Example: Calculating Instantaneous Acceleration

- What is the rover's instantaneous acceleration at t = 2 s (magnitude and direction, angle to $+x$)? Speed up?

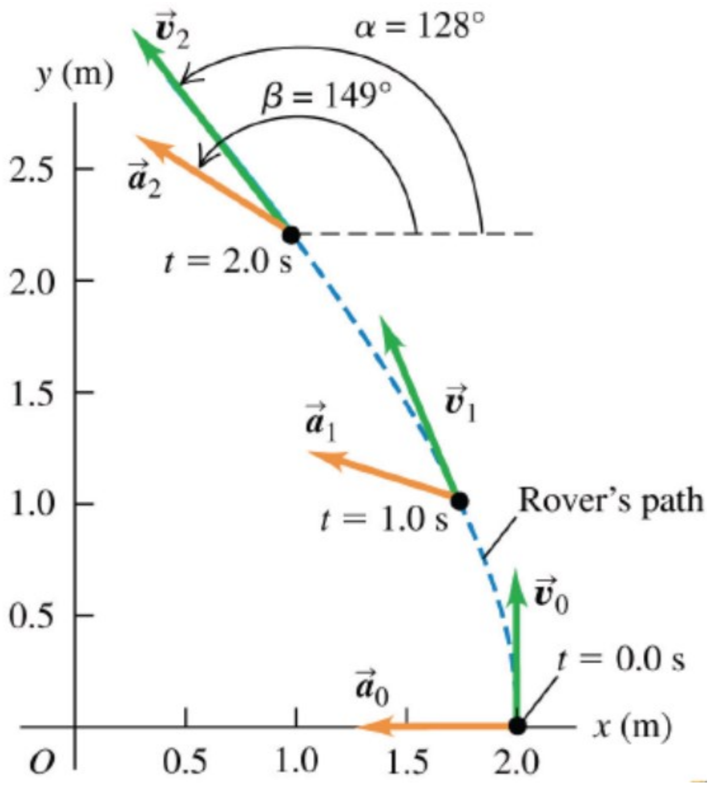
$$x = 2.0\,m - (0.25 m/s^2)t^2$$
$$y = (1.0\,m/s)t + (0.025\,m/s^3)t^3$$

$$a_x = \frac{d}{dt}\left(\frac{dx}{dt}\right) = -0.5\ m/s^2 \qquad a_y = \frac{d}{dt}\left(\frac{dy}{dt}\right) = 0.15\ m/s^2(t)$$

$$a_x = -0.5\ m/s^2 \qquad a_y = 0.3\,m/s^2$$

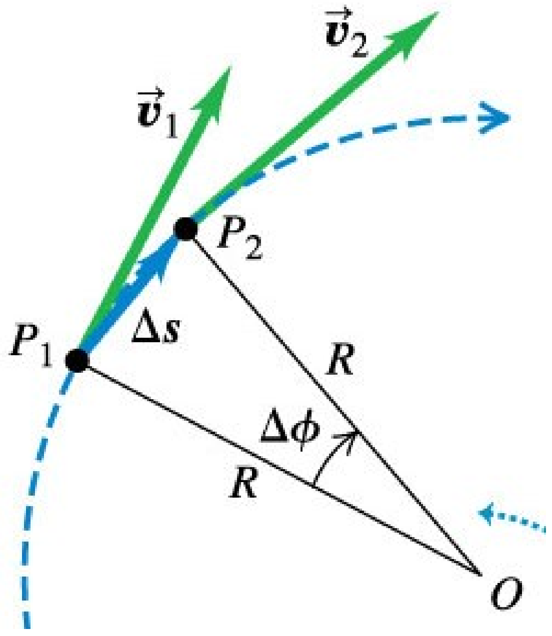
$$|a| = \sqrt{(-0.5)^2+(0.3)^2} = 0.58\,m/s^2$$

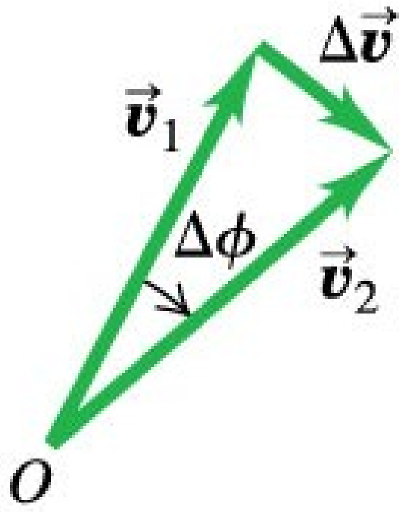
$$tan\theta = a_y/a_x = -0.6 \qquad \theta = 149°$$

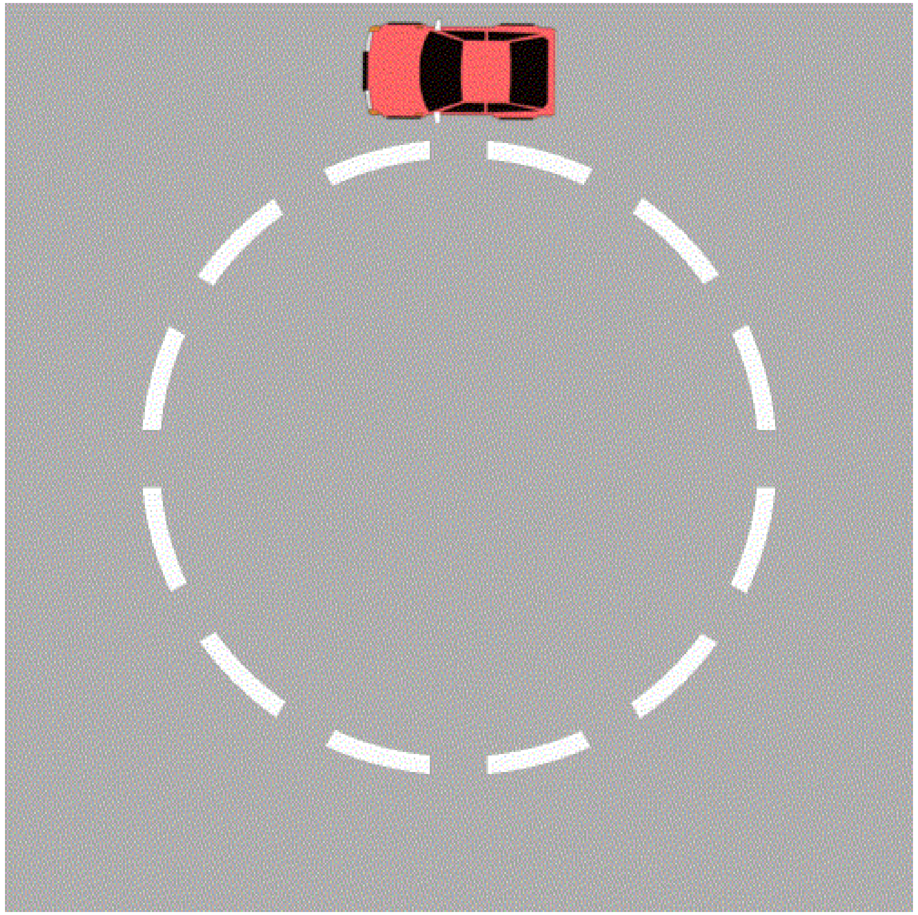


Acceleration for uniform circular motion

- **Q:** What is the magnitude of the acceleration?







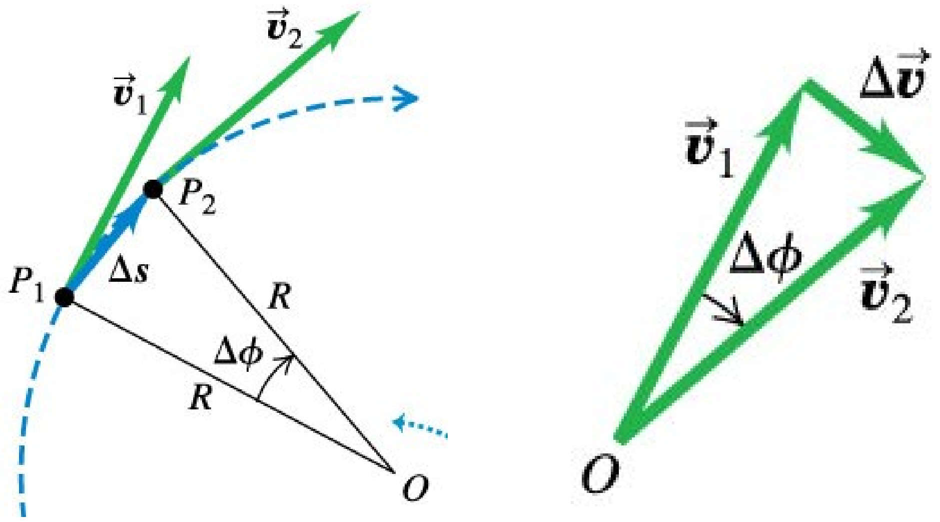
The black triangle and the green triangle are similar.

Acceleration for uniform circular motion

The black triangle and the green triangle are similar.

$$\frac{\Delta s}{R} = \frac{\Delta v}{v} = \Delta\phi \qquad \Delta v = \frac{v\Delta s}{R}$$

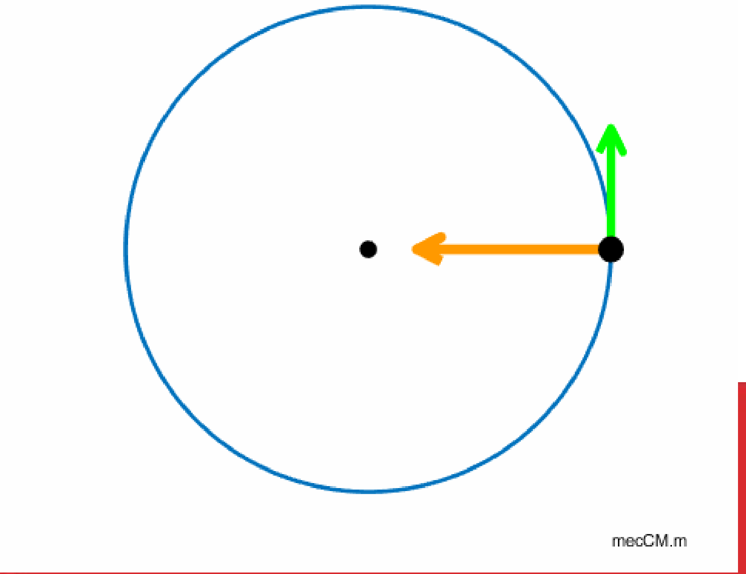
$$a = \frac{\Delta v}{\Delta t} = \frac{v}{R}\left(\frac{\Delta s}{\Delta t}\right) = \frac{v^2}{R}$$



- The magnitude of the acceleration is $a_{rad} = v^2/R$.

Q: What is the period T? $T = 2\pi R/v$ $v = 2\pi R/T$

- We also have $a_{rad} = 4\pi^2 R/T^2$.



Example:

An object moves in a horizontal circle (dimeter 2 m) at constant speed v m/s. In order to have an acceleration of π^2 m/s^2, what is the speed that the object must have? How fast (T) is the object turning in the unit of revolution/min?

A. Speed is π m/s, T is $2\sqrt{2}$ s.

B. Speed is $\frac{\pi}{\sqrt{2}}$ m/s, T is $4\sqrt{2}$ s.

C. Speed is π m/s, T is 2 s. ✓

D. Speed is 2π m/s, T is 2 s.

E. none of the above

Q1: Known and unknown quantities?

$$a_{rad} = 4\pi^2 R/T^2$$

$$a_{rad} = v^2/R$$

$$T = 2\pi R/v$$

$$a_{rad} = \pi^2 = v^2/(1\text{ m})$$

v = ? m/s $\quad T$ =?

$v = \pi$ m/s $\quad T = 2\pi R/v = 2\pi(1\text{ m})/\pi = 2$ s

Example:

An object moves in a horizontal circle (dimeter 2.0 m) at constant speed v (in units of m/s). It takes the object 10.0 s to complete one revolution. What is the speed of this object? What is the radial acceleration?

A. Speed is $\frac{\pi}{5}$ m/s, radial acceleration is $\frac{\pi^2}{25}$ m/s^2 ✓

B. Speed is $\frac{\pi^2}{25}$ m/s, radial acceleration is $\frac{\pi}{5}$ m/s^2

C. Speed is $\frac{2\pi}{5}$ m/s, radial acceleration is $\frac{2\pi^2}{25}$ m/s^2

D. Speed is $\frac{2\pi^2}{25}$ m/s, radial acceleration is $\frac{2\pi}{5}$ m/s^2

$a_{rad} = 4\pi^2 R/T^2$

$T = 2\pi R/v$

$a_{rad} = v^2/R$

$T = 10 = 2\pi R/v = 2\pi 1/v$

$a_{rad} = 4\pi^2 R/T^2 = 4\pi^2 1/100$

Nonuniform circular motion

- If the speed varies, the motion is *nonuniform circular motion*.
- The radial acceleration component is still $a_{rad} = v^2/R$.
- But there is also a tangential acceleration component a_{tan} that is *parallel* to the instantaneous velocity.

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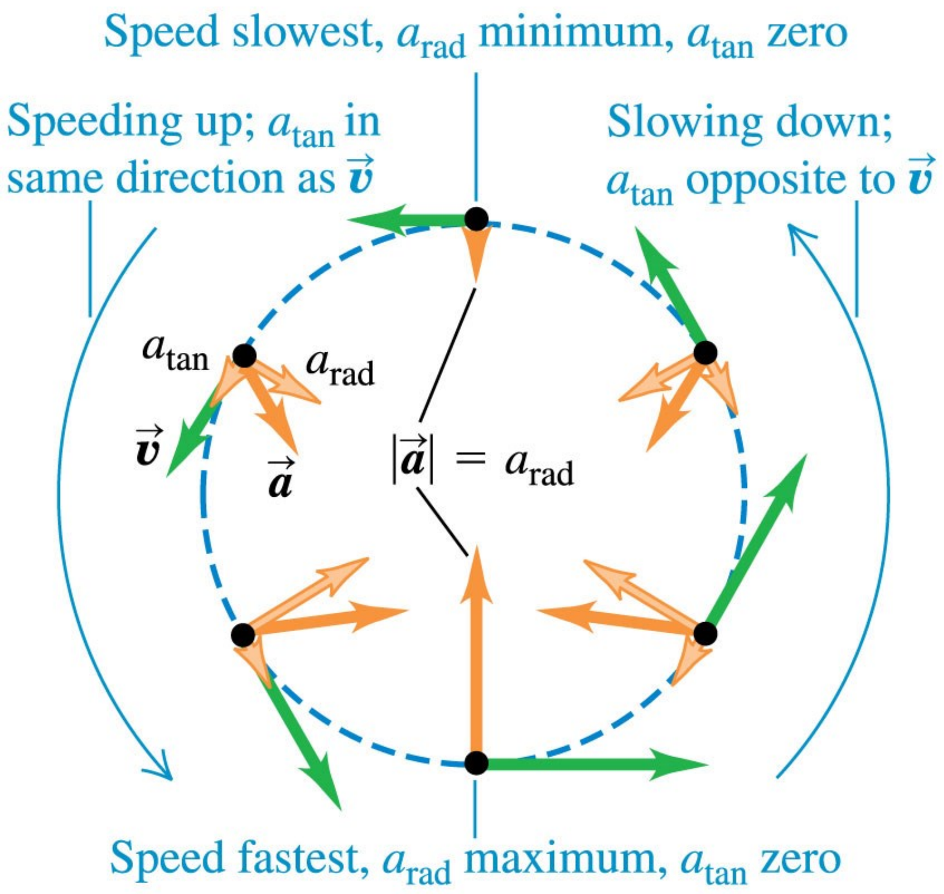


2-D motion under constant acceleration

- Constant acceleration equations in vector form of

$$\vec{v} = \vec{v}_0 + \vec{a}t \qquad \vec{r} - \vec{r}_0 = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 \qquad \vec{v}^2 = \vec{v}_0^2 + 2\vec{a}(\vec{r} - \vec{r}_0)$$

- Motions in two dimensions are **independent** components
- Constant acceleration equations **hold in each dimension**

$$v_x = v_{0x} + a_x t, \qquad v_y = v_{0y} + a_y t$$

$$x - x_0 = v_{0x} t + \frac{1}{2} a_x t^2, \qquad y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2$$

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0), \qquad v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

✓ $t = 0$ beginning of the process

✓ $\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath}$ where a_x and a_y are constant

✓ Initial velocity $\vec{v}_0 = v_{0x}\hat{\imath} + v_{0y}\hat{\jmath}$ initial displacement $\vec{r}_0 = x_0\hat{\imath} + y_0\hat{\jmath}$

$$v = v_0 + at$$

$$x - x_0 = v_0 t + \tfrac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$x - x_0 = \tfrac{1}{2}(v + v_0)t$$

$$x - x_0 = vt - \tfrac{1}{2}at^2$$

Hints for solving problems

- Define coordinate system. Make sketch showing **axes**, origin.
- List **known** quantities. Find v_{0x}, v_{0y}, a_x, a_y et al. Show **initial** conditions on sketch. List **unknown** quantities and which quantities you want to calculate.
- List equations of motion to see which ones to use.
- **Time *t* is the same for *x* and *y* direction!**

$$x_0 = x(t=0), \quad y_0 = y(t=0), \quad v_{0x} = v_x(t=0), \quad v_{0y} = v_y(t=0).$$

- Have an axis point along the direction of $\vec{a}$ if it is constant.

$$v_x = v_{0x} + a_x t,$$
$$x - x_0 = v_{0x} t + \frac{1}{2} a_x t^2,$$
$$v_x^2 = v_{0x}^2 + 2a_x (x - x_0),$$

$$v_y = v_{0y} + a_y t$$
$$y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2$$
$$v_y^2 = v_{0y}^2 + 2a_y (y - y_0)$$

List the quantities and equations for *x* and *y* direction, separately.

Projectile motion: constant acceleration

- A ball rolls off a table of height h. The ball has horizontal velocity v_0 when it leaves the table.
- **Q:** How long (s) does it take to reach the ground?

Q1: What is the type of motion along x direction?

Uniform velocity along x direction

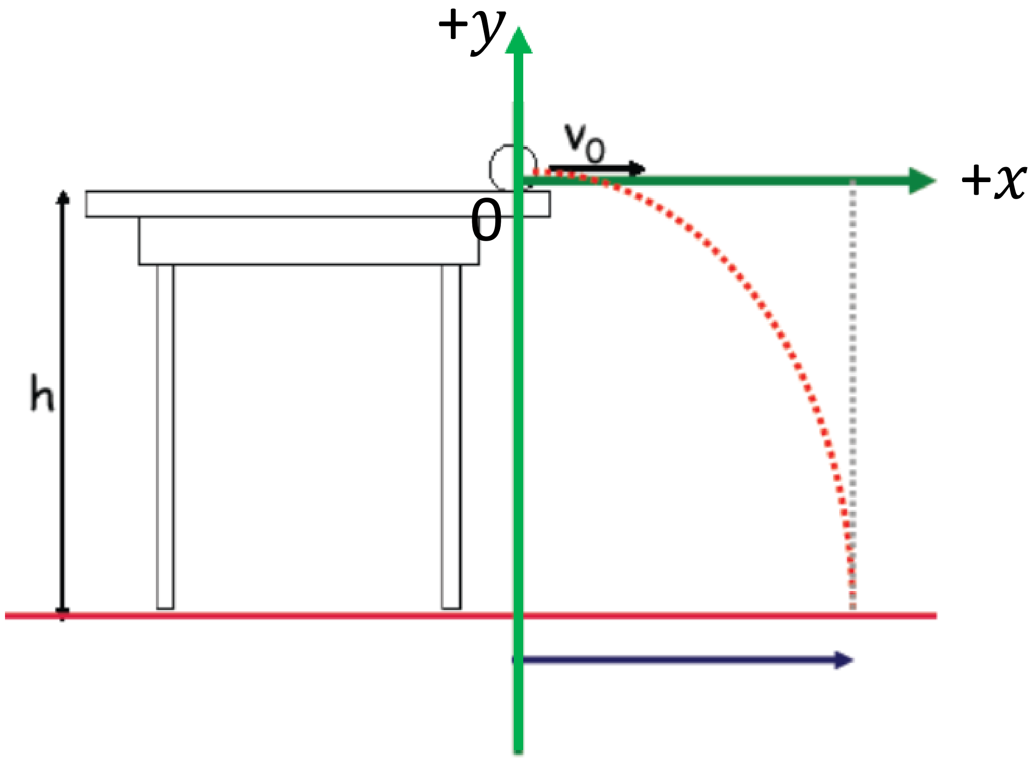
$$x_0 = ?, x = ?, v_{0x} = ?, v_x = ?, a_x = ?$$

$$x_0 = 0, x = v_0 t, v_{0x} = v_0, v_x = v_0, a_x = 0,$$

Q2: What is the type of motion along y direction?

Free fall motion along y direction

$$y_0 = ?, y = ?, v_{0y} = ?, v_y = ?, a_y = ?,$$



$$y_0 = 0, y = -h,$$

$$v_{0y} = 0, v_y = -gt, a_y = -g,$$

Projectile motion: constant acceleration

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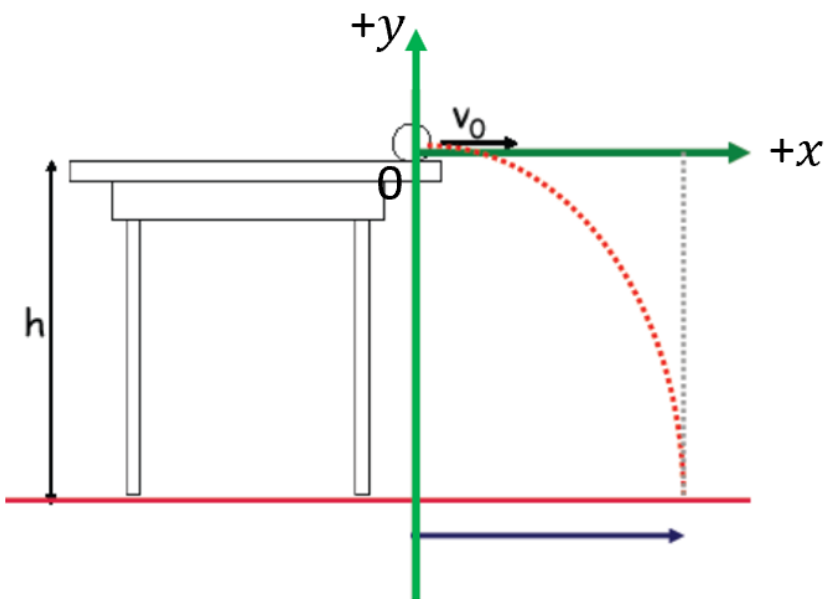


For x direction: $x_0 = 0$, $x = v_0 t$,
$v_{0x} = v_0$, $v_x = v_0$, $a_x = 0$,

For y direction: $y_0 = 0$, $y = -h$,
$v_{0y} = 0$, $v_y = -gt$, $a_y = -g$,
Vertical free fall: $y - y_0 = -\frac{1}{2} g t^2 = -h$

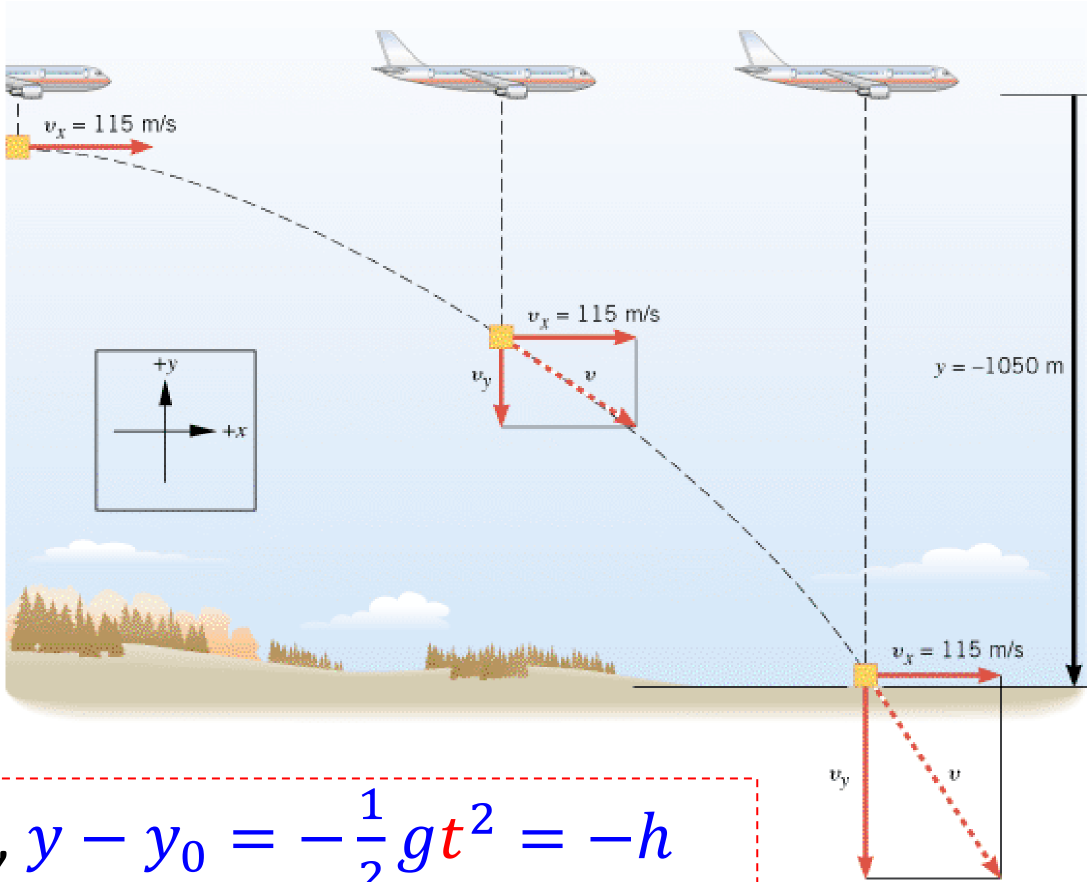
Use y direction to solve $t = \sqrt{2h/g}$

Use t to solve x direction: $x - x_0 = v_0 t = v_0 \sqrt{2h/g}$

How far (m) away does it strike the ground.

Example: **Projectile motion - airplane drops package**

An airplane is flying horizontally at a speed of 115 m/s. At a certain moment, it drops a package. How far (m) away does package strike the ground?



Q1: What is the type of motion along *x* direction?

For *x* direction: $x_0 = 0$, $x = v_0 t$,

$v_{0x} = v_0$, $v_x = v_0$, $a_x = 0$,

Q2: What is the type of motion along *y* direction?

$y_0 = 0, y = -h, v_{0y} = 0, v_y = -gt, a_y = -g, y - y_0 = -\frac{1}{2}gt^2 = -h$

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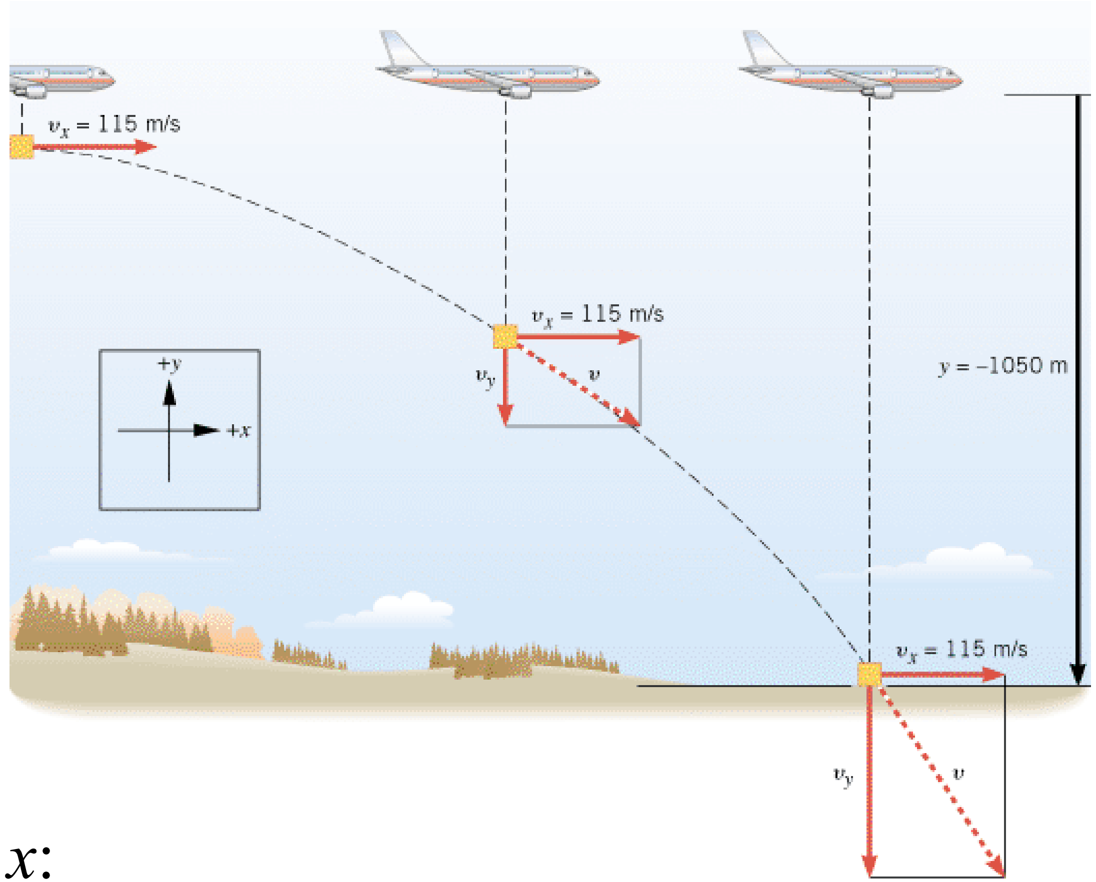


For *x* direction: $x_0 = 0$, $x = v_0 t$,
$v_{0x} = v_0$, $v_x = v_0$, $a_x = 0$,

$y_0 = 0$, $y = -h$, $v_{0y} = 0$, $v_y = -gt$, $a_y = -g$,
$y - y_0 = -\frac{1}{2}gt^2 = -h$

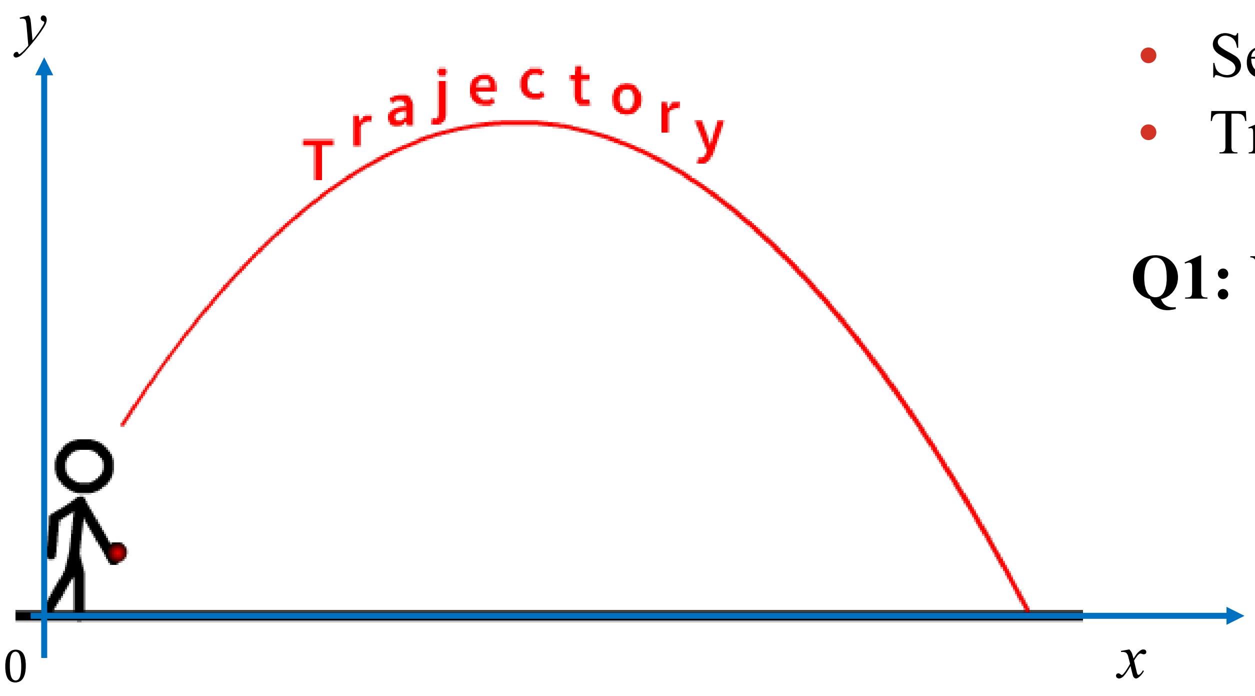
Use *y* direction to solve *t*

$$t = \sqrt{2h/g} = 14.64 \text{ s}$$

Use *t* to solve *x*:

$$x = v_0 t = 1.683 \text{ km}$$

Projectile motion: general case



- Set horizontal x horizontal, and y vertical.
- Try to pick $x_0 = 0, y_0 = 0$ at $t = 0$

Q1: What is the type of motion along x direction?

Uniform velocity along x direction

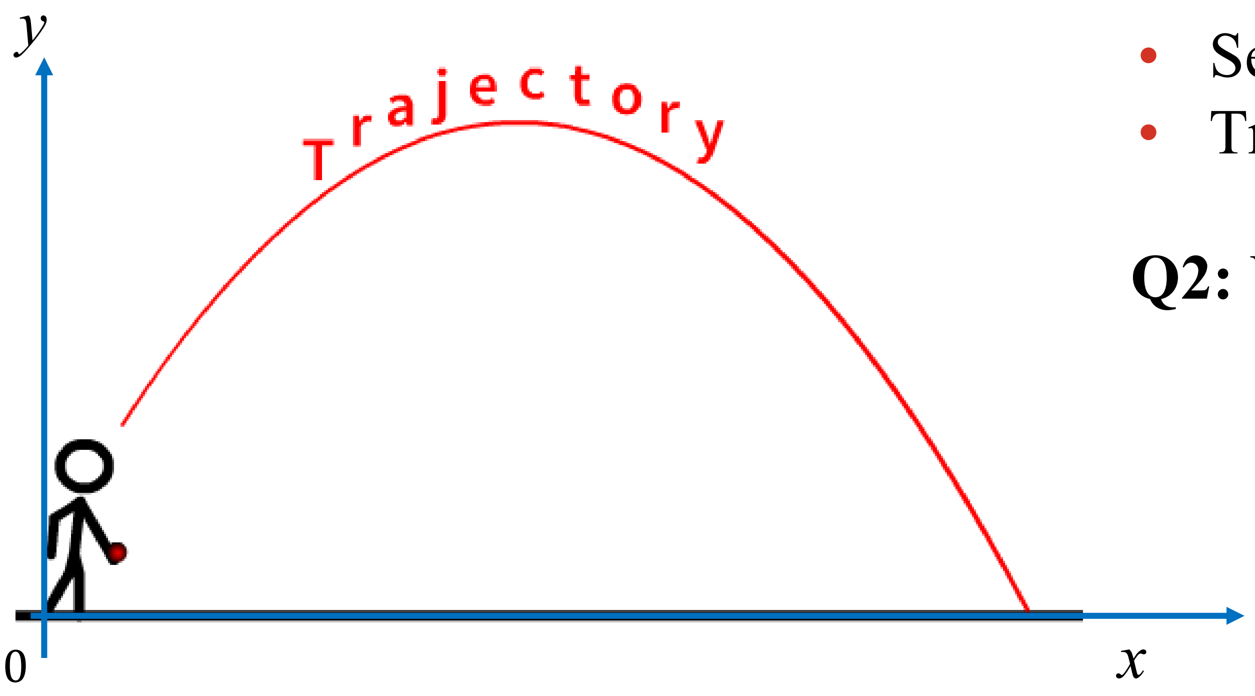
Horizontal: $a_x = 0, v_{0x} \neq 0$

$$v_x = v_{0x} + a_x t = v_{0x}$$

$$x - x_0 = v_{0x} t + \frac{1}{2} a_x t^2 = v_{0x} t$$

$$v_x^2 = v_{0x}^2 + 2a_x (x - x_0) = v_{0x}^2$$

Projectile motion: general case



- Set horizontal x horizontal, and y vertical.
- Try to pick $x_0 = 0, y_0 = 0$ at $t = 0$

Q2: What is the type of motion along y direction?

Vertical up-and-down free fall

Vertical: $a_y = -9.8\, m/s^2$, $v_{0y} \neq 0$

$$v_y = v_{0y} + a_y t$$

$$y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2$$

$$v_y^2 = v_{0y}^2 + 2a_y (y - y_0)$$

Projectile motion: general case

- Try to pick $x_0 = 0, y_0 = 0$ at $t = 0$
- Horizontal: $a_x = 0, v_{0x} \neq 0$
- Vertical: $a_y = -9.8\, m/s^2, v_{0y} \neq 0$

Horizontal $$v_x = v_{0x} + a_x t = v_{0x}$$

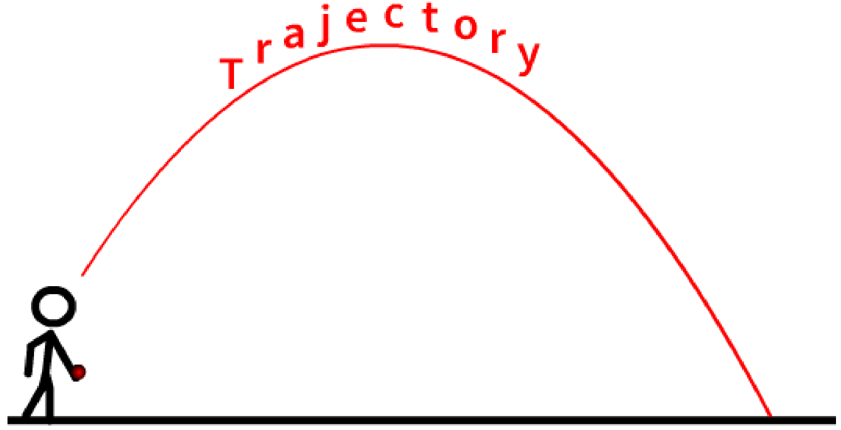
$$x - x_0 = v_{0x} t + \frac{1}{2} a_x t^2 = v_{0x} t$$

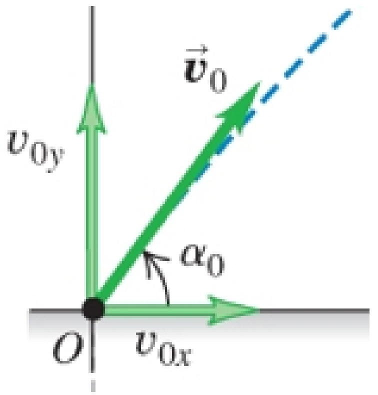
$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0) = v_{0x}^2$$

Vertical $$v_y = v_{0y} + a_y t$$

$$y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2$$

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$





For initial velocity, both $v_{0x} \neq 0$ and $v_{0y} \neq 0$. We need to decompose v_0 to v_{0x} and v_{0y}.

- If α_0 angle and v_0 are given, the x and y components of v_0 can be written as:

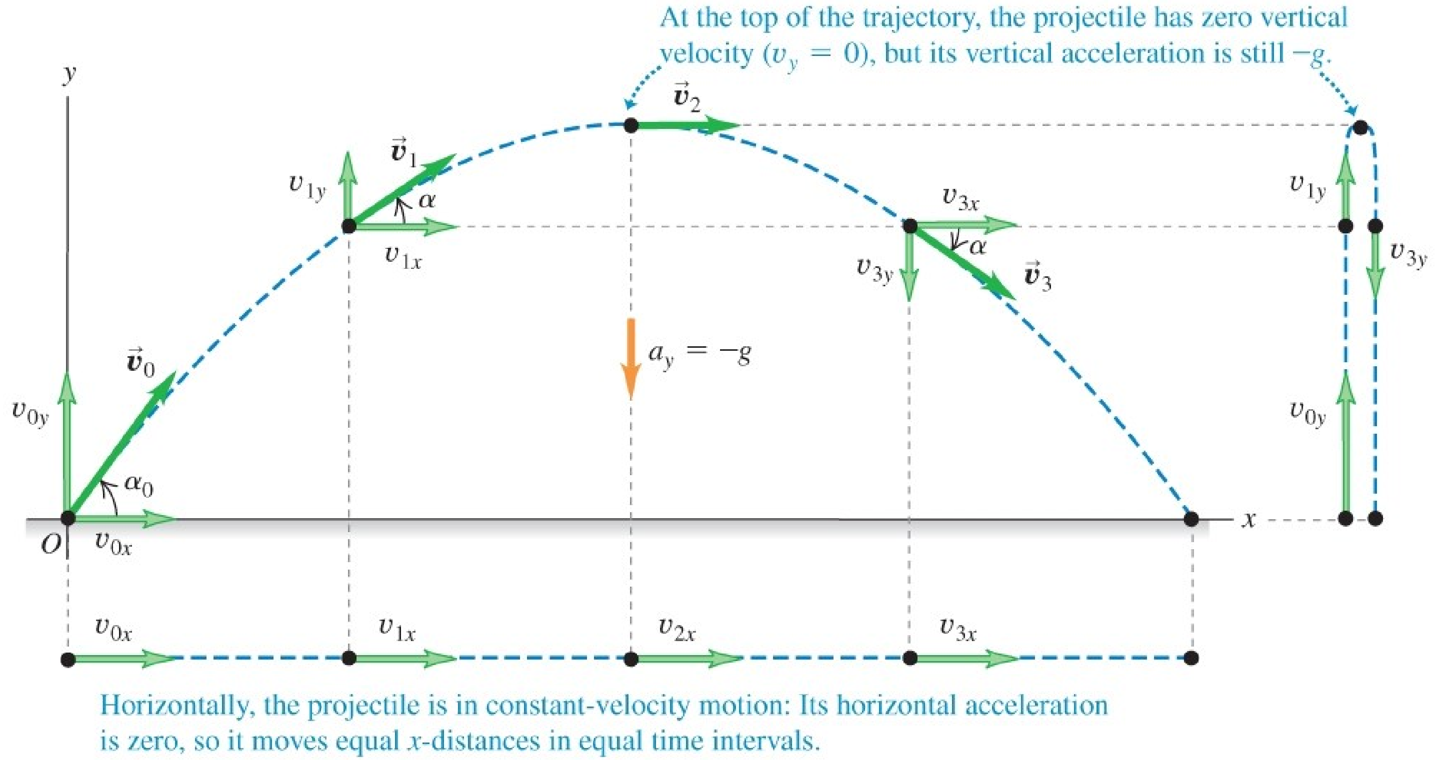
$$v_{0x} = v_0 cos\alpha_0 \qquad v_{0y} = v_0 sin\alpha_0$$

Projectile motion: general case

- If α_0 angle and v_0 are given, the x and y components of v_0 can be written as:

$$v_{0x} = v_0 cos\alpha_0$$
$$x = x_0 + v_{0x}t = (v_0 cos\alpha_0)t$$

$$v_{0y} = v_0 sin\alpha_0$$
$$v_y = v_{0y} + a_y t = v_0 sin\alpha_0 - gt$$
$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$
$$= (v_0 sin\alpha_0)t - \frac{1}{2}gt^2$$



Example:

A projectile is fired from the ground with a velocity of **110.0 m/s** at an angle of **40.0 degrees** above the horizontal. What will be the range (**the horizontal displacement**) of this projectile?

Both α_0 angle and v_0 are given

$$v_{0x} = v_0 cos\alpha_0$$
$$x = x_0 + v_{0x}t$$

$$v_{0x} = 110 \times cos40° = 84.3 \text{ m/s}$$
$$x = 0 + 84.3t = 1216 \text{ m}$$

Use t to solve x

$$v_{0y} = v_0 sin\alpha_0$$
$$v_y = v_{0y} + a_y t$$
$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

$$v_{0y} = 110 \times sin40° = 70.7 \text{ m/s}$$
$$0 = 0 + 70.7 \times t - \frac{1}{2}9.8t^2$$
$$t = 14.4 \text{ s}$$

Use y to solve time

Maximum height and range of a projectile

- Angle not given, what initial angle will give the maximum range of a projectile?

$$v_{0x} = v_0 cos\alpha_0 \qquad x = (v_0 cos\alpha_0)t$$

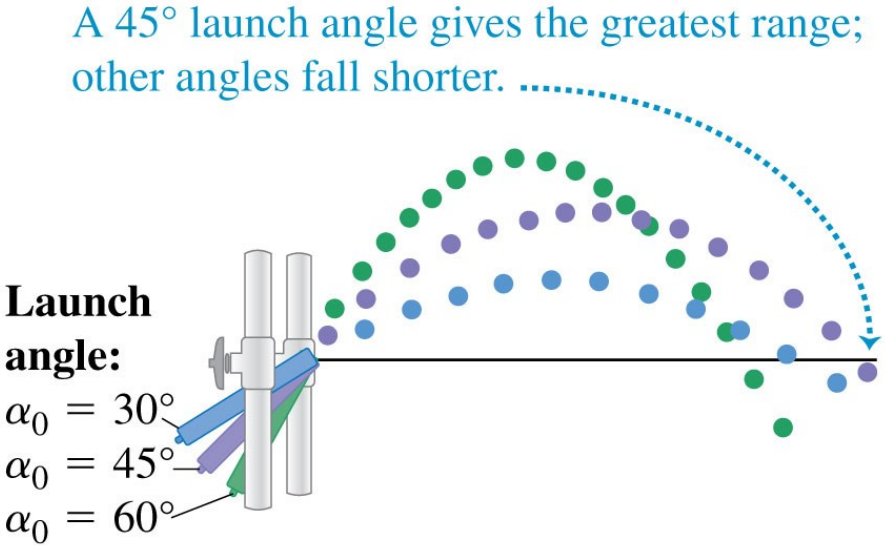
$$v_{0y} = v_0 sin\alpha_0 \qquad v_y = v_0 sin\alpha_0 - gt$$
$$y = (v_0 sin\alpha_0)t - (1/2)gt^2$$

$$y = (v_0 sin\alpha_0)t - (1/2)gt^2 = 0$$

Use *y* to solve time $t = 2v_0 sin\alpha_0/g$

Use *t* to solve *x*:
$$x = v_0 cos\alpha_0 t = (v_0 cos\alpha_0)(2v_0 sin\alpha_0)/g$$

$$x = (v_0^2/g)\sin(2\alpha_0)$$



Maximum height/longest fly time: α_0?

$\alpha_0 = 45$ degrees