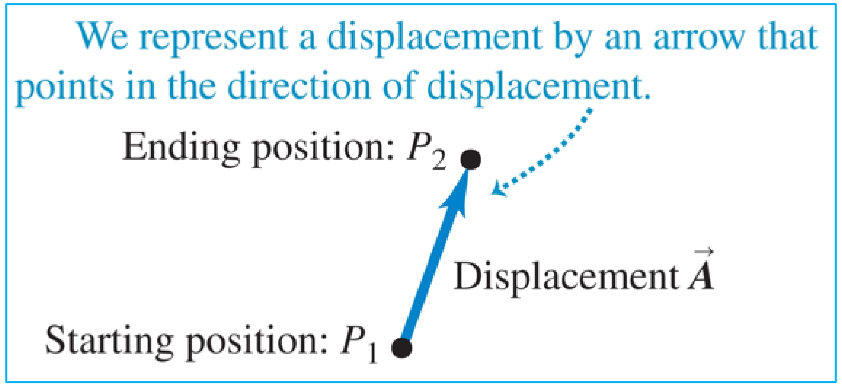


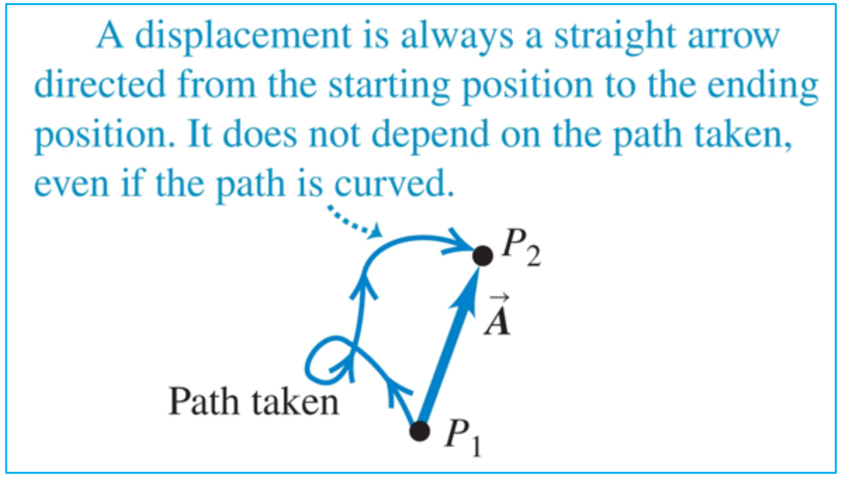


Vectors: Important Notation

- For a more general case, to describe vectors we will use: The bold font: vector A is **A**, and/or an arrow above the vector: $\vec{A}$
- In the pictures, we will always show vectors as arrows.
- Arrows point the direction.

- To describe the magnitude of a vector we will use the absolute value sign: $|\vec{A}|$ or just A.
- Magnitude is always positive; the magnitude of a vector is equal to the length of a vector.





Properties of Vectors

- Equality of two vectors?

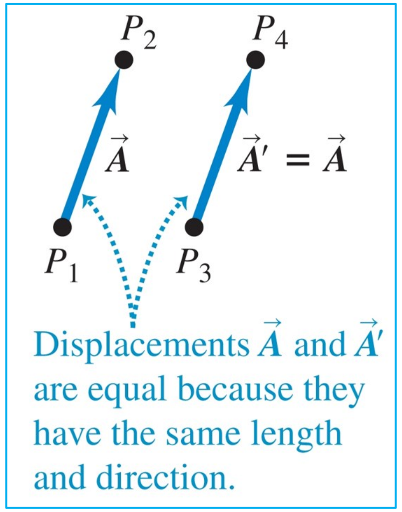
  Two vectors are equal if they have the same magnitude and the same direction.
- Negative of a vector?

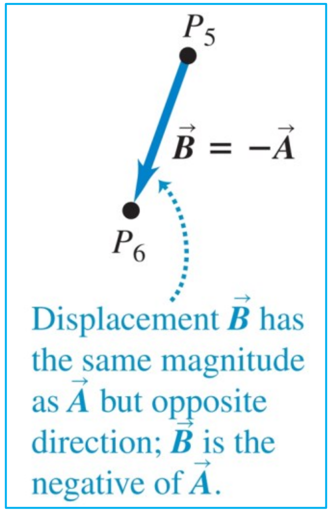
  A vector the negative of another if they have the same magnitude but are 180° apart (opposite directions)

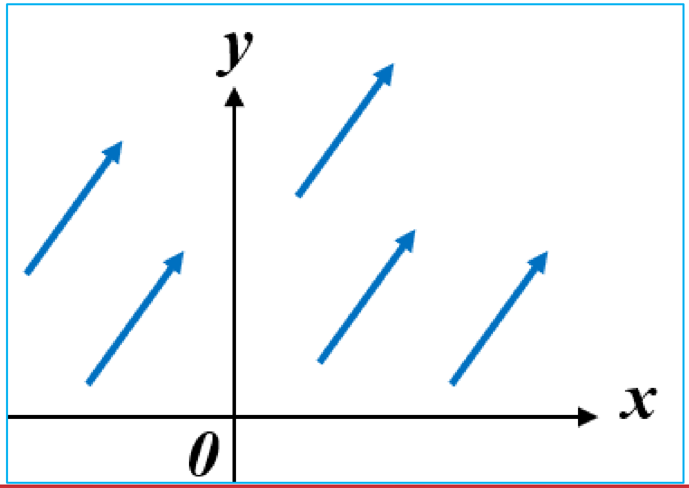
$$\vec{A} = -\vec{B} \text{ or } \vec{B} = -\vec{A}$$

- Movement of vectors in a diagram:

  Any vector can be moved parallel to itself without being affected. (Position does not change equality.)



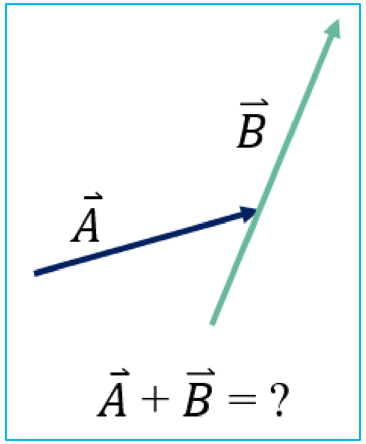


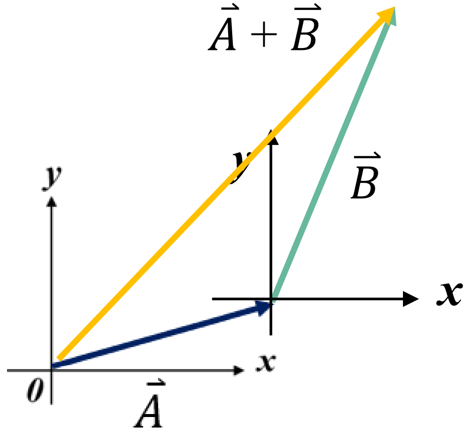


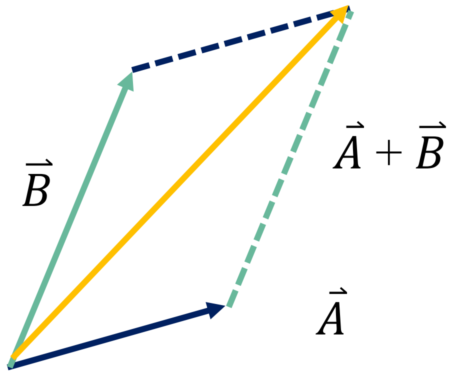
Adding Vectors Graphically

- Draw the first vector $\vec{A}$ with the appropriate length and in the direction specified, with respect to a coordinate system.
- Draw the next vector $\vec{B}$ whose origin is the end of the vector $\vec{A}$ and parallel to the coordinate system used for $\vec{A}$: "tip-to-tail".
- The resultant is drawn from the origin of $\vec{A}$ to the end of the last vector $\vec{B}$.

- We can also add two vectors by placing them tail to tail and constructing a parallelogram.

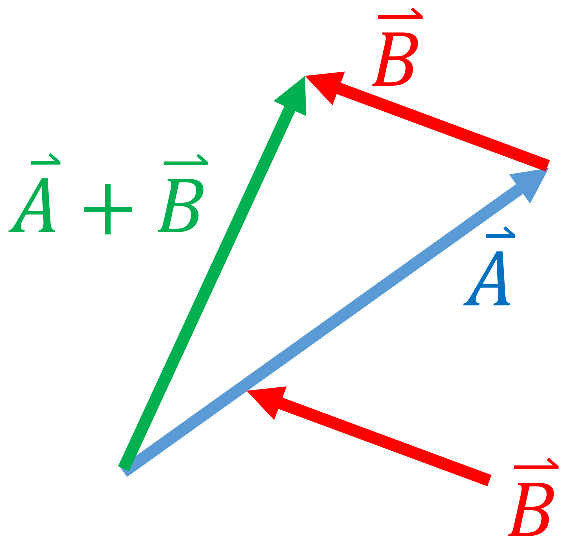






Practice:

Draw $\vec{A} + \vec{B}$?

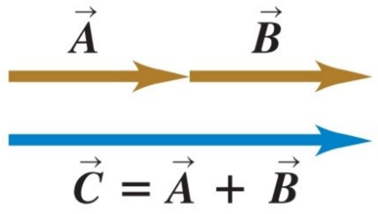


Question:

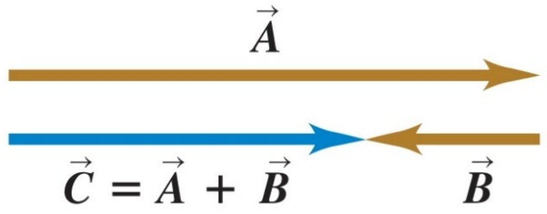
Graphical addition not convenient, can I simply add A and B like scalar?

Such as C = A + B.

(a) Only when vectors $\vec{A}$ and $\vec{B}$ are parallel does the magnitude of their vector sum $\vec{C}$ equal the sum of their magnitudes: $C = A + B$.



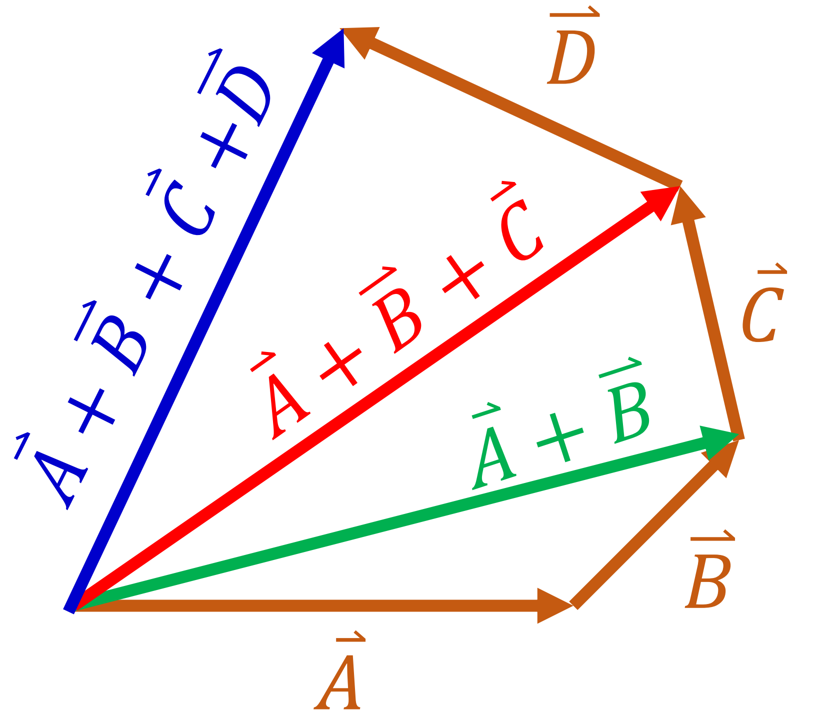
(b) When $\vec{A}$ and $\vec{B}$ are antiparallel, the magnitude of their vector sum $\vec{C}$ equals the *difference* of their magnitudes: $C = |A - B|$.



Adding Many Vectors Graphically

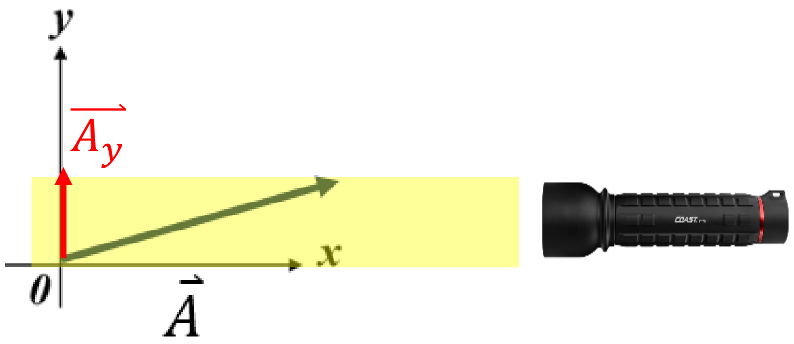
- When we have many vectors?
 just keep repeating the process until all are included

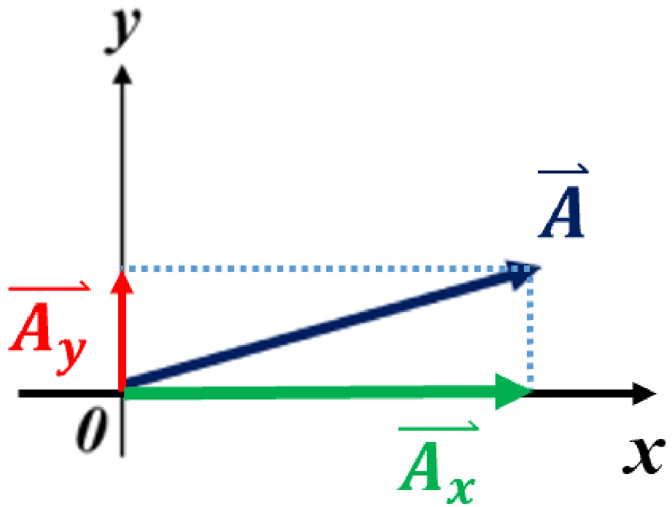
- The resultant is still drawn from the origin of the first vector to the end of the last vector.



Other method for adding vectors?

- A vector $\vec{A}$ is with respect to a "*xyz*" coordinate system.
- We can "decompose" a vector to its "*x*", "*y*" and "*z*" **component**, then add the components, respectively.
- Like shinning a light to see where the shadow lies.
- **Components** of a vector are the **projections** of the vector along the *x*- and *y*- axes.
- Then vector $\vec{A}$ can be written as $\vec{A} = \overrightarrow{A_x} + \overrightarrow{A_y}$
- $\overrightarrow{A_x}$ and $\overrightarrow{A_y}$ are **component vectors.**
- A_x and A_y are **components** (magnitudes of component vectors).

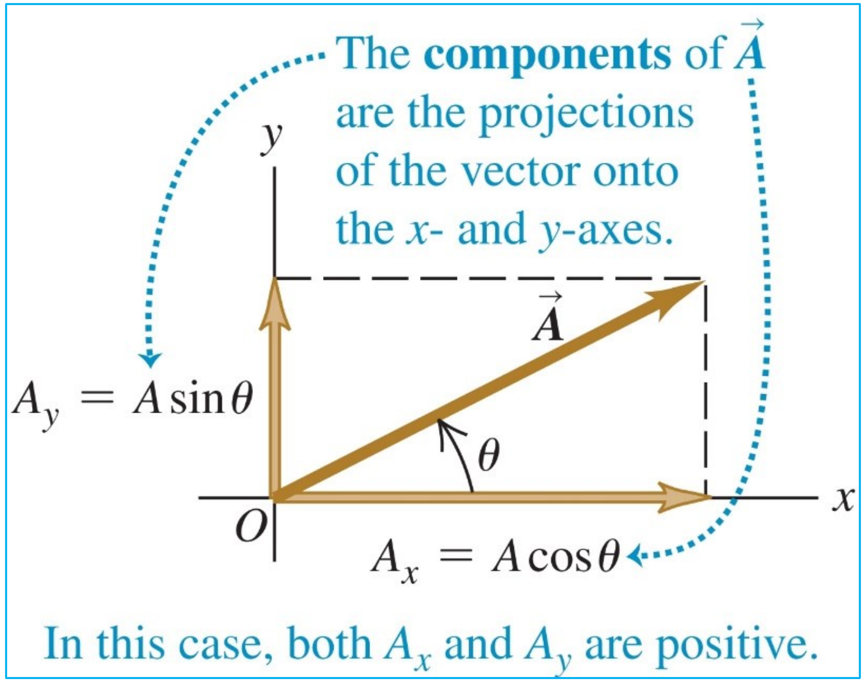


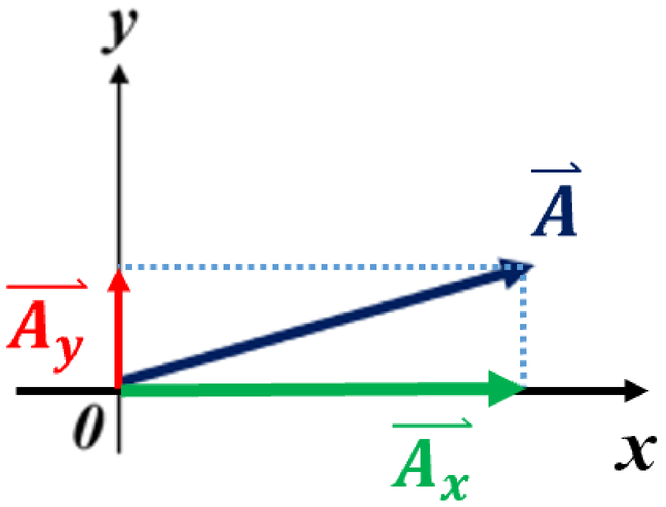


Other method for adding vectors?

Question: values of component A_x and A_y?

- $A_x = Acos\theta$ and $A_y = Asin\theta$





Angle θ is defined as counterclockwise from the x axis to the vector.

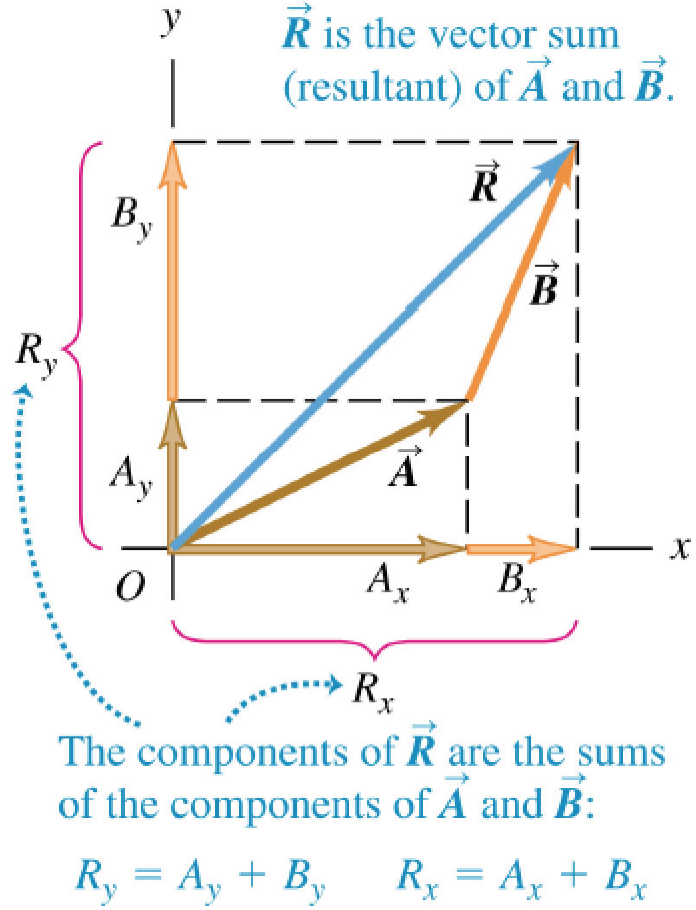
Using Components: Vector Addition

- Now we can use the components of a set of vectors to find the component of their sum:

$$\vec{R} = \vec{A} + \vec{B}$$

- Components of vector $\vec{A}$: A_x and A_y
- Components of vector $\vec{B}$: B_x and B_y
- Components of vector $\vec{R}$:

$$R_x = A_x + B_x, R_y = A_y + B_y$$
$$\vec{R} = \overrightarrow{R_x} + \overrightarrow{R_y}$$



Example:

$\vec{R} = \vec{A} + \vec{B}$

$A_x = 1$ (m), $A_y = 2$ (m),

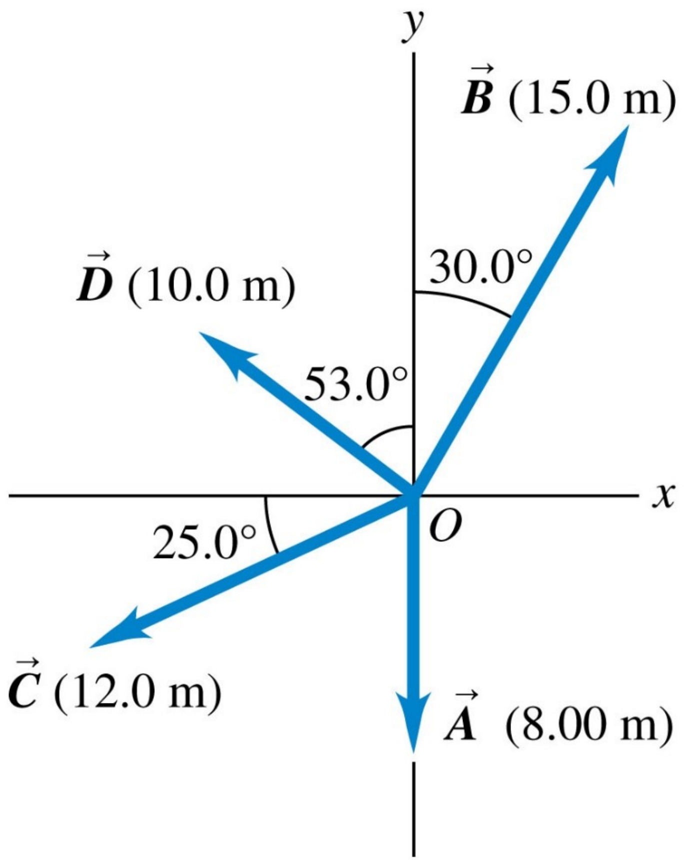
$B_x = 0$ (m), $B_y = 1$ (m),

What are the R_x and R_y?

$$R_x = A_x + B_x = 1 \text{ (m)}$$
$$R_y = A_y + B_y = 3 \text{ (m)}$$

Example:

What are the components of the vector $\vec{E} = \vec{A} + \vec{D}$?



A. $E_x = -8.00$ m, $E_y = -2.00$ m ✓

B. $E_x = -8.00$ m, $E_y = +2.00$ m

C. $E_x = -6.00$ m, $E_y = 0$

D. $E_x = -6.00$ m, $E_y = +2.00$ m

$A_x = 0.00$ m,
$A_y = -8.00$ m,

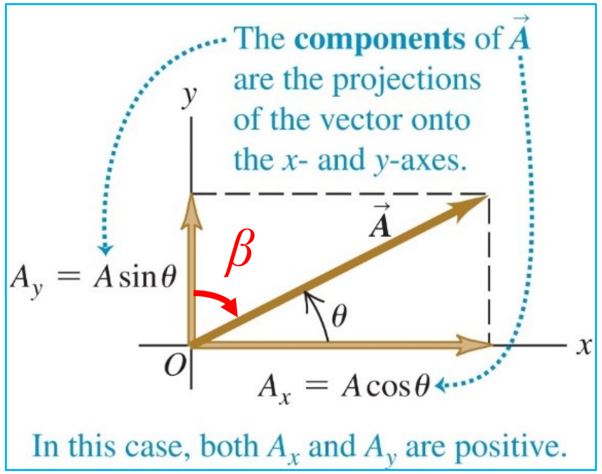
$D_x = 10.0 \times \cos(90 + 53)° = -8.00$ m,
$D_y = 10.0 \times \sin(90 + 53)° = 6.00$ m,

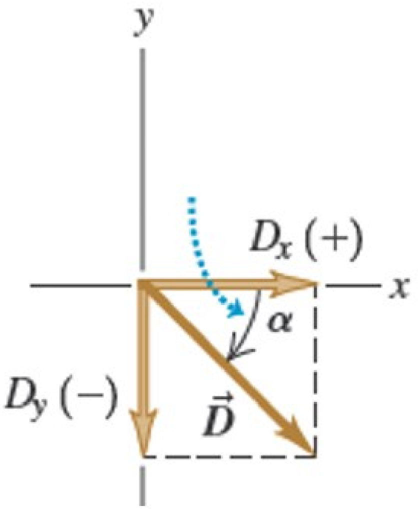
$E_x = -8.00 + 0 = -8.00$ m,
$E_y = 6.00 - 8.00 = -2.00$ m,

More on components of a Vector

- $\vec{A} = \overrightarrow{A_x} + \overrightarrow{A_y}$, $A_x = Acos\theta$ and $A_y = Asin\theta$
- θ is measured with respect to the $+x$-axis counter-clockwisely.

- What are the A_x and A_y if β is given?
  We must use "θ" which is measured from the $+x$-axis toward the $+y$-axis: $\theta = 90 - \beta$



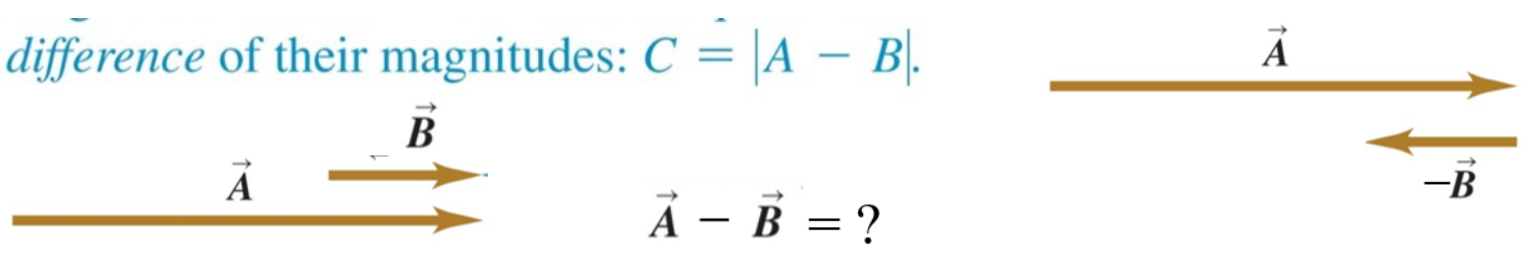


$A_x = Acos(90 - \beta) = Asin\beta$,
$A_y = Asin(90 - \beta) = Acos\beta$

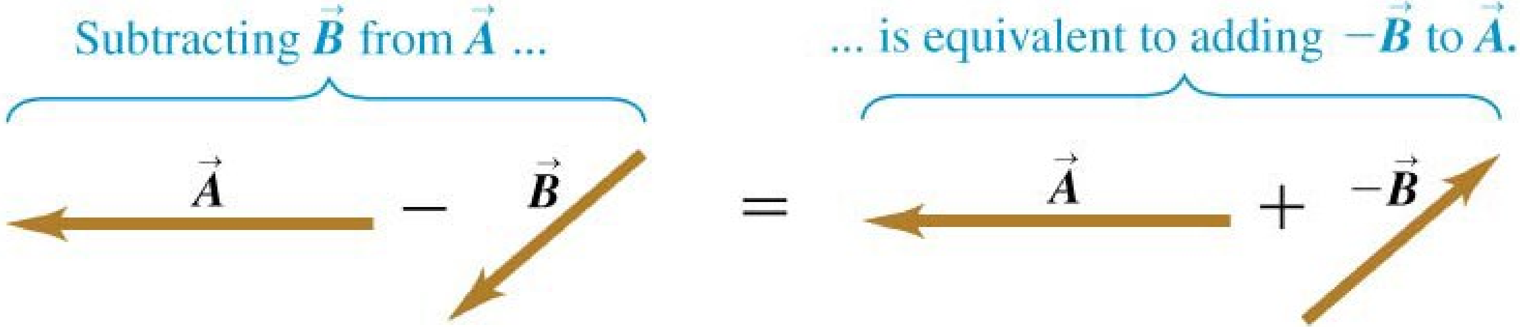
$A_x = Acos(360 - \alpha) = Acos\alpha$,
$A_y = Asin(360 - \alpha) = -Asin\alpha$

Vector Subtraction

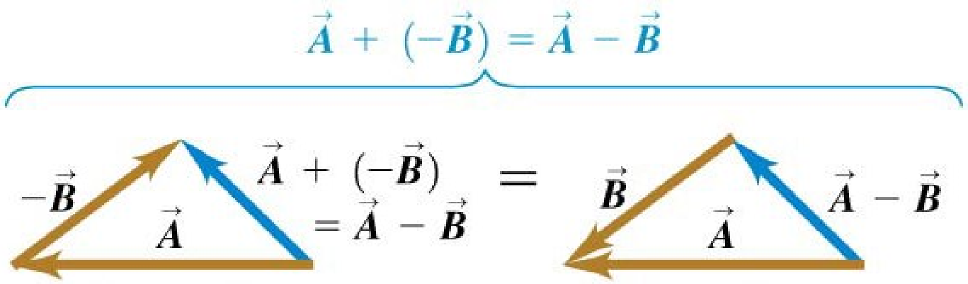
- Simplest subtraction:



- More general case:

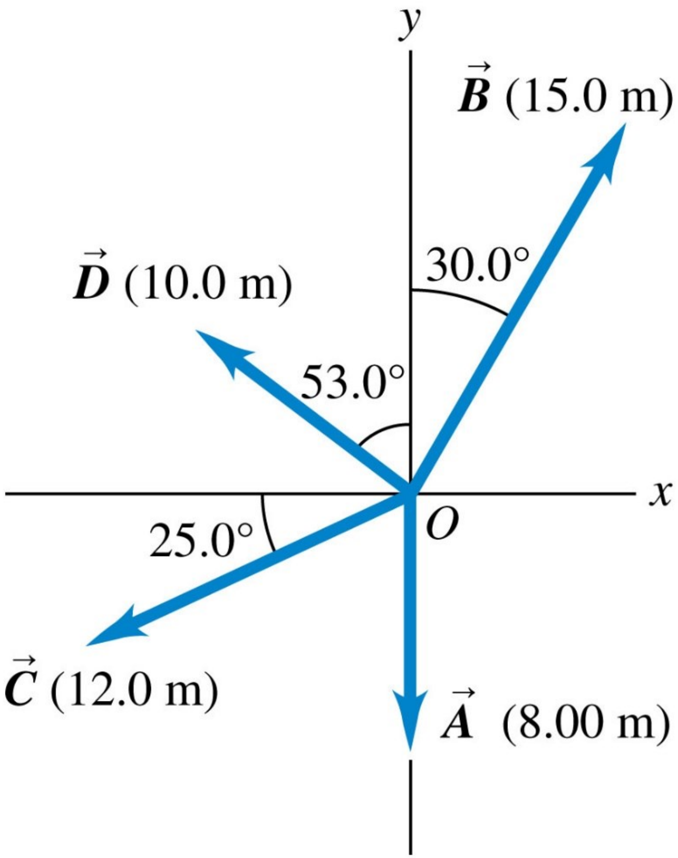


- Continue with stand vector addition procedure



Example:

Find the components of $\vec{E} = \vec{A} - \vec{B}$?



$$A_x = 0.00 \text{ m},$$
$$A_y = -8.00 \text{ m},$$

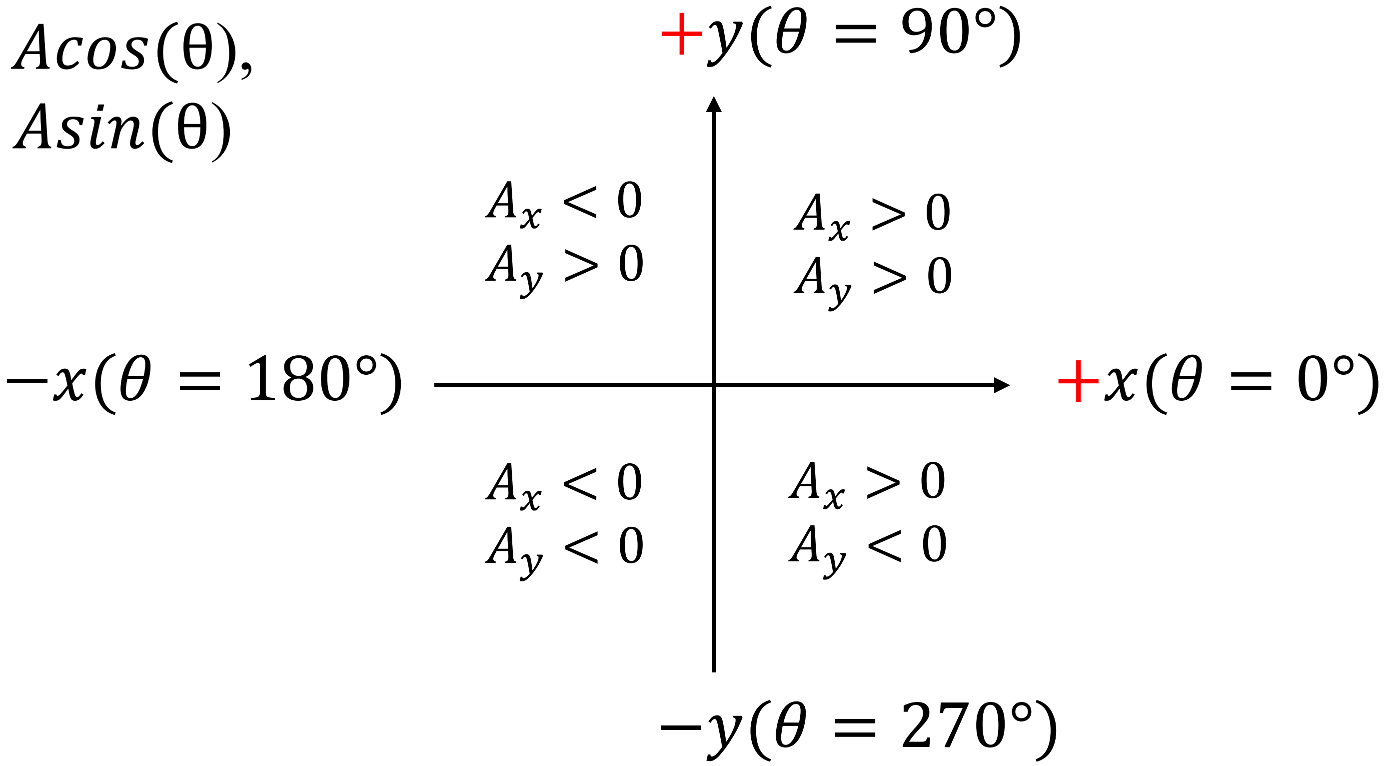
$$B_x = 15.0 \times \cos(90 - 30)° = 7.50 \text{ m},$$
$$B_y = 15.0 \times \sin(90 - 30)° = 12.99 \text{ m},$$

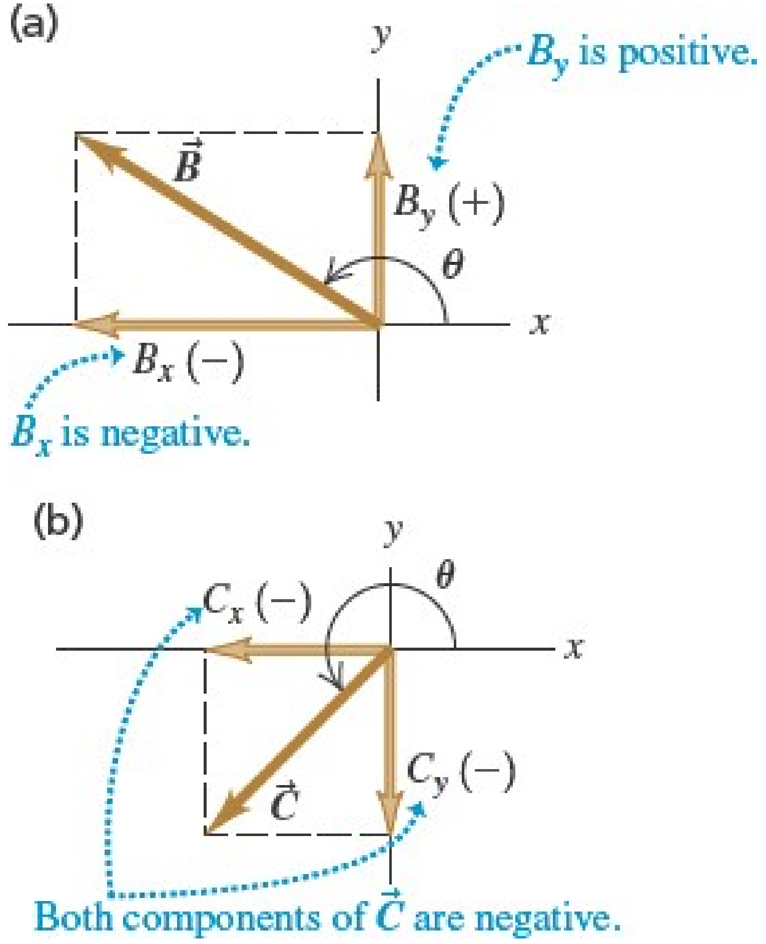
$$E_x = 0 - 7.50 = -7.50 \text{ m},$$
$$E_y = -8 - 12.99 = -20.99 \text{ m},$$

Sign of component vector

- The components can be positive or negative and will have the same units as the original vector.

$A_x = A\cos(\theta)$,
$A_y = A\sin(\theta)$



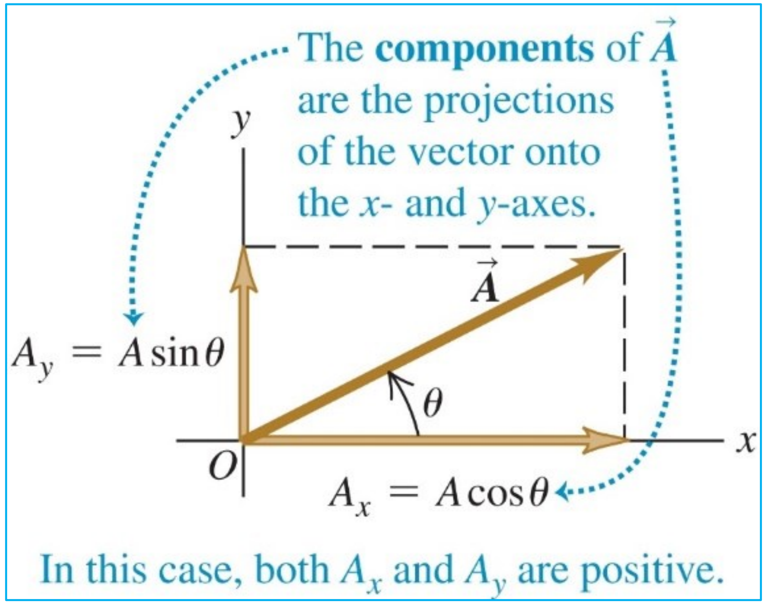


More on components of a Vector

- $\vec{A} = \overrightarrow{A_x} + \overrightarrow{A_y}$, if the magnitude A and θ are known:
  We can calculate the components
  $A_x = A\cos\theta$ and $A_y = A\sin\theta$

- If the A_x and A_y are known? But θ is *not* known.

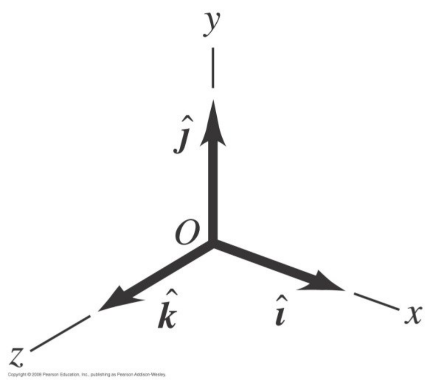
  We can use the components of a vector to find its magnitude and direction:

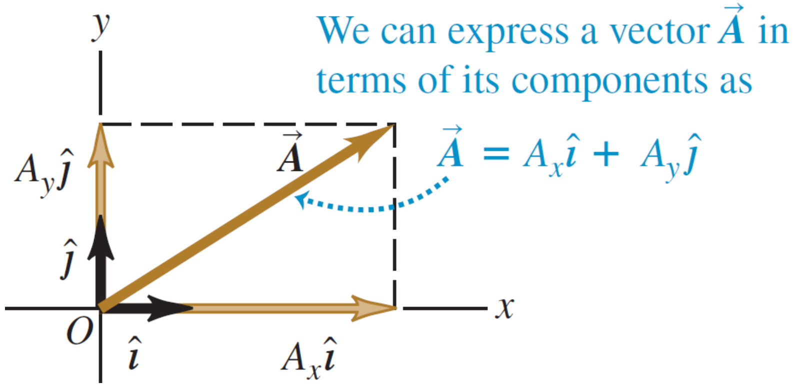


$$\tan\theta = \frac{A_y}{A_x} \qquad \theta = \arctan\frac{A_y}{A_x} \qquad A = \sqrt{A_x^{\,2} + A_y^{\,2}}$$

Unit Vectors

- $\vec{A} = \overrightarrow{A_x} + \overrightarrow{A_y}$, $A_x = A cos\theta$ and $A_y = A sin\theta$, we can rewrite vector $\vec{A} = A_x\vec{x} + A_y\vec{y}$
- Then we set the magnitude of $\vec{x}$ and $\vec{y}$ equals 1 with no units.
- Unit vectors: $\vec{x} \rightarrow \hat{\imath}$, points in the +*x*-direction,
  $\vec{y} \rightarrow \hat{\jmath}$, points in the +*y*-direction,
  $\vec{z} \rightarrow \hat{k}$, points in the +*z*-direction,
- Unit vectors used to specify direction and they have a magnitude of 1.
- Then $\vec{A} = \overrightarrow{A_x} + \overrightarrow{A_y} = A_x\,\hat{\imath} + A_y\hat{\jmath}$
- We also write $\vec{A} = (A_x, A_y, A_z)$





Adding Vectors Algebraically

- Consider two vectors:

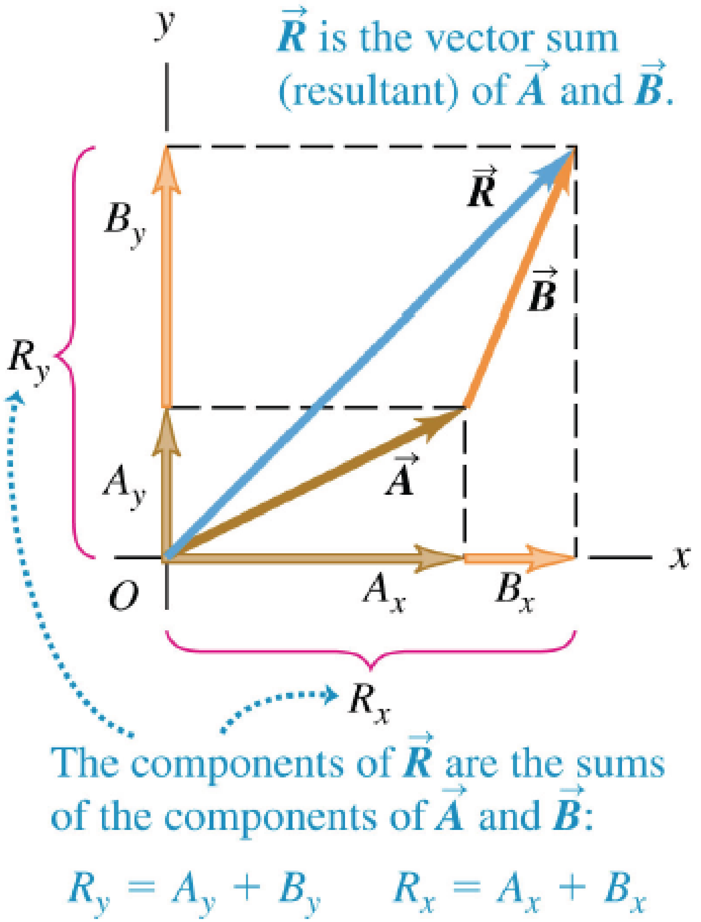
  $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath},$

  $\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath}$
- Then $\vec{A} + \vec{B} = \left(A_x\hat{\imath} + A_y\hat{\jmath}\right) + \left(B_x\hat{\imath} + B_y\hat{\jmath}\right)$

  $= (A_x + B_x)\hat{\imath} + (A_y + B_y)\,\hat{\jmath}$
- Then $\vec{R} = \vec{A} + \vec{B} = R_x\hat{\imath} + R_y\hat{\jmath}$

  $R_x = A_x + B_x,\ R_y = A_y + B_y$

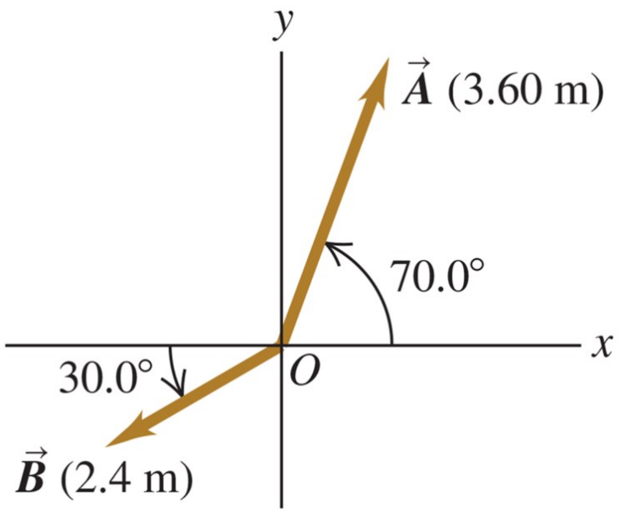
  Unit vectors are more important for "multiplication" of vectors



Example:

(a) Write each vector in terms of the unit vectors $\hat{\imath}$ and $\hat{\jmath}$;

(b) Use unit vectors to express vector $\vec{C}$, where $\vec{C} = 3.00\vec{A} - 4.00\vec{B}$;



$$\vec{A} = (3.60\text{ m})\cos 70.0°\hat{\imath} + (3.60\text{ m})\sin 70.0°\hat{\jmath} = (1.23\text{ m})\hat{\imath} + (3.38\text{ m})\hat{\jmath}$$

$$\vec{B} = -(2.40\text{ m})\cos 30.0°\hat{\imath} - (2.40\text{ m})\sin 30.0°\hat{\jmath} = (-2.08\text{ m})\hat{\imath} + (-1.20\text{ m})\hat{\jmath}$$

$$\vec{C} = (3.00)\,\vec{A} - (4.00)\,\vec{B}$$

$$= (3.00)(1.23\text{ m})\hat{\imath} + (3.00)(3.38\text{ m})\hat{\jmath} - (4.00)(-2.08\text{ m})\hat{\imath} - (4.00)(-1.20\text{ m})\hat{\jmath}$$

$$\vec{C} = (12.01\text{ m})\hat{\imath} + (14.94\text{ m})\hat{\jmath}$$

Next week

Chapter 2: Motion in One Dimension