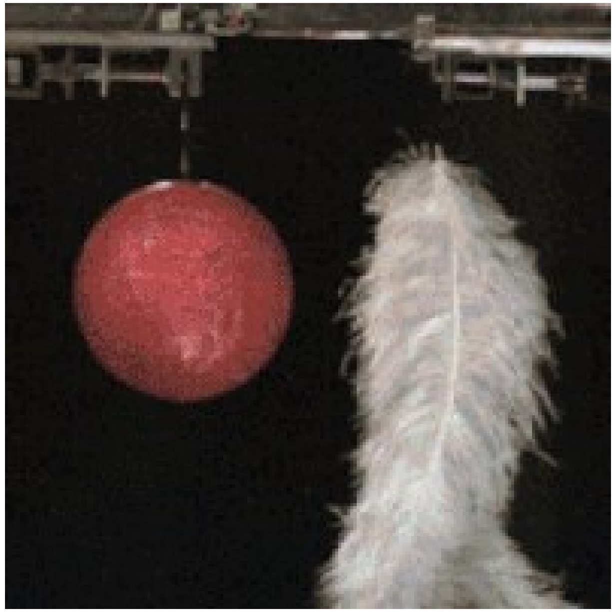
Free Fall Acceleration

- One main reason we focus on uniform acceleration: Earth gravity provides a constant acceleration.
- Free-fall is the most important case of constant acceleration. Magnitude: $|a| = g = 9.8\, m/s^2$
- Direction: always downward, so a_g is **negative** if we define "**up**" as **positive**, $a_g = -g = -9.8\, m/s^2$
- Free-fall is **independent of mass**.
- Try to pick origin so that $y_i = 0$, and use the previously derived equations, we get:



$v = v_0 + at$ $\Delta x = v_0 t + \frac{1}{2} a t^2$

$v = v_0 - gt$ $y - y_0 = v_0 t - \frac{1}{2} g t^2$

Begin with $t_0 = 0$, $v_0 = 0$, $y_0 = 0$
so $t^2 = 2y/g$ same for any objects.

Example:

An apple falls from the apple tree, and it strikes the ground in 1.0 s. You may ignore air resistance, what is the height of the apple on the tree? What is the magnitude of the apple's velocity just before it reached the ground? $|a| = g = 10.0\ m/s^2$

Q1: What quantities are given, and which are unknown?

Given quantities		**Unknow** quantities	
$g = 10\ \text{m/s}^2$	$t = 1.0\ \text{s}$	$v = ?\ \text{m/s}$	$y = ?\ \text{m}$
$v_0 = 0\ \text{m/s}$	$y_0 = 0\ \text{m}$		

Q2: Draw a diagram?

Q3: Choose the appropriate kinematic equation?

$$v = v_0 - gt \qquad y - y_0 = v_0 t - \frac{1}{2} g t^2$$

Q4: Solve unknown quantity?

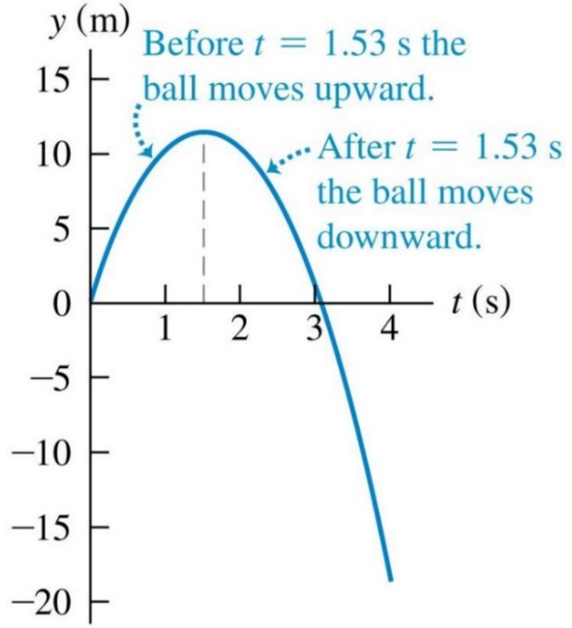
$$v = 0 - (10)\times 1 = -10\ \text{m/s}$$

$$y - 0 = 0\times 1 - 0.5\times(10)\times 1^2 = -5\ \text{m}$$

Up-And-Down Motion in Free Fall (Basic)

On rooftop of Tiernan Hall (20 m tall) I throw an apple upward with an initial speed of 15.0 m/s.

a) The time needed for the apple to reach its maximum height?



Q1: What quantities are given, and which are unknown?

Given quantities

$g = 10$ m/s^2 $\quad v_0 = +15$ m/s

$y_0 = 0$ m

Unknow quantities

$t = ?$ $\quad y = ?$

$v = ?$

Q2: Meaning of maximum height? $\quad v = 0$

Q3: Draw a diagram?

Q4: Choose the appropriate kinematic equation? $\quad v = v_0 - gt \quad\quad y - y_0 = v_0 t - \frac{1}{2} g t^2$

Use $v = v_0 - gt$, 0 m/s = 15 m/s – (10 m/s^2)$\times t$, we get $t = 1.5$ s.

Up-And-Down Motion in Free Fall (Basic)

On rooftop of Tiernan Hall (20 m tall) I throw an apple upward with an initial speed of 15.0 m/s.

a) The maximum height?

Given quantities

g = 10 m/s^2 v_0 = +15 m/s

y_0 = 0 m v = 0 t = 1.5

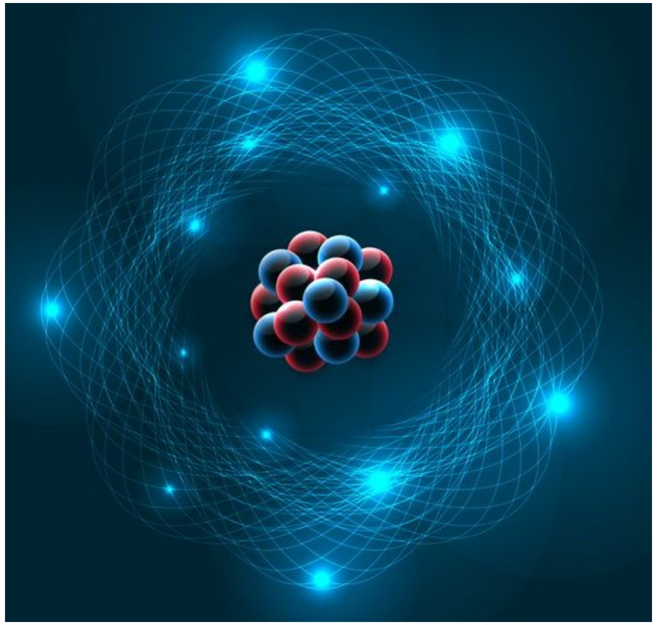
Unknow quantities

y = ?

Q4: Choose the appropriate kinematic equation? $v = v_0 - gt$ $y - y_0 = v_0 t - \frac{1}{2} g t^2$

Use $y - y_0 = v_0 t - (1/2)gt^2$, y = (15 m/s)×(1.5 s) – 0.5×(10 m/s^2)×(1.5 s)^2, we get y = 11.25 m.

Physics 111: Mechanics
Chapter 3



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Summary: 1D motion

Kinematic variables in one dimension

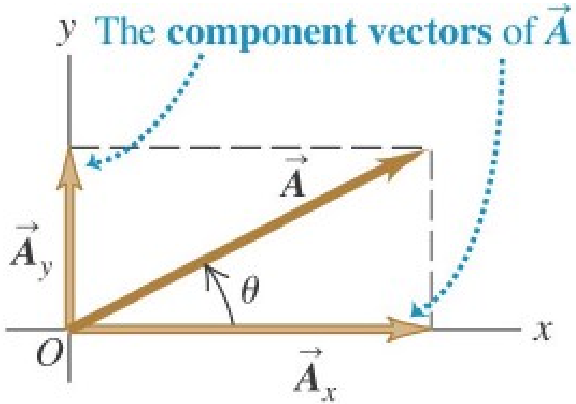
✓ Position	$x(t)$	m	L
✓ Velocity	$v(t)$	m/s	L/T
✓ Acceleration	$a(t)$	m/s^2	L/T^2

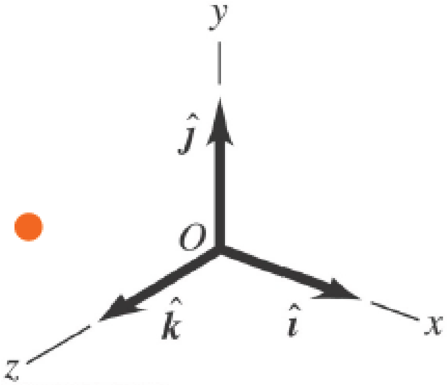
- ✓ All depend on time
- ✓ All are vectors: magnitude and direction vector

Equations for motion with constant acceleration:	missing quantities
✓ $v = v_0 + at$	$x - x_0$
✓ $x - x_0 = v_0 t + \frac{1}{2}at^2$	v
✓ $v^2 = v_0^2 + 2a(x - x_0)$	t
✓ $x - x_0 = \frac{1}{2}(v + v_0)t$	a
✓ $x - x_0 = vt - \frac{1}{2}at^2$	v_0

2D & 3D motion

- If an object is going around a curve, is it accelerating?
- We need to extend our description of motion to two and three dimensions.

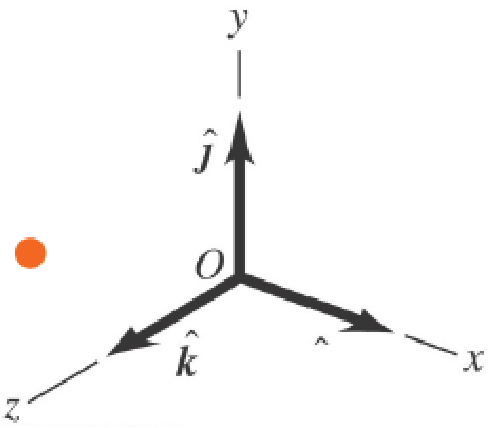




Motion in 2D and 3D

- Kinematic variables in one dimension
  - ✓ Position: $x(t)$ m
  - ✓ Velocity: $\vec{v}(t)$ m/s
  - ✓ Acceleration: $\vec{a}(t)$ m/s²

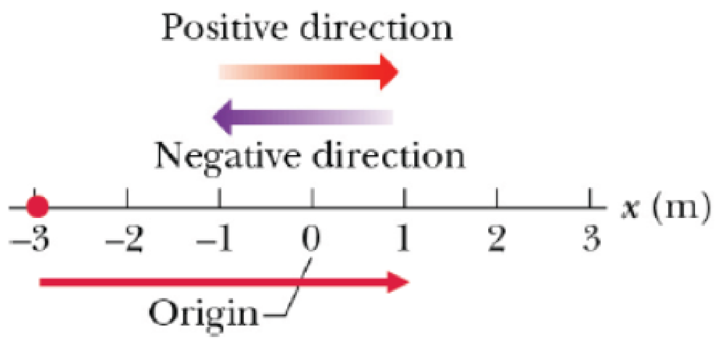
- Kinematic variables in three dimensions
  - ✓ Position: $\vec{r}(t) = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ m
  - ✓ Velocity: $\vec{v}(t) = v_x\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k}$ m
  - ✓ Acceleration: $\vec{a}(t) = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$ m/s²



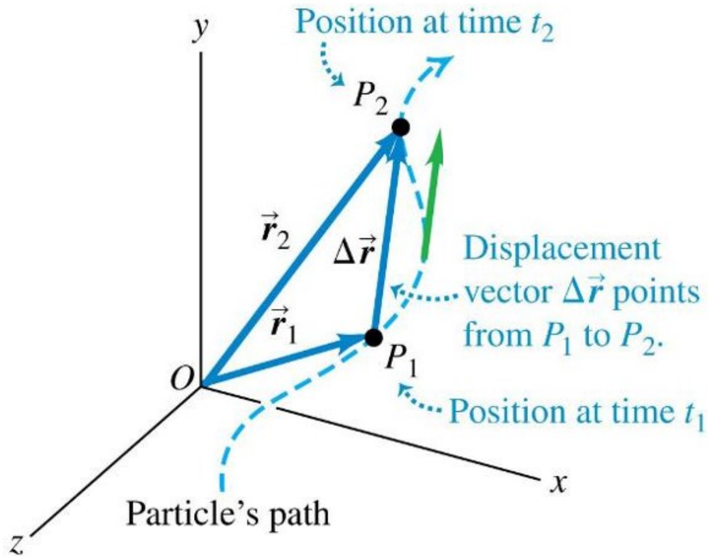
Position and Displacement

- In one dimension

  $\Delta\vec{x} = x_f(t_f) - x_i(t_i)$



- In two or three dimension
  - ✓ Position: the position of an object is described by its position vector $\vec{r}(t)$ always points to particle from origin.
  - ✓ Displacement: $\Delta\vec{r} = \vec{r_f}(t_f) - \vec{r_i}(t_i)$



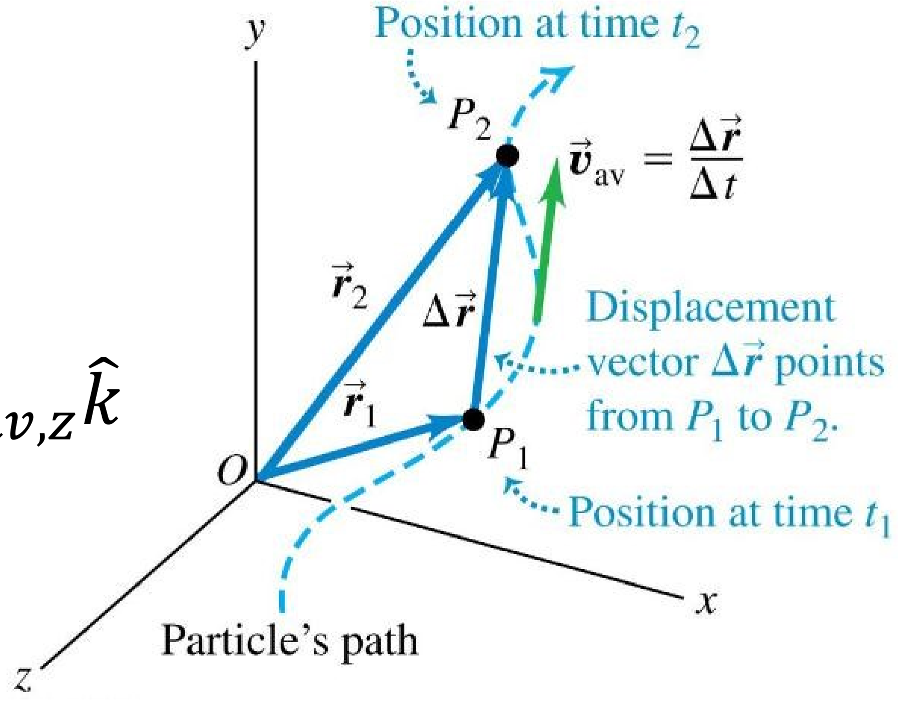
Average velocity

- The average velocity between two points is the displacement divided by the time interval between the two points.

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$$

$$\vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\Delta x\hat{\imath} + \Delta y\hat{\jmath} + \Delta z\hat{k}}{\Delta t}$$

$$= \frac{\Delta x}{\Delta t}\hat{\imath} + \frac{\Delta y}{\Delta t}\hat{\jmath} + \frac{\Delta z}{\Delta t}\hat{\jmath} = v_{av,x}\hat{\imath} + v_{av,y}\hat{\jmath} + v_{av,z}\hat{k}$$



- The average velocity has the same direction as the displacement

Instantaneous velocity

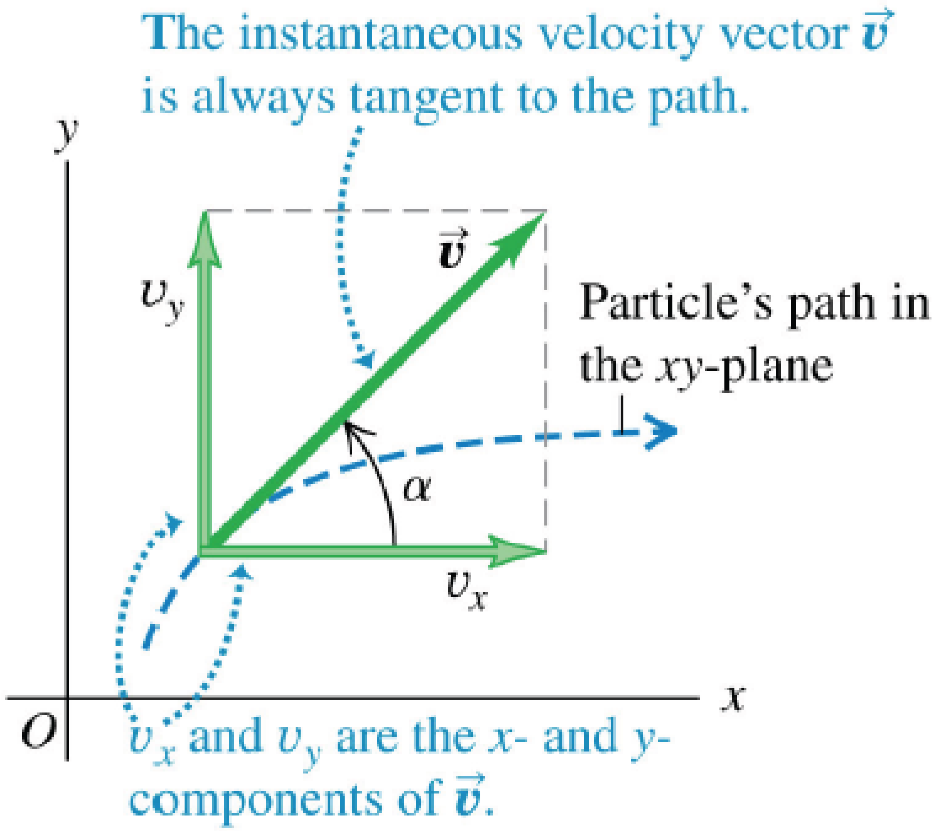
- The instantaneous velocity is the instantaneous rate of change of position vector with respect to time

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

- The component of the instantaneous velocity are

$$v_x = \frac{dx}{dt}, v_y = \frac{dy}{dt}, v_z = \frac{dz}{dt}$$

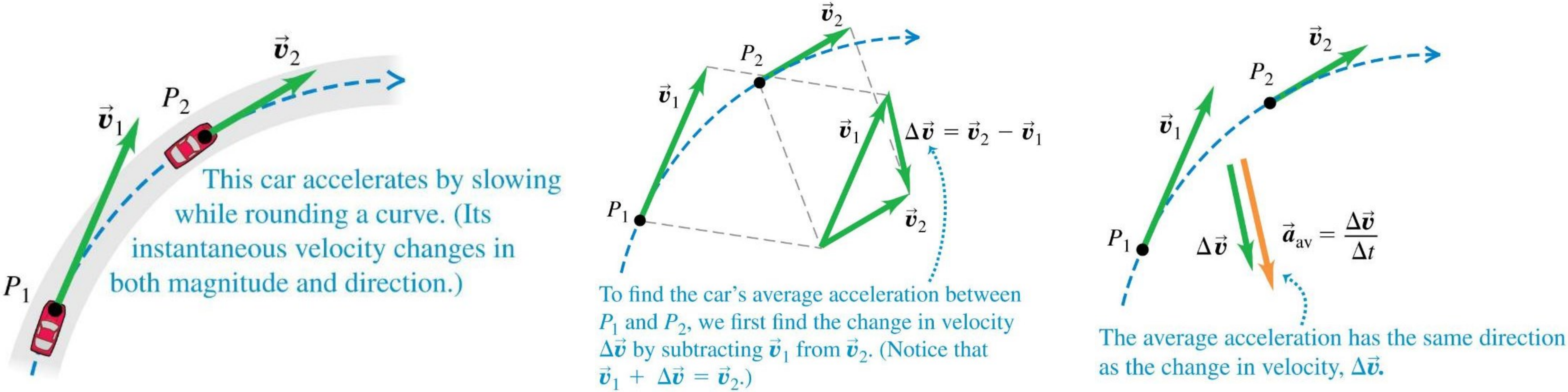
- **The instantaneous velocity of a particle is always tangent to its path.**



Average acceleration

- The average acceleration: the velocity change $\Delta\vec{v}$ divided by time interval Δt

$$\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t}$$



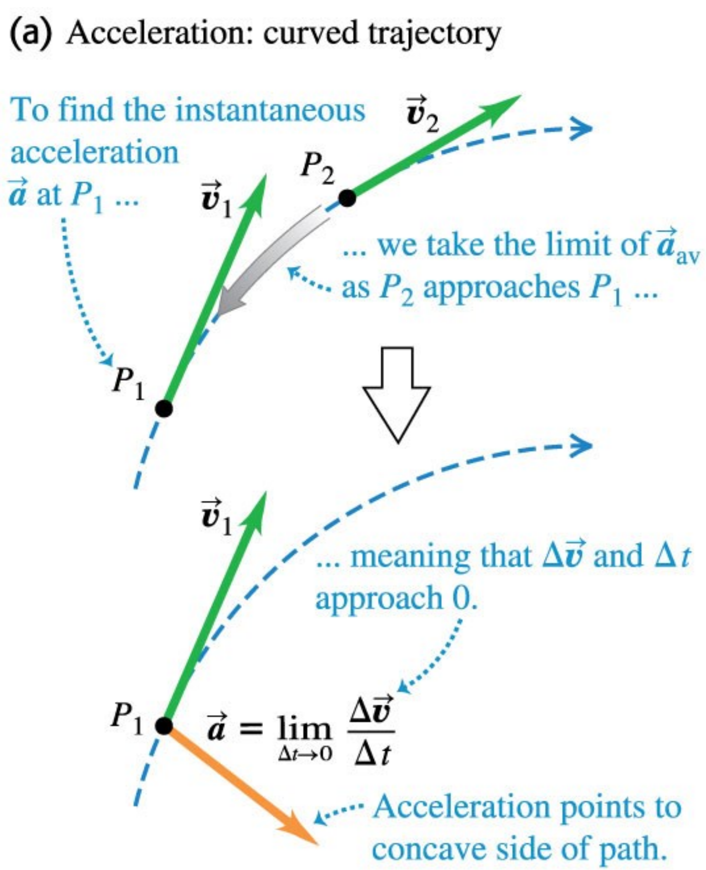
Instantaneous acceleration

- The instantaneous acceleration is the instantaneous rate of change of the velocity with respect to time.

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

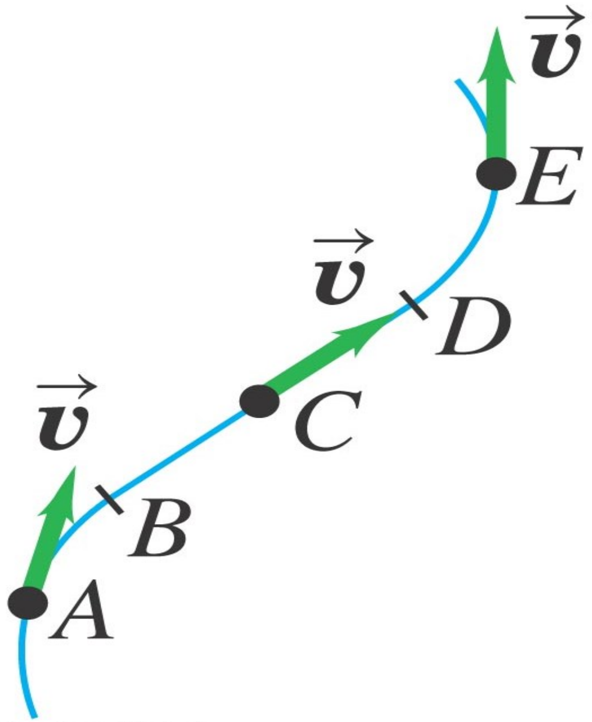
- *Any particle following a curved path is accelerating, even if it has constant speed.*
- The component of the instantaneous acceleration are

$$a_x = \frac{dv_x}{dt},\ a_y = \frac{dv_y}{dt},\ a_z = \frac{dv_z}{dt}$$



Example:

The motion diagram shows an object moving along a curved path at constant speed. At which of the points *A*, *C*, and *E* does the object have *zero* acceleration?



A. point *A* only

B. point *C* only ✓

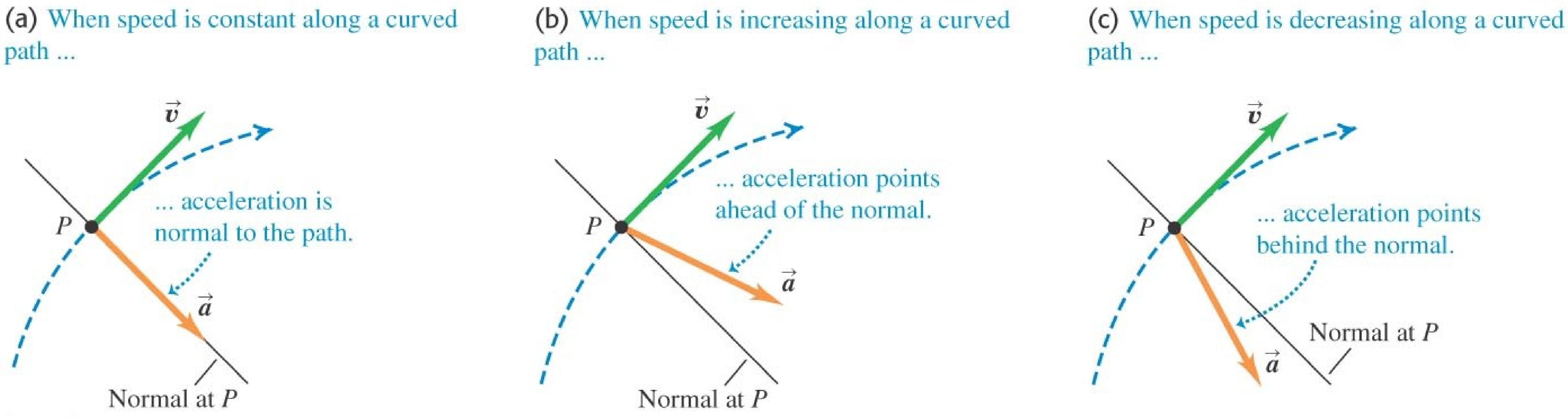
C. point *E* only

D. points *A* and *C* only

E. points *A*, *C*, and *E*

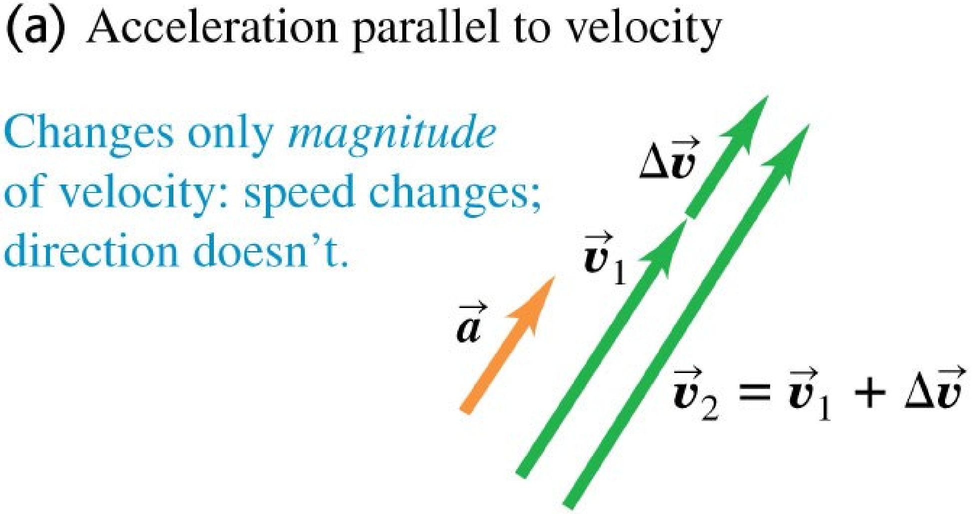
Direction of the acceleration vector

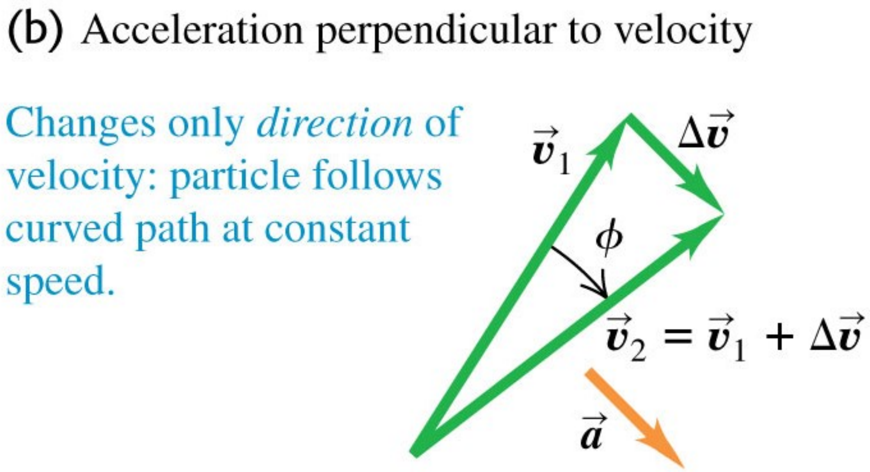
- The direction of the acceleration vector depends on whether the speed is constant, increasing, or decreasing.

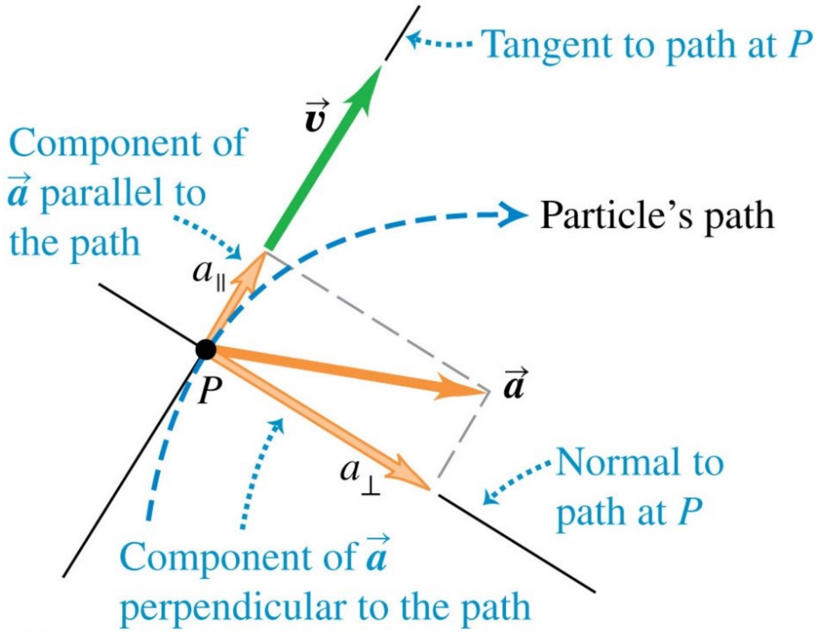


Direction of the acceleration vector

- Another useful way to think about instantaneous acceleration is in terms of **component parallel** or **perpendicular** to the velocity.
- **Parallel component** tells us about changes in the particle's **magnitude of velocity**.
- **Perpendicular component** tells us about changes in the particle's **direction of motion**.

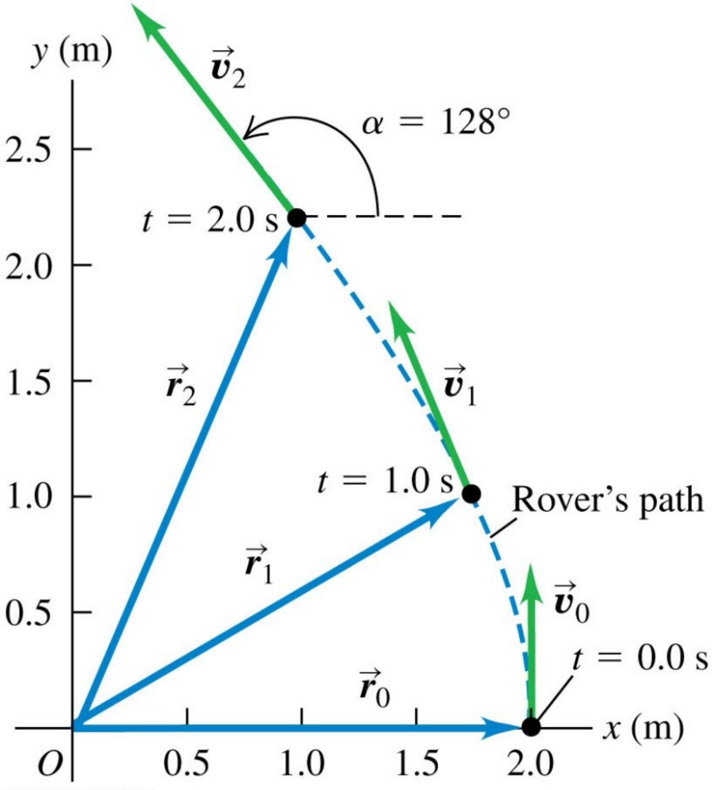






Example:

This illustration shows the path of a robotic vehicle, or rover. What is the direction of the rover's average acceleration vector for the time interval from $t = 0.0$ s to $t = 2.0$ s?



A. up and to the left ✓

B. up and to the right

C. down and to the left

D. down and to the right

E. none of the above

Example: Calculating Average Velocity

A rover is exploring the surface of Mars, which we represent as a point. It has *x*- and *y*-coordinates that vary with time:

$$x = 2.0\,m - (0.25 m/s^2)t^2$$

$$y = (1.0\,m/s)t + (0.025\,m/s^3)t^3$$

Find the displacement and average velocity vectors for the interval $t = 0.0$ s to $t = 2.0$ s.

Q1: Definition of displacement and average velocity?

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} \qquad \Delta\vec{r} = \vec{r}_2 - \vec{r}_0 = (0.0\,m)\hat{\imath} + (0.2\,m)\hat{\jmath}$$

Q2: Vectors of $\vec{r}_0$ and $\vec{r}_2$? $\vec{r}_0 = (2.0\,m)\hat{\imath} + (0.0\,m)\hat{\jmath}$ $\vec{r}_2 = (1.0\,m)\hat{\imath} + (2.2\,m)\hat{\jmath}$

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} = \frac{(-1.0\,m)\hat{\imath} + (2.2\,m)\hat{\jmath}}{2\,s\ - 0\,s} = (-0.5\,m/s)\,\hat{\imath} + (1.1\,m/s)\,\hat{\jmath}$$

Example: Calculating Instantaneous Velocity

- Find a general expression for the rover's instantaneous velocity vector $\vec{v}$.

$$x = 2.0\,m - \left(0.25 m/s^2\right)t^2$$
$$y = (1.0\,m/s)t + (0.025\,m/s^3)t^3$$

$$v_x = \frac{dx}{dt} = -\left(0.25\ m/s^2\right)(2t)$$

$$v_y = \frac{dy}{dt} = 1.0\ m/s + \left(0.025\ m/s^3\right)(3t^2)$$

$$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}$$

Example: Calculating Instantaneous Velocity

- What is the rover's instantaneous velocity at $t = 2$ s (magnitude and direction, angle to $+x$)?

$$x = 2.0\, m - (0.25 m/s^2)t^2$$
$$y = (1.0\, m/s)t + (0.025\, m/s^3)t^3$$

$$v_x = \frac{dx}{dt} = -(0.25\ m/s^2)(2t)$$

$$v_y = \frac{dy}{dt} = 1.0\ m/s + (0.025\ m/s^3)(3t^2)$$

$$\vec{v}(t = 2\ s) = (-1.0\, m/s)\hat{\imath} + (1.3\, m/s)\hat{\jmath}$$

$$|v| = \sqrt{(-1.0\, m/s)^2 + (1.3\, m/s)^2} = 1.64 \text{ m/s}$$

$$tan\theta = v_y/v_x = -1.3$$

$$\theta = 128°$$