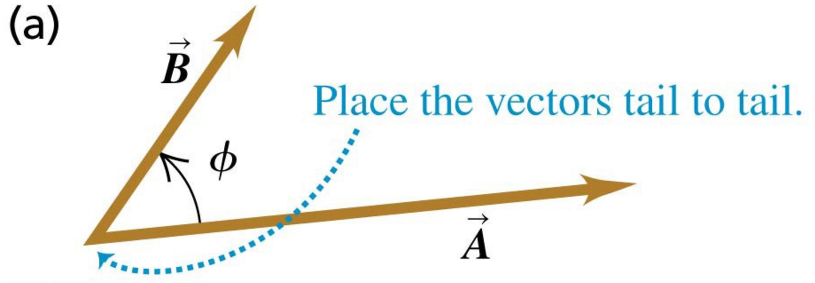
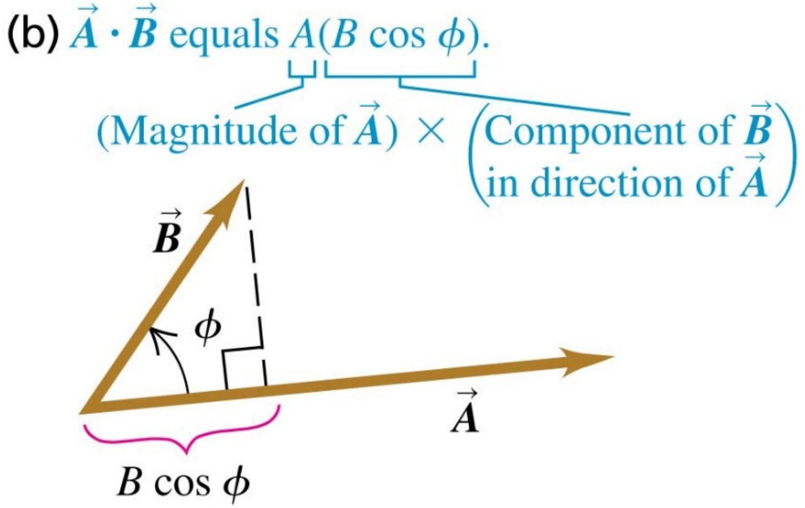
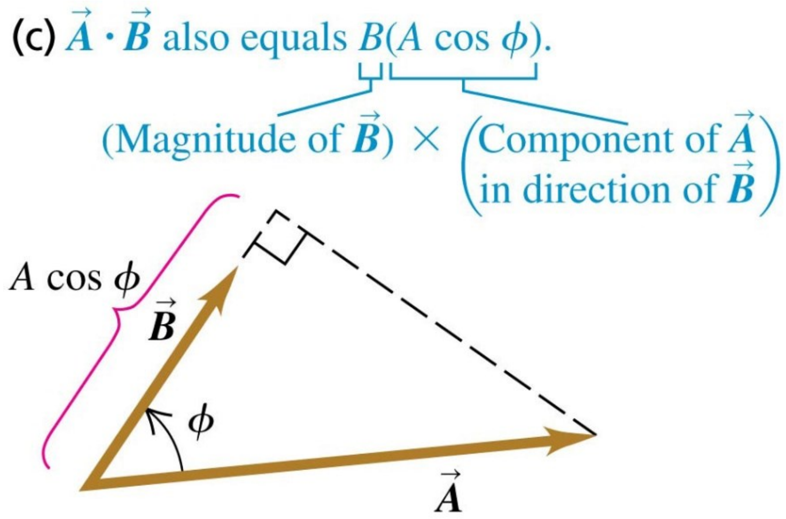
“Multiplication” of vectors: scalar product

- The *scalar product* (“dot product”) of two vectors is $\vec{A} \cdot \vec{B} = ABcos\phi$



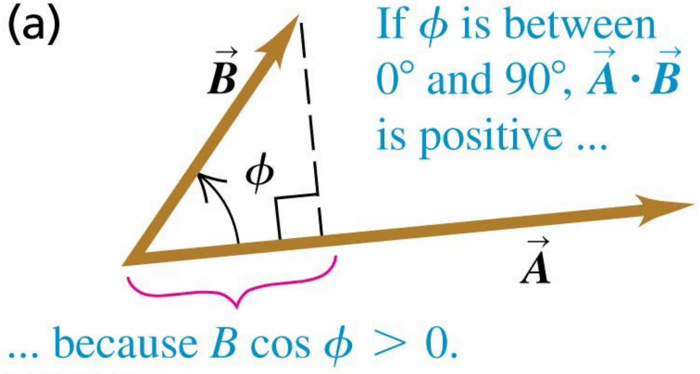
Understand the *scalar product* graphically

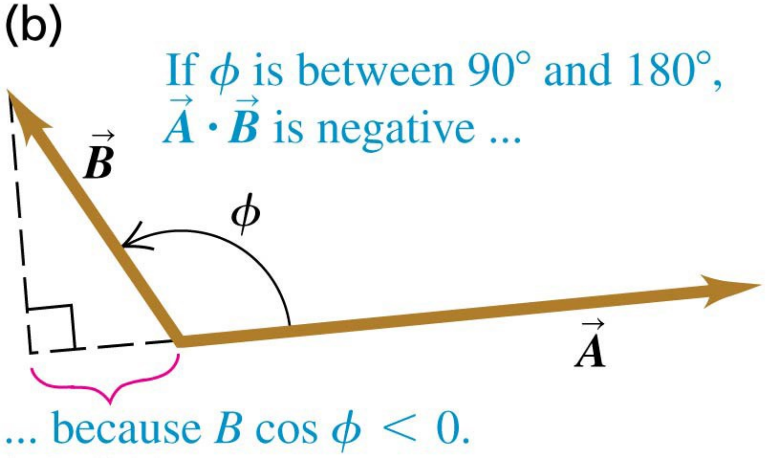


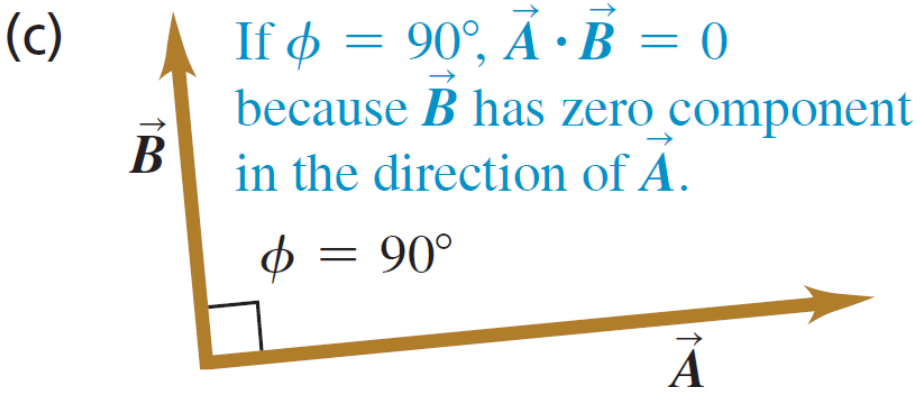


Scalar product: graphically

- $\vec{A} \cdot \vec{B} = ABcos\phi$: Is the result a scalar or vector?
- As it is named, the result is a scalar.
- However, the value can be positive, negative, or zero, depending on the angle between $\vec{A}$ and $\vec{B}$

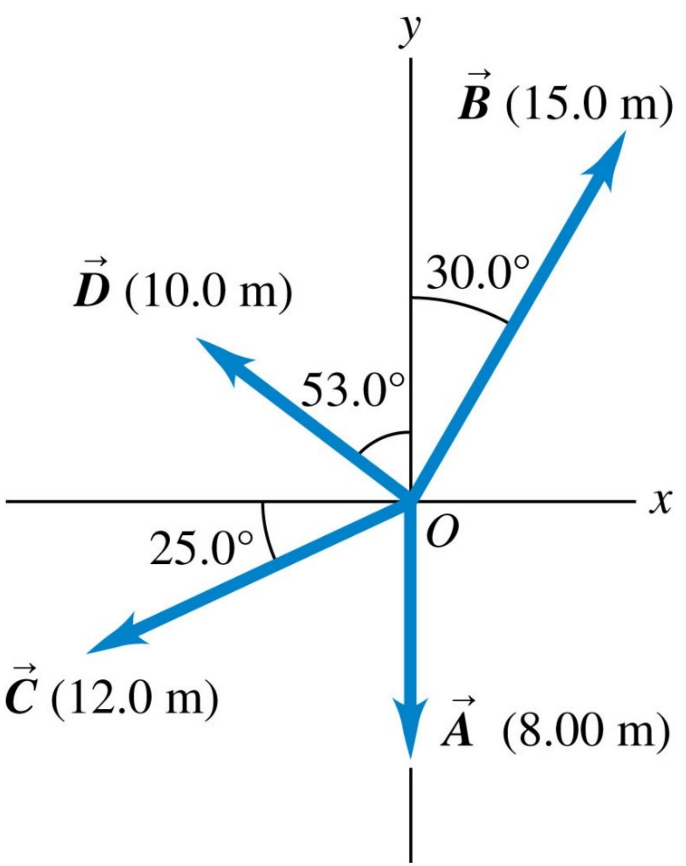






Example:

Consider the vectors shown. What is the dot product $\vec{C} \cdot \vec{D}$?



A. (120 m^2) cos 78.0°

B. (120 m^2) sin 78.0°

C. (120 m^2) cos 62.0° ✓

D. (120 m^2) sin 62.0°

E. none of these

$$\vec{C} \cdot \vec{D} = CDcos\phi$$

Which are known?
Which are unknown?

C = 12 m, D = 10 m

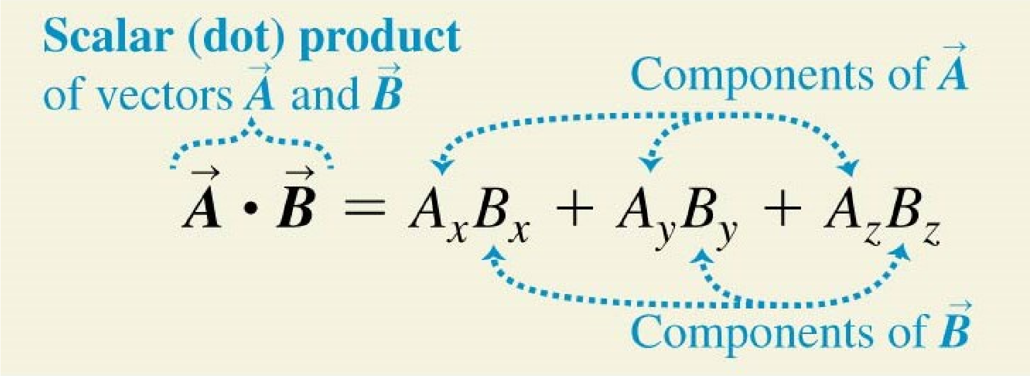
ϕ is unknown.

ϕ = 25°+(90-53)°=62°

$$\vec{C} \cdot \vec{D} = 120cos62°$$

Scalar product: algebraically

- $\vec{A} \cdot \vec{B} = ABcos\phi$, in terms of components:



- The scalar product of two vectors is the sum of the products of their respective components.

$$\vec{A} = (1.23)\hat{i} + (3.38)\hat{j}$$

$$\vec{B} = (-2.08)\hat{i} + (-1.20)\hat{j}$$

$$\vec{A} \cdot \vec{B} = ?$$

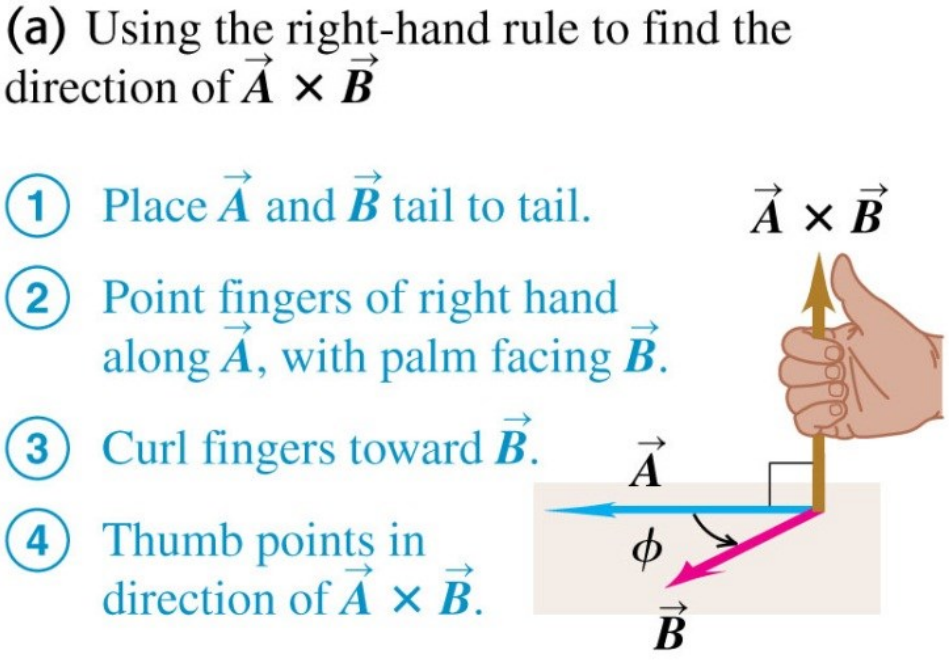
$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

$$= 1.23 \times (-2.08) + 3.38 \times (-1.2)$$

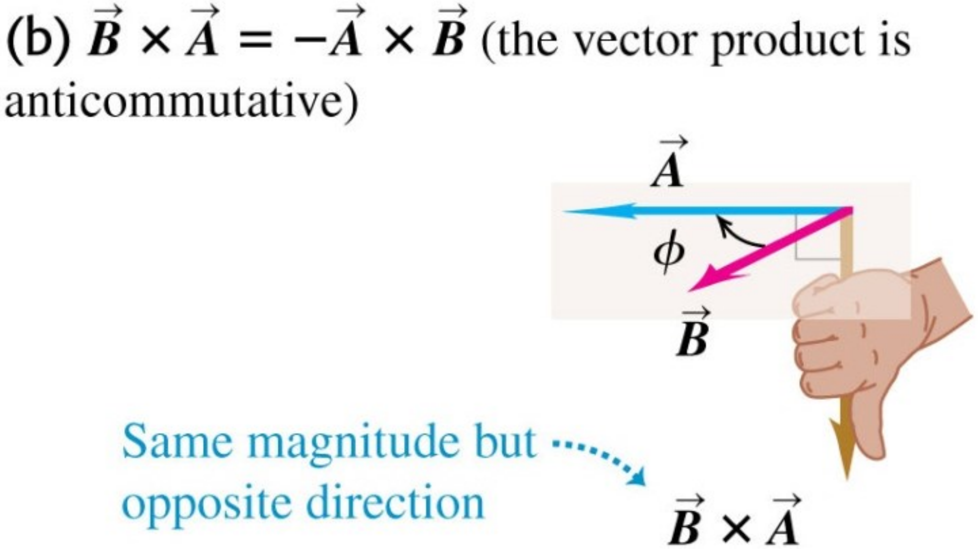
$$= -6.6144$$

Multiplication: The vector product

- The vector product ("cross product") $\vec{C} = \vec{A} \times \vec{B}$ has **magnitude** $\left|\vec{A} \times \vec{B}\right| = ABsin\phi$
- The result is a vector, and the *right-hand rule* gives its direction.

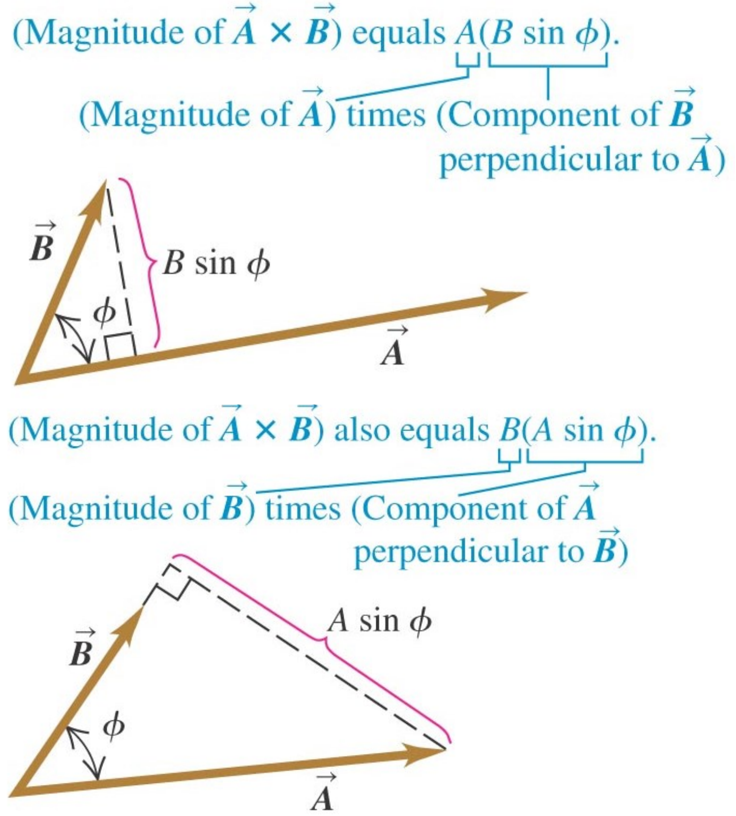


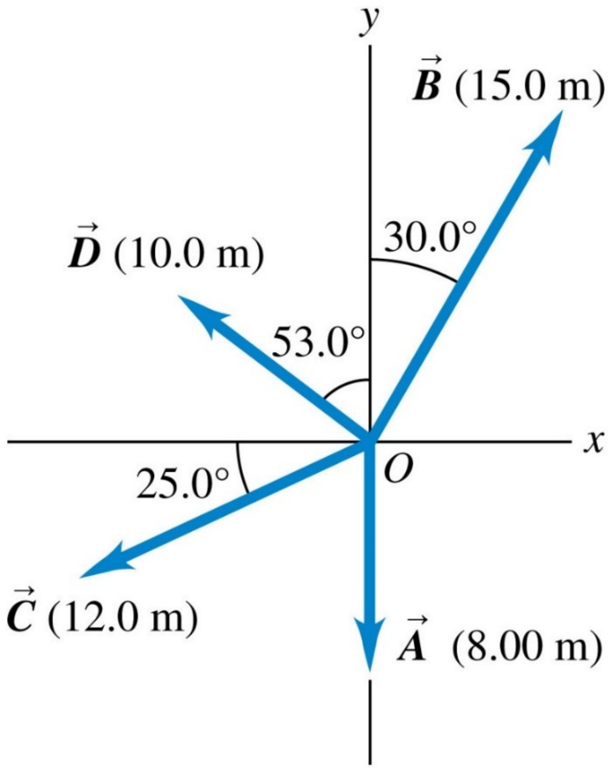
Direction of $\vec{C} = \vec{B} \times \vec{A}$?



Multiplication: The vector product

- Calculate $\vec{A} \times \vec{B}$ graphically: $ABsin\phi$





What is the cross product $\vec{A} \times \vec{C}$?

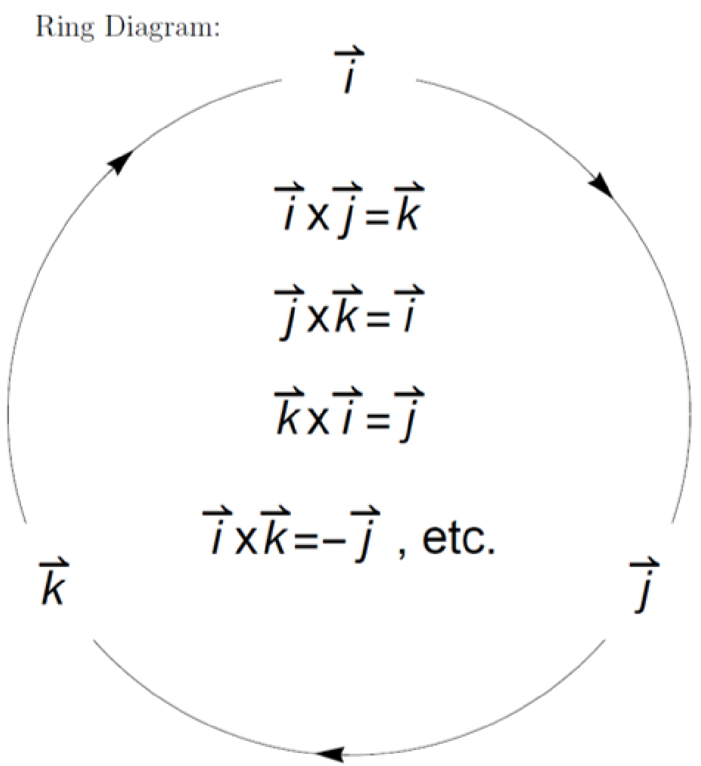
$$\left|\vec{A} \times \vec{C}\right| = ACsin\phi$$

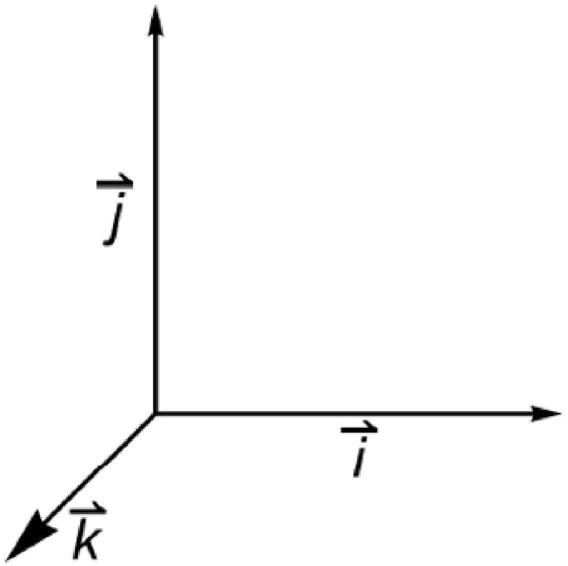
$$\vec{A} \times \vec{C}$$
$$= (8 \times 12)\sin(90 - 25)°(-\hat{k})$$
$$= (96)cos25°(-\hat{k})$$

Multiplication: The vector product

- Calculate $\vec{A} \times \vec{B}$ algebraically using unit vectors

$$\left|\vec{A} \times \vec{B}\right| = ABsin\phi$$





$$\hat{\imath} \times \hat{\imath} = 0,$$
$$\hat{k} \times \hat{k} = 0,$$
$$\hat{\jmath} \times \hat{\jmath} = 0,$$

$$\phi = 0$$

$$\hat{\imath} \times \hat{\jmath} = +\hat{k},$$
$$\hat{k} \times \hat{\jmath} = -\hat{\imath},$$
$$\hat{\jmath} \times \hat{\imath} = -\hat{k},$$

etc

$$\phi = 90$$

Example:

$\vec{A} = -2\hat{\imath} + \hat{\jmath}, \vec{B} = \hat{\imath} + 2\hat{\jmath}$. Find the cross product $\vec{A} \times \vec{B}$

$$\hat{\imath} \times \hat{\imath} = 0,$$
$$\hat{k} \times \hat{k} = 0,$$
$$\hat{\jmath} \times \hat{\jmath} = 0,$$

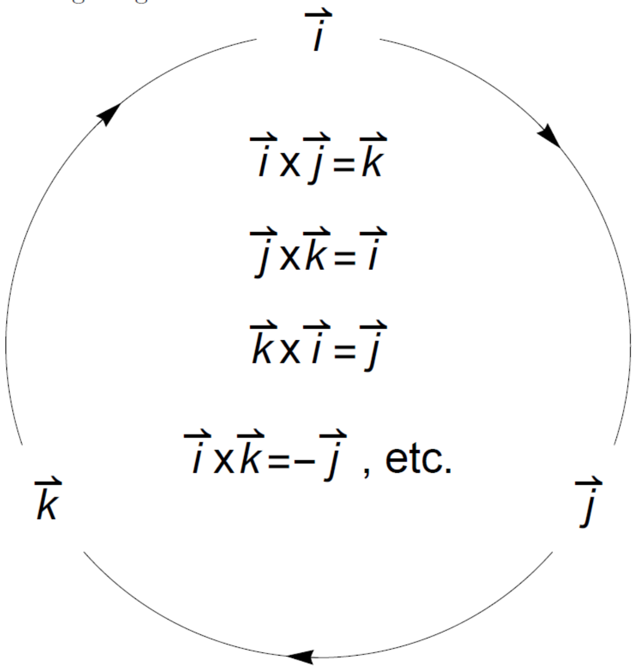
$$\vec{A} \times \vec{B} = (-2\hat{\imath} + \hat{\jmath}) \times (\hat{\imath} + 2\hat{\jmath})$$

$$= (-2\hat{\imath}) \times (\hat{\imath}) + (-2\hat{\imath}) \times (2\hat{\jmath}) + (\hat{\jmath}) \times (\hat{\imath}) + (\hat{\jmath}) \times (2\hat{\jmath})$$

$$= (-4)(\hat{\imath} \times \hat{\jmath}) + \hat{\jmath} \times \hat{\imath}$$

$$= -4\,\hat{k} - \hat{k} = -5\hat{k}$$

Ring Diagram:



Example:

Two vectors $\vec{A} = -2\hat{\imath} + \hat{\jmath}, \vec{B} = \hat{\imath} + 2\hat{\jmath}, \vec{C} = \vec{A} + \vec{B}$.

Question: Find the angle between $\vec{B}$ and $\vec{C}$

Q1: Which formula can we use to find the angle between $\vec{B}$ and $\vec{C}$?

Addition? Subtraction? Multiplication?

Scalar product! Since $\vec{B} \cdot \vec{C} = BC\cos\phi$, we get $\cos\phi = \frac{\vec{B}\cdot\vec{C}}{BC}$

Q2: Which quantities should we calculate first?

Value of $\vec{B} \cdot \vec{C}$ and value of B and value of C

$\vec{B} = \hat{\imath} + 2\hat{\jmath}$ $\quad$ $\vec{C} = -\hat{\imath} + 3\hat{\jmath}$

$$\vec{B} \cdot \vec{C} = B_x C_x + B_y C_y + B_z C_z$$

$$\vec{B} \cdot \vec{C} = 1(-1) + 2 \cdot 3 = 5$$

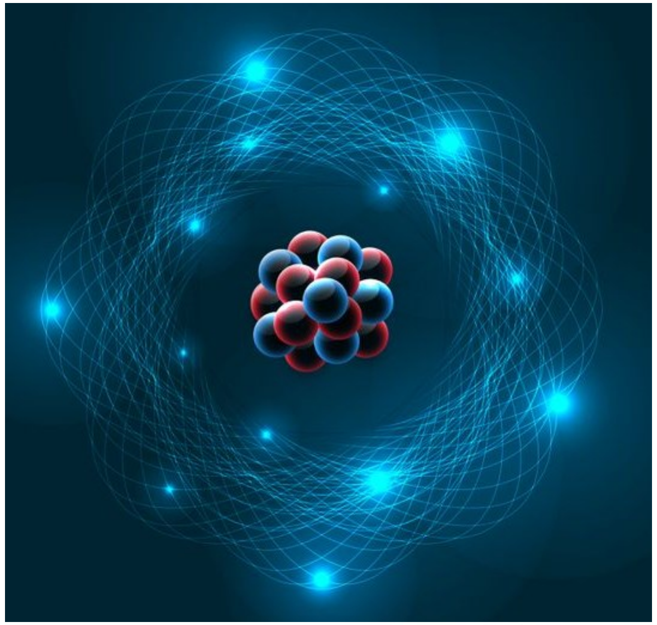
$$B = \sqrt{B_x^{\,2} + B_y^{\,2}} = \sqrt{1^2 + 2^2} = \sqrt{5}$$

$$C = \sqrt{(-1)^2 + 3^2} = \sqrt{10}$$

$$\cos\phi = \frac{5}{\sqrt{5}\sqrt{10}} = \frac{1}{\sqrt{2}}$$

$$\phi = 45^\circ$$

Physics 111: Mechanics
Chapter 02



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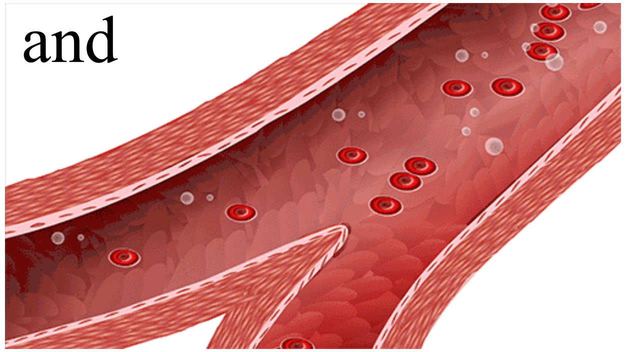
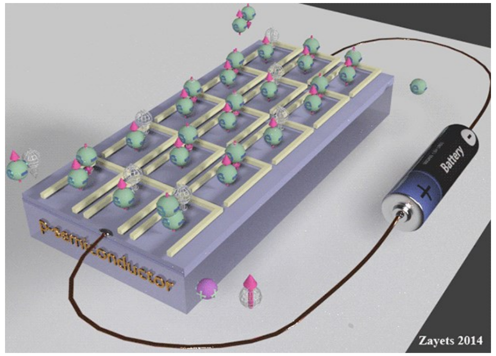
1D Motion

- Everything moves!
- There are some "common" features for different moving objects.
- The physics of motion is one of the main topic in PHYS111.
- Simplification: Consider a moving object as particle, i.e. a "**point object**".
- Motion can be studied with **Kinematics** and **Dynamics.**





Kinematics *vs* Dynamics

- A bungee jumper speeds up during the first part of his fall and then slows to a halt.

- **Kinematics** is the study of jumper's motion: Such as how the jumper's vertical position and speed change with time.

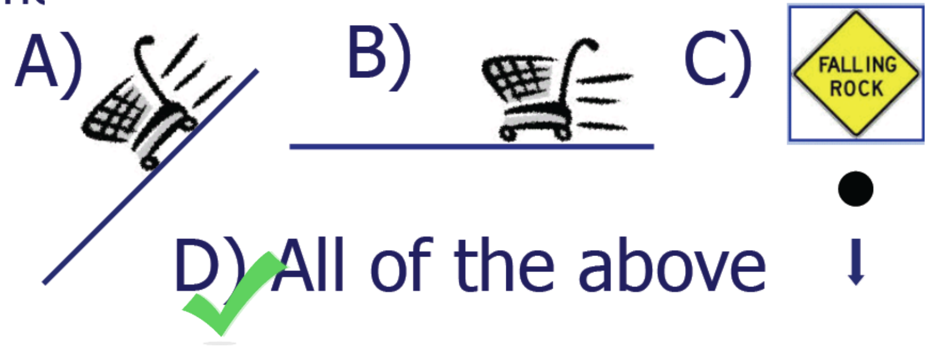
  Displacement, **velocity** and **acceleration** are important physical quantities for kinematics.

- **Dynamics** is the study of "driving force" (reason) for the jumper's motion: such as why the jumper falls down rather ran flies up.

Straight Line: 1D motion

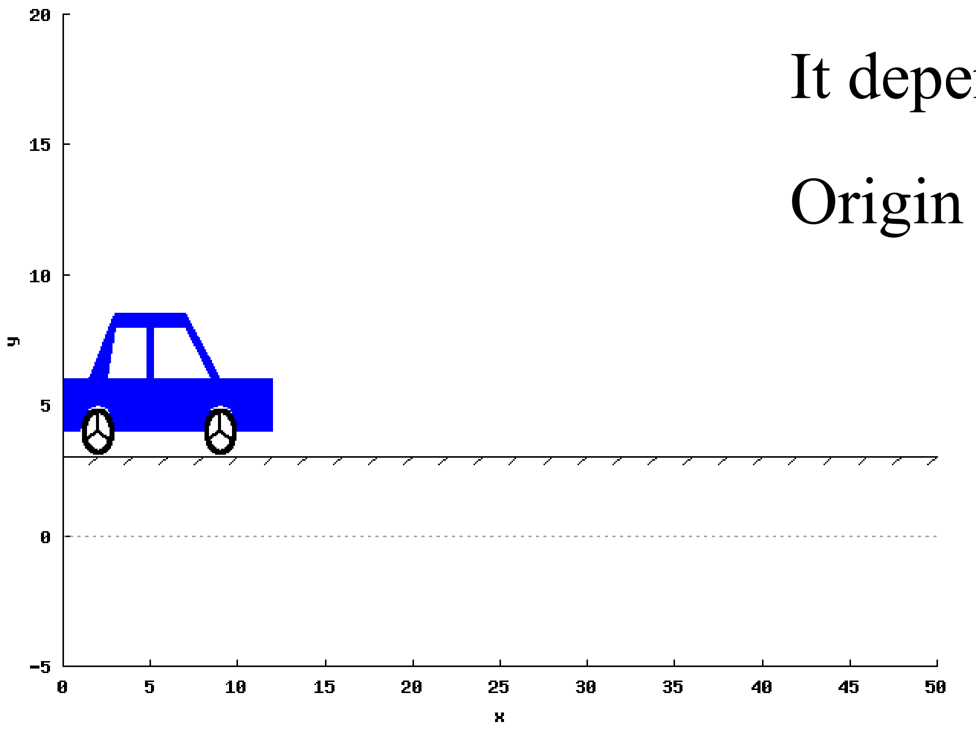
- Motion is complicated in 3-dimensional space (*xyz* space).
- In the spirit of taking things apart for study, then putting them back together, we will first consider only motion along a straight line: **one dimensional motion** (*x*).
- This is the simplest type of motion.
- Straight line can be oriented along any direction: Horizontal, vertical, or at some angle.
- It lays the groundwork for more complex motion.

Which case is a motion along a straight line?



Basic Quantities in Kinematics

- Motion can be defined as the change of **position** over **time**.
- **Position definition**: A particle moving along the *x*-axis has a coordinate *x*.



It depends on time: $x(t)$, position-time function/graph.

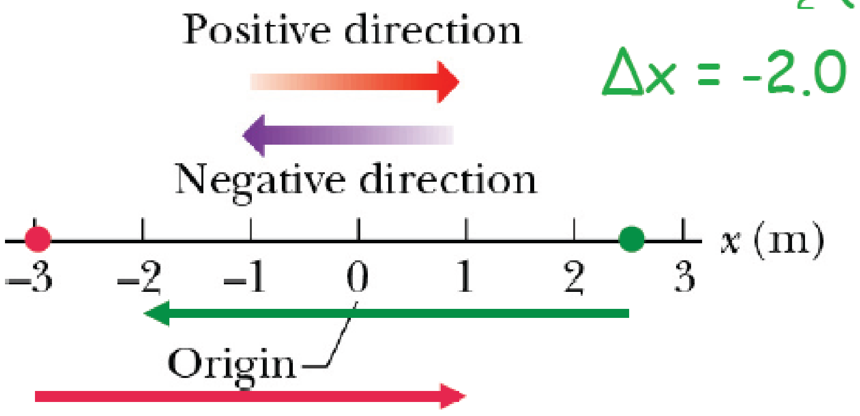
Origin (x=0): the starting point, then x is relative to origin

Sign: positive (usually right or up),
negative (usually left or down)

Displacement

- Define a new quantity "displacement": a **change of position** in time.
- **Displacement** (math language): $\Delta x = x_f(t_f) - x_i(t_i)$ *f*: final, *i*: initial
- Is the displacement a vector or scalar?

$x_1\,(t_1) = +\,2.5\text{ m}$
$x_2\,(t_2) = -\,2.0\text{ m}$
$\Delta x = -2.0\text{ m} - 2.5\text{ m} = -4.5\text{ m}$

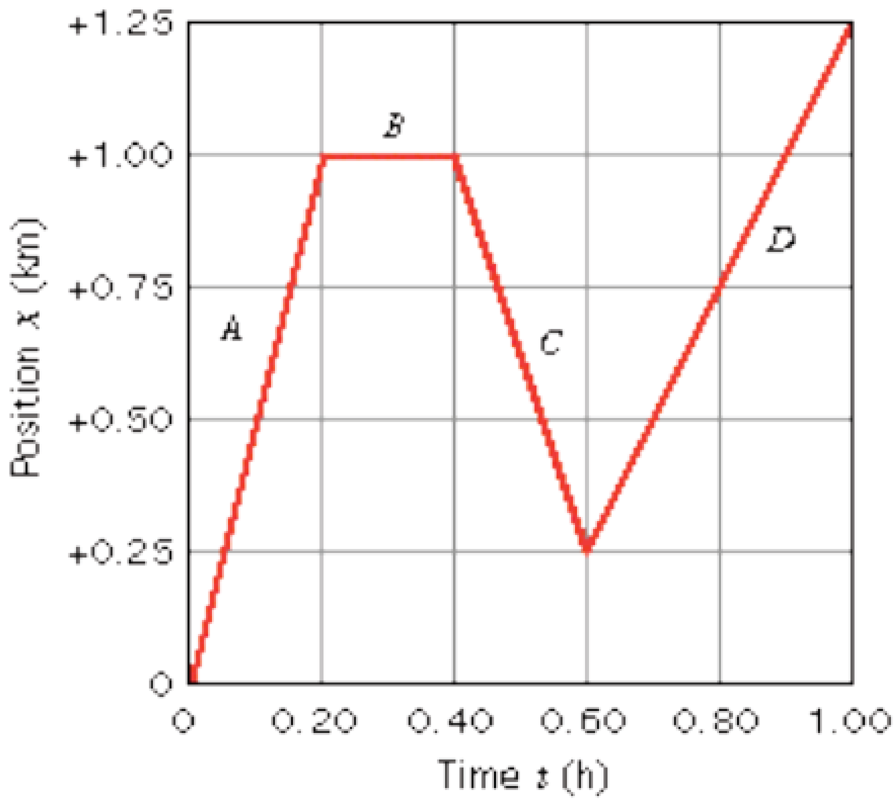


$x_1\,(t_1) = -\,3.0\text{ m}$
$x_2\,(t_2) = +\,1.0\text{ m}$
$\Delta x = +1.0\text{ m} + 3.0\text{ m} = +4.0\text{ m}$

- It is a vector quantity.
- Has both magnitude and direction (+ or - sign).
- SI unit: meter.

Position-time graph

- $\boldsymbol{x = f(t)}$ or the **position-time** graph shows the particle's position x as a function of time t.
- Calculate displacement using $x = f(t)$ or the graph.



This graph represents that a person walks for exercise.

(a) Which segment represents that the person is at rest?

(b) In 0.6 hr, what's the person's displacement?

(A) 0.25 km, (B) 1.0 km, (C) 1.75 km, (D) 0.75 km ✔

$$\Delta x = x_f(t_f) - x_i(t_i)$$

Displacement is only determined by the final and initial states.

Velocity

- Velocity is the **changing rate of displacement** (the rate of change of position).

$$\text{Velocity} = \frac{Displacement}{time}$$

- Is the velocity a scalar or vector?
- Velocity is a vector quantity, and it has both magnitude and direction.
- The SI unit for velocity is meter/second (m/s).
- **Average velocity** defined as:

(math language)

$$v_{\text{av-}x} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

Average *x*-velocity of a particle in **straight-line motion** during time interval from t_1 to t_2

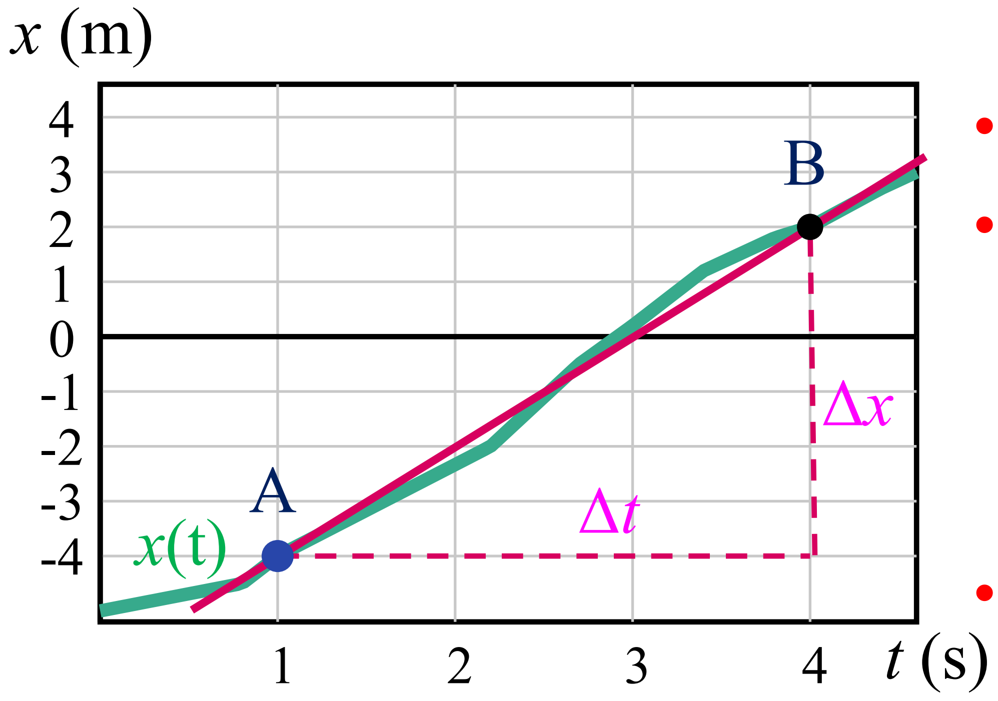
x-component of the particle's displacement

Final *x*-coordinate minus initial *x*-coordinate

Time interval

Final time minus initial time

Average Velocity



- What is the average velocity from A to B?
- It is the **slope of the line segment** between end points on the graph.

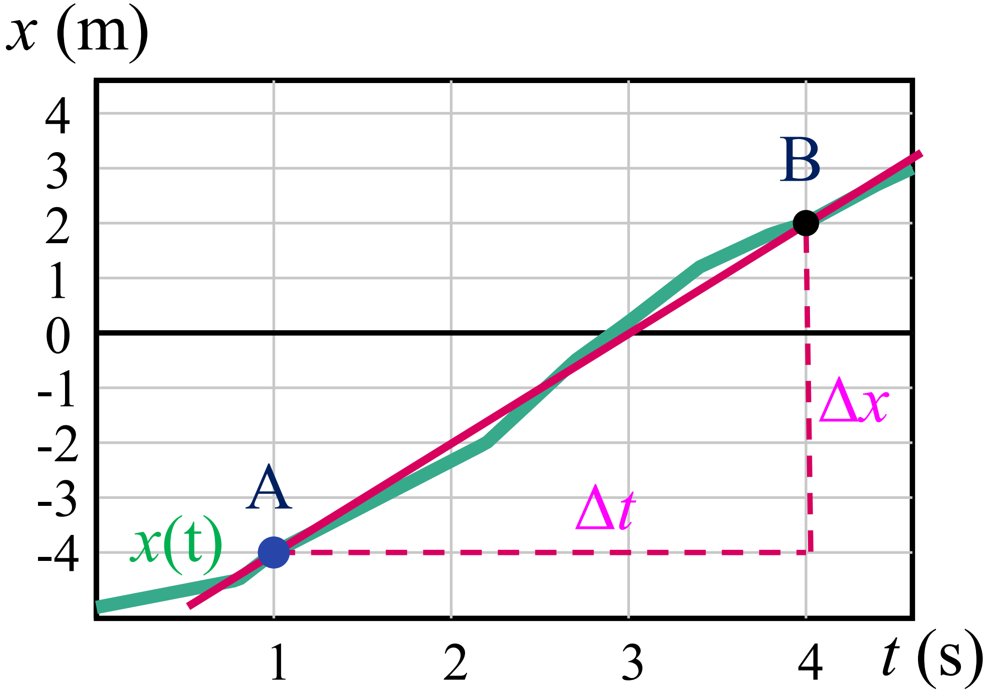
$$v_{\text{av-}x} = \frac{\Delta x}{\Delta t}$$

- It is a vector (i.e. is signed), and displacement direction sets its sign.

Average velocity from A to B? $v_{avg} = \frac{[2-(-4)]=+6\ m}{3\ s} = +2$ m/s

Average velocity from B to A? $v_{avg} = \frac{[(-4)-2]=-6\ m}{3\ s} =$ -2 m/s

Average Speed vs Average Velocity



- Average speed: $S_{avg} = \frac{\textit{total distance}}{\Delta t}$
- Dimension: length/time, [m/s].
- **Scalar**: No direction involved.
- Let compare the average velocity and speed.
- The average speed is **not** the magnitude of the average velocity! Not necessarily close to v_{avg}.

Q1: Average velocity and speed from A to B?

$v_{avg} = +2$ m/s $\quad S_{avg} = \frac{6\ m}{3\ s} = 2$ m/s

Q2: A to B to A? $v_{avg} = ? = \frac{0\ m}{3\ s} = 0$ m/s

$S_{avg} = \frac{6\ m + 6\ m}{3\ s + 3\ s} = 2$ m/s

Instantaneous Velocity

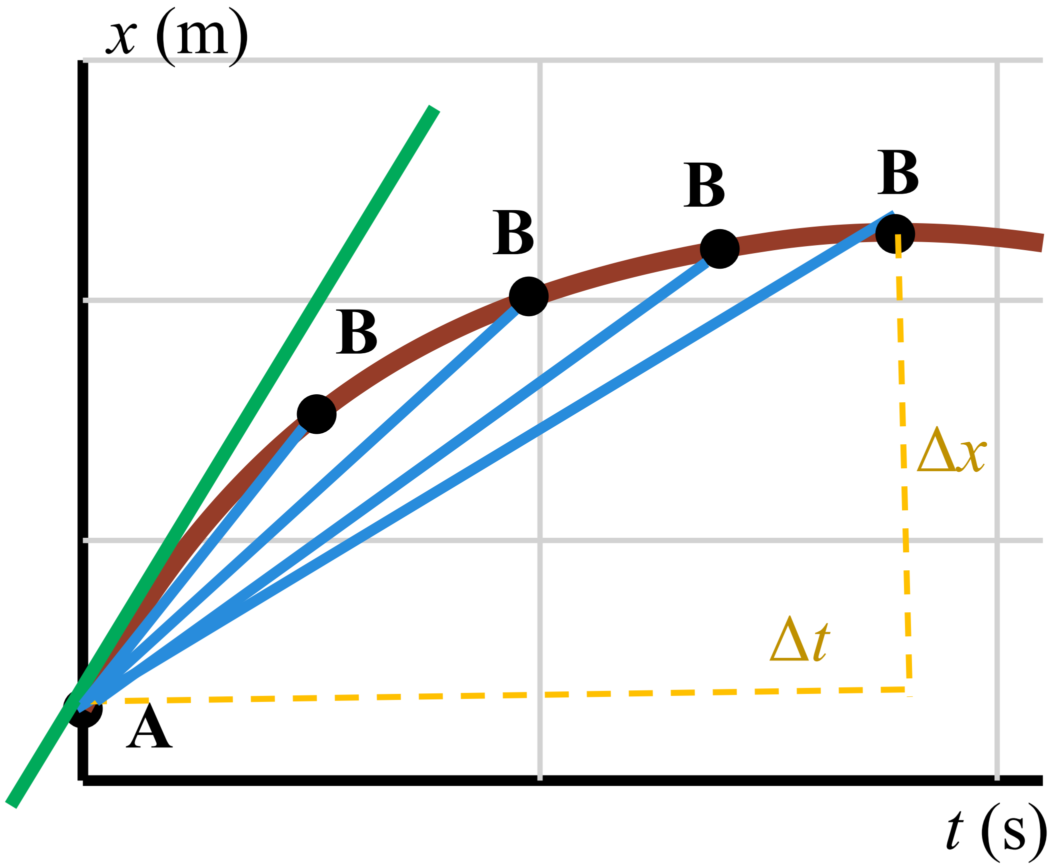
- Instantaneous means "at some given instant". The **instantaneous velocity** indicates what is happening at every point of time: $\boldsymbol{v_x}(t)$

- Connection to average velocity and $x(t)$:

$$\boldsymbol{v_x} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

- Understand instantaneous velocity graphically:

Chords approach the tangent as $\Delta t \to 0$
Slope measure rate of change of position



Green line: instantaneous velocity at A.