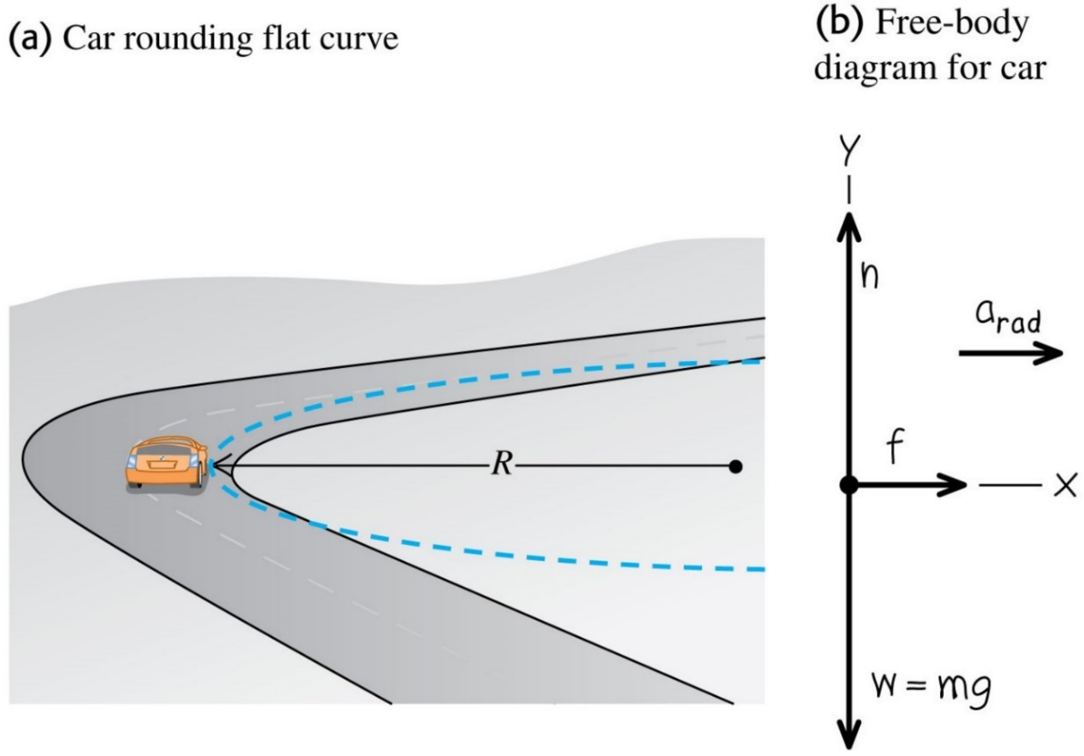
Example: A car rounds a flat curve

A car rounds a flat unbanked curve (radius ***R***), coefficient of **static friction** between tires and road is $\boldsymbol{\mu_s}$**.** What is its **maximum speed**?



$$\sum F_x = f = ma_{rad} = m\frac{v^2}{R}$$

$$\sum F_y = n + (-mg) = 0$$

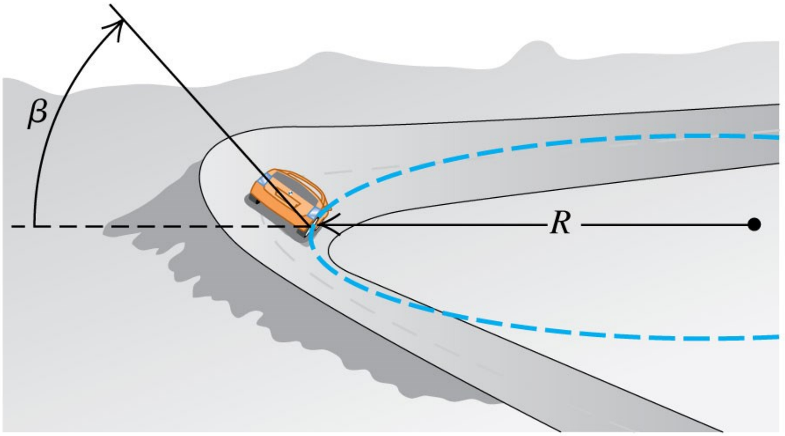
$$f_{max} = \mu_s n = \mu_s mg$$

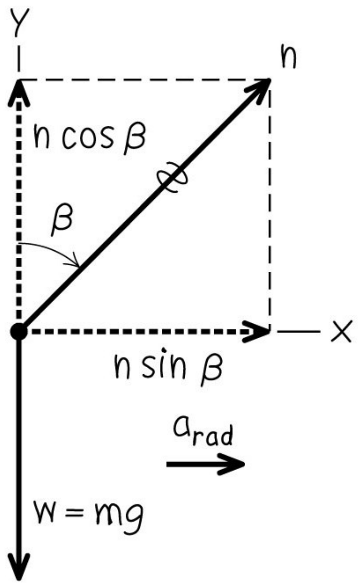
$$\mu_s mg = m\frac{v_{max}^2}{R} \qquad v_{max} = \sqrt{\mu_s gR}$$

What will happen if the speed is larger than v_{max}?

Example: A car rounds a banked curve

- At what angle should a curve be banked so a car can make the turn even with **no friction**?





$$\sum F_x = n\ sin\beta = ma_{rad}$$

$$\sum F_y = n\ cos\beta + (-mg) = 0$$

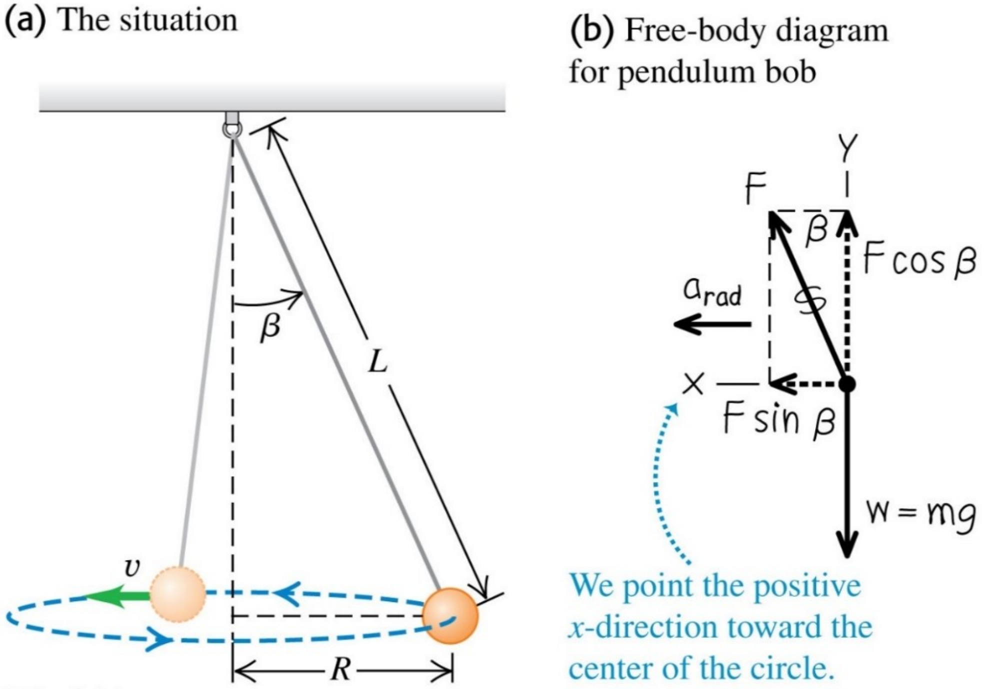
n= $mg/cos\beta$, so $(sin\beta\ mg)/cos\beta = ma_{rad}$

$$a_{rad} = g\ tan\beta \qquad a_{rad} = \frac{v^2}{R}$$

$$tan\beta = \frac{a_{rad}}{g} = \frac{v^2}{gR} \qquad \beta = \arctan(\frac{v^2}{gR})$$

Example: A conical pendulum

A pendulum clock has a bob with mass ***m*** at the end of a thin wire of length ***L***. Instead of swinging back and forth, the bob is to move in a **horizontal circle** with **constant speed *v***, with the wire making a fixed **angle *β*** with the vertical direction. Find the tension ***F*** in the wire and the period ***T*** (the time for one revolution of the bob.)



$$\sum F_x = F\,sin\beta = ma_{rad} \qquad \sum F_y = F\,cos\beta + (-mg) = 0$$

$$F = mg/cos\beta, \text{ so } (sin\beta\, mg)/cos\beta = ma_{rad}$$

$$a_{rad} = g\,tan\beta \qquad a_{rad} = \frac{4\pi^2 R}{T^2}, \text{ so } T^2 = \frac{4\pi^2 R}{a_{rad}}$$

$$T = 2\pi\sqrt{\frac{R}{g\,tan\beta}} \qquad R = L\,sin\beta \qquad T = 2\pi\sqrt{\frac{L\,cos\beta}{g}}$$

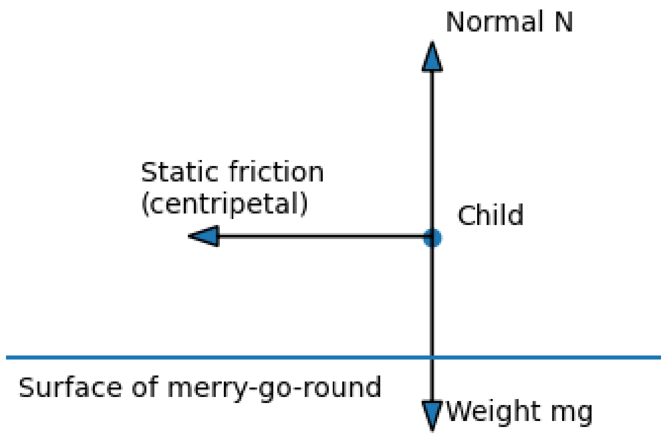
Examples:

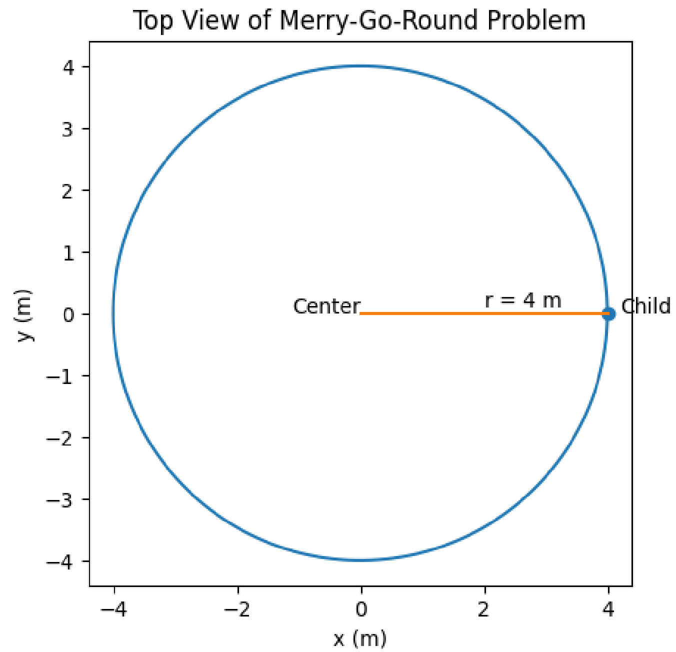
A child is on the edge of a playground merry-go-round (radius 4.00 m). The coefficient of static friction between the child and the surface of the merry-go-round is 0.3, what is the minimum time for one revolution if the child is not to slide off?

$$\sum F_x = f_{max} = m a_{rad} = m\frac{v^2}{R}$$

$$f_{max} = \mu_s n = \mu_s m g$$

$$\mu_s m g = m\frac{v_{max}^2}{R}$$

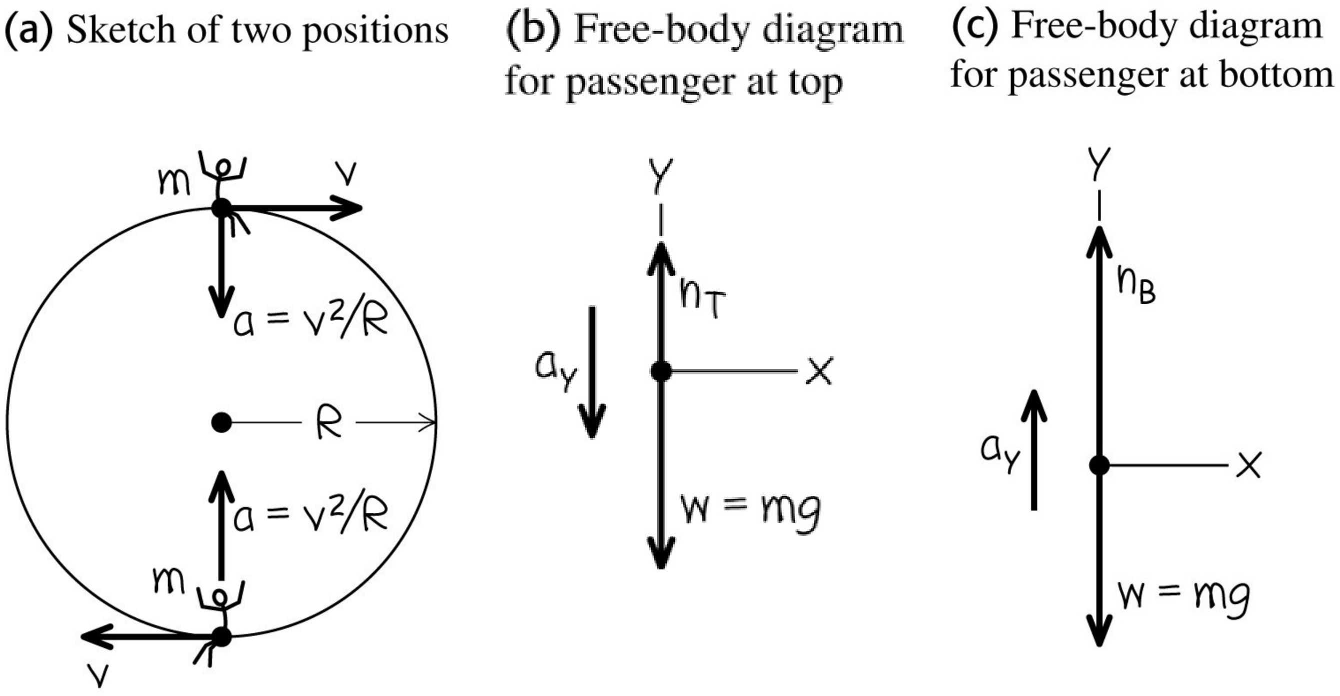




$$v_{max} = \sqrt{\mu_s g R} = 3.4 \qquad T = \frac{2\pi R}{v_{max}} = 7.32\ s$$

Example: motion in a vertical circle

A passenger on a wheel moves in a vertical circle. Radium is ***R***, **constant speed *v***. Find the expressions for the **force** the wheel exerts on the passenger at the top of the circle and at the bottom.



Top $a_y = -v^2/R$

$$\sum F_y = n_T + (-mg) = -mv^2/R$$

$$n_T = mg\left(1 - \frac{v^2}{gR}\right)$$

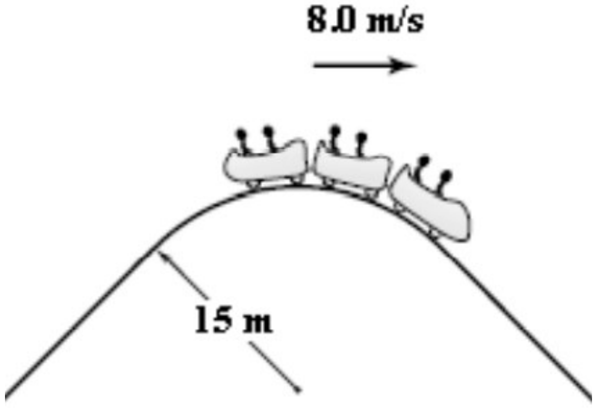
Bottom $a_y = +v^2/R$

$$\sum F_y = n_B + (-mg) = +mv^2/R$$

$$n_B = mg\left(1 + \frac{v^2}{gR}\right)$$

Examples:

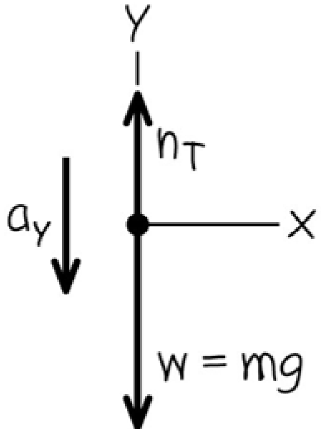
A roller coaster track has a hill with a circular curve of radius r = 15 m. What is the normal force on the 60-kg passenger when the roller coaster passes the hill at 8 m/s?



$$\sum F_y = n_T + (-mg) = ma_y \qquad a_y = -v^2/R$$

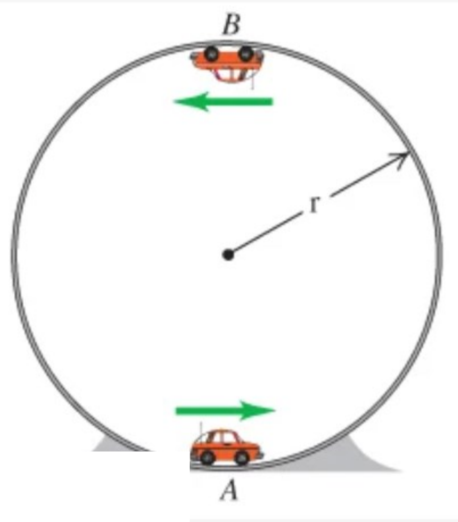
$$\sum F_y = n_T + (-mg) = -mv^2/R$$

$$n_T = mg\left(1 - v^2/gR\right) = 332\ N$$



Examples:

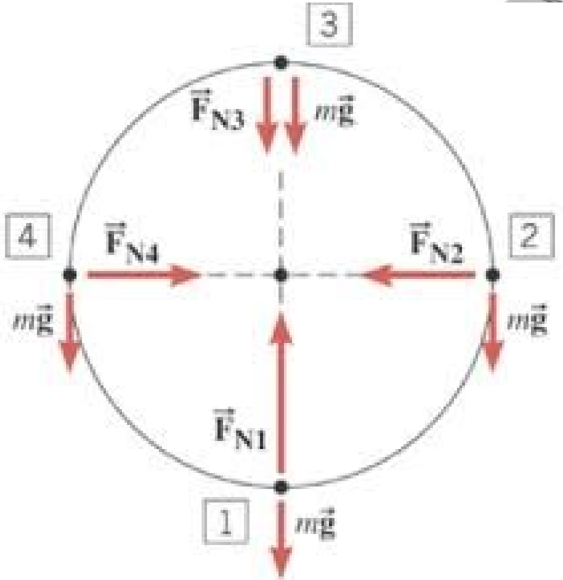
A car on a vertical circle. Radium is R, constant speed v. Find the expressions for the force the car exerts on the circle at the top and at the bottom.



Top $\sum F_y = -n_T + (-mg) = ma_y$ $\quad a_y = -v^2/R$

$$-n_T + (-mg) = -mv^2/R$$

$$n_T = mg\left(v^2/gR - 1\right)$$



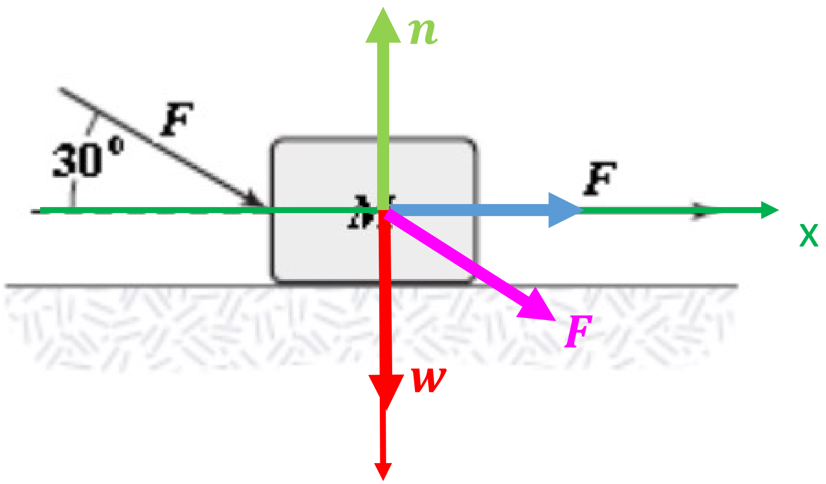
Bottom $\sum F_y = n_B + (-mg) = ma_y$ $\quad a_y = +v^2/R$

$$n_B + (-mg) = +mv^2/R$$

$$n_B = mg\left(1 + v^2/gR\right)$$

Examples:

The horizontal surface on which the block slides is frictionless. If F = 20 N and M = 5.0 kg, what is the magnitude of the resulting acceleration of the block?



$$\sum F_x = F + Fcos30 = 20 + 20cos30 = 37.32 = Ma_x$$

$$a_x = \frac{37.32}{5} = 7.46$$

$$\sum F_y = Mg + Fsin30 - n = 0$$

Examples:

At an instant when a 4.0-kg object has an acceleration equal to $(5\hat{\imath} + 3\hat{\jmath})$ m/s^2, one of the two forces acting on the object is known to be $(12\hat{\imath} + 22\hat{\jmath})$ N. Determine the magnitude of the other force acting on the object.

$$\vec{F}_{net} = m\vec{a} = 4 \times (5\vec{\imath} + 3\vec{\jmath}) = 20\vec{\imath} + 12\vec{\jmath}$$

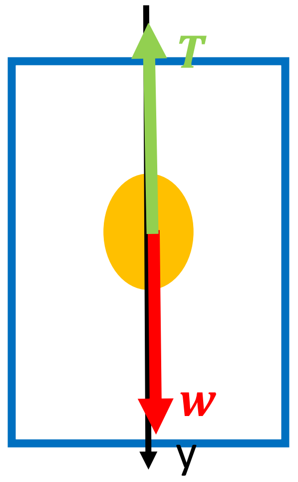
$$\vec{F}_{net} = \vec{F}1 + \vec{F}2$$

$$\vec{F}2 = \vec{F}_{net} - \vec{F}1 = (20\vec{\imath} + 12\vec{\jmath}) - (12\vec{\imath} + 22\vec{\jmath}) = 8\vec{\imath} - 10\vec{\jmath}$$

$$F = \sqrt{8^2 + 10^2} = 12.81 \text{ N}$$

Examples:

The tension is a string from which a 4.0-kg object is suspended in an elevator is equal to 28 N. What is the acceleration of the elevator?



$$\sum F_y = w - T = 4 \times 9.8 - 28 = 11.2 = ma = 4a$$

$$a = 2.8$$

Examples:

A worker lifts (constant speed) a weight w by pulling down on a rope with a force $\vec{F}$. The upper pulley is attached to the ceiling by a chain, and the lower pulley is attached to the weight by another chain. Find the tension in each chain and the magnitude of $\vec{F}$, in terms of w. Ropes, pulleys, and chains have negligible weights.