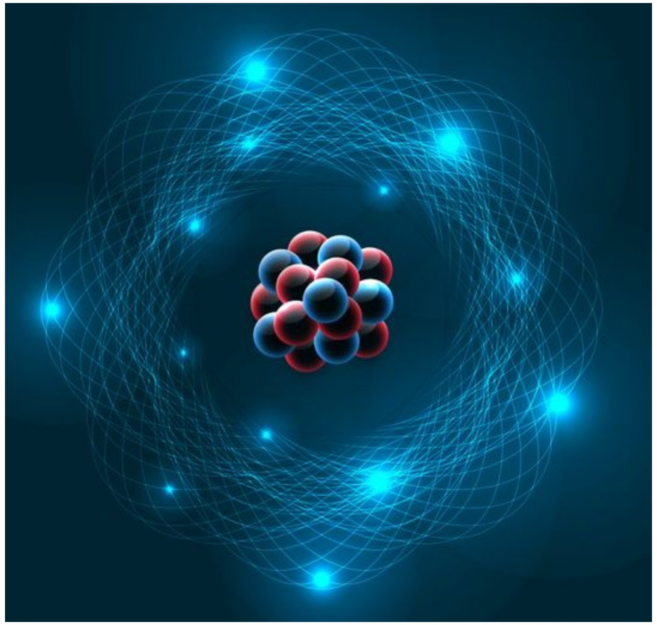
Physics 111: Mechanics Chapter05



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Example

An object with mass $m = 2$ kg is moving along the x-axis according to the equation $x(t) = \alpha t^3 - 2\beta t$, where $\alpha = 3$ and $\beta = 2$. What is the magnitude of the net force on the object at time $t = 0$ and $t = 2$ s?

$v_x = dx/dt$ $\qquad a_x = dv_x/dt.$ $\qquad \sum F_x = ma_x$

$v_x = 3\alpha t^2 - 2\beta$ $\qquad a_x = 6\alpha t = 18t$

$t = 0$ $\qquad a_x = 18(t = 0) = 0$ $\qquad \sum F_x = ma_x = 0$

$t = 2$ $\qquad a_x = 18(t = 2) = 36$ $\qquad \sum F_x = ma_x = 2 \times 36 = 72$

Applications of Newton's laws

- To use Newton's first law for bodies in **equilibrium**
- To use Newton's second law for **accelerating bodies**
- To study the types of **friction** and **fluid resistance**
- To solve problems involving **circular motion**

Object in Equilibrium

- Objects that are either **at rest** or moving with **constant velocity** are said to be in **equilibrium**.
- Acceleration of an object in equilibrium: $\vec{a} = 0$
- Mathematically, the **net force** acting on the object is **zero**

$$\vec{F}_{net} = \sum \vec{F} = 0$$

- Equivalent to the set of component equations given by

$$F_{net,x} = \sum F_x = 0, \qquad F_{net,y} = \sum F_y = 0$$

Equilibrium Example

A traffic light weighing 100 N hangs from a vertical cable tied to two other cables that are fastened to a support. The upper cables make angles of 37° and 53° with the horizontal beam. Find the tension in each of the three cables.

Step 1. Draw free body diagram

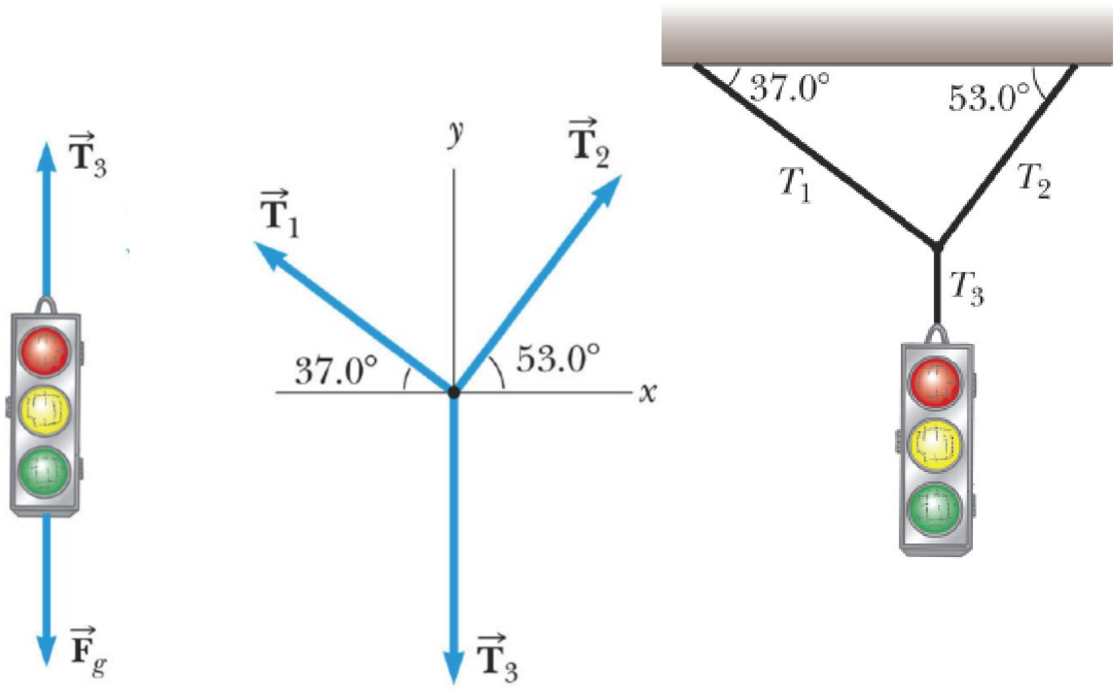
Step 2. Find net force along *x* and *y*.

Equilibrium, so $\vec{F}_{net} = \sum \vec{F} = 0$

For both objects:

$F_{net,x} = \sum F_x = 0,$

$F_{net,y} = \sum F_y = 0$



Equilibrium Example

- Apply equilibrium equations.
- The light: $\sum F_y = 0, \quad \vec{T}_3 + \vec{F}_g = 0, \quad T_3 = F_g = 100 \text{ N}$
- The knot: $\sum F_x = T_{1x} + T_{2x} = -T_1 \cos(37°) + T_2 \cos(53°) = 0$

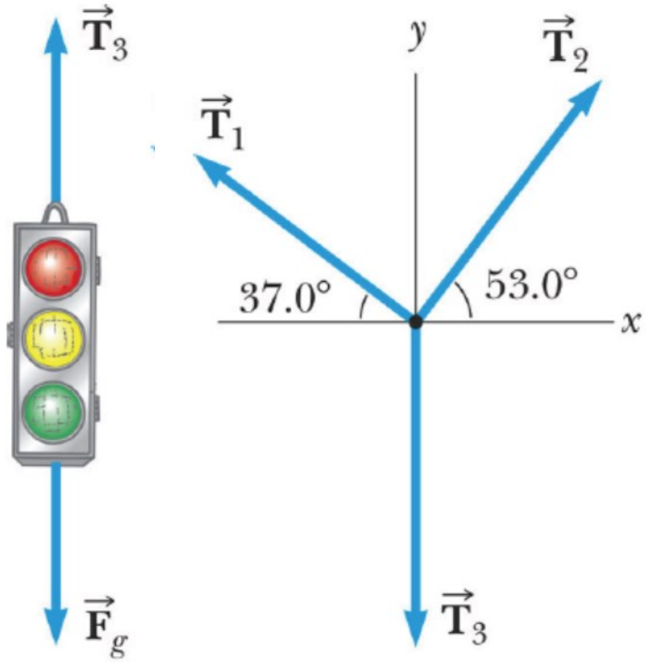
$$T_2 = T_1 \frac{\cos(37°)}{\cos(53°)} = 1.33 T_1$$

$$\sum F_y = T_{1y} + T_{2y} + T_{3y}$$

$$= T_1 \sin(37°) + T_2 \sin(53°) - 100 \text{ N}$$

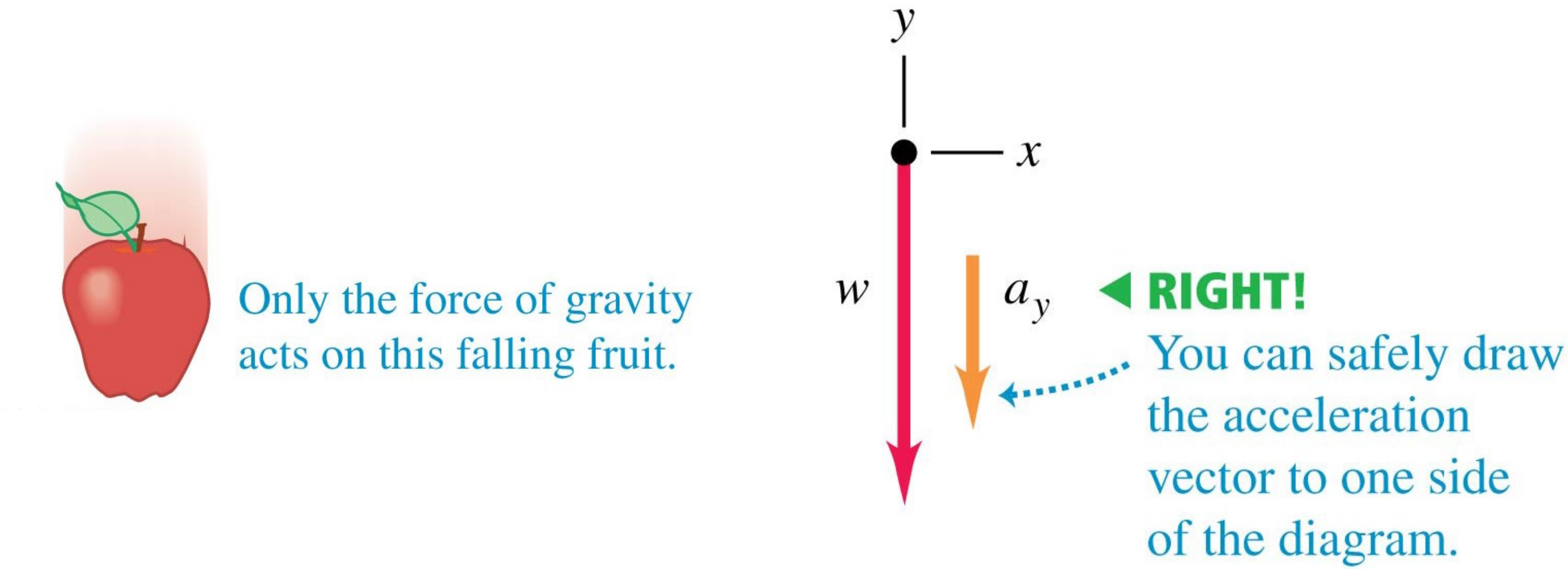
$$= T_1 \sin(37°) + 1.33 T_1 \sin(53°) - 100 \text{ N} = 0 \qquad T_1 = 60 \text{ N}$$

$$T_2 = 1.33 T_1 = 80 \text{ N}$$



Accelerating Objects

- An object has an acceleration: there must be a **nonzero net force** acting on it.
- Important first step: Draw a free-body diagram
- Apply Newton's Second law in component form



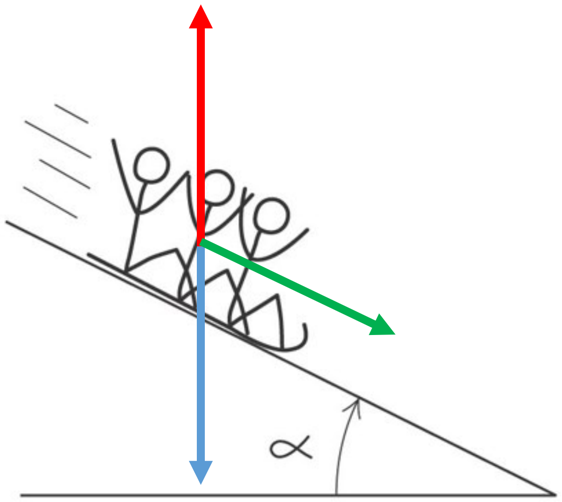
$$\sum \vec{F} = m\vec{a}$$

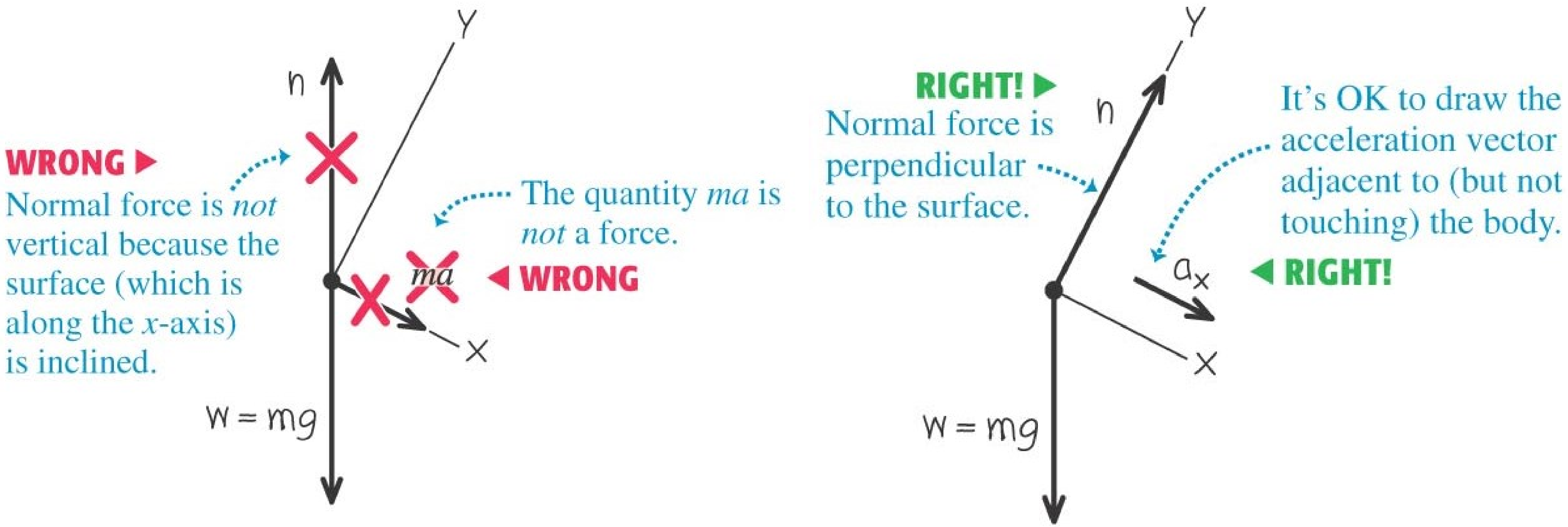
$$\sum F_x = ma_x$$

$$\sum F_y = ma_y$$

Two common free-body diagram errors

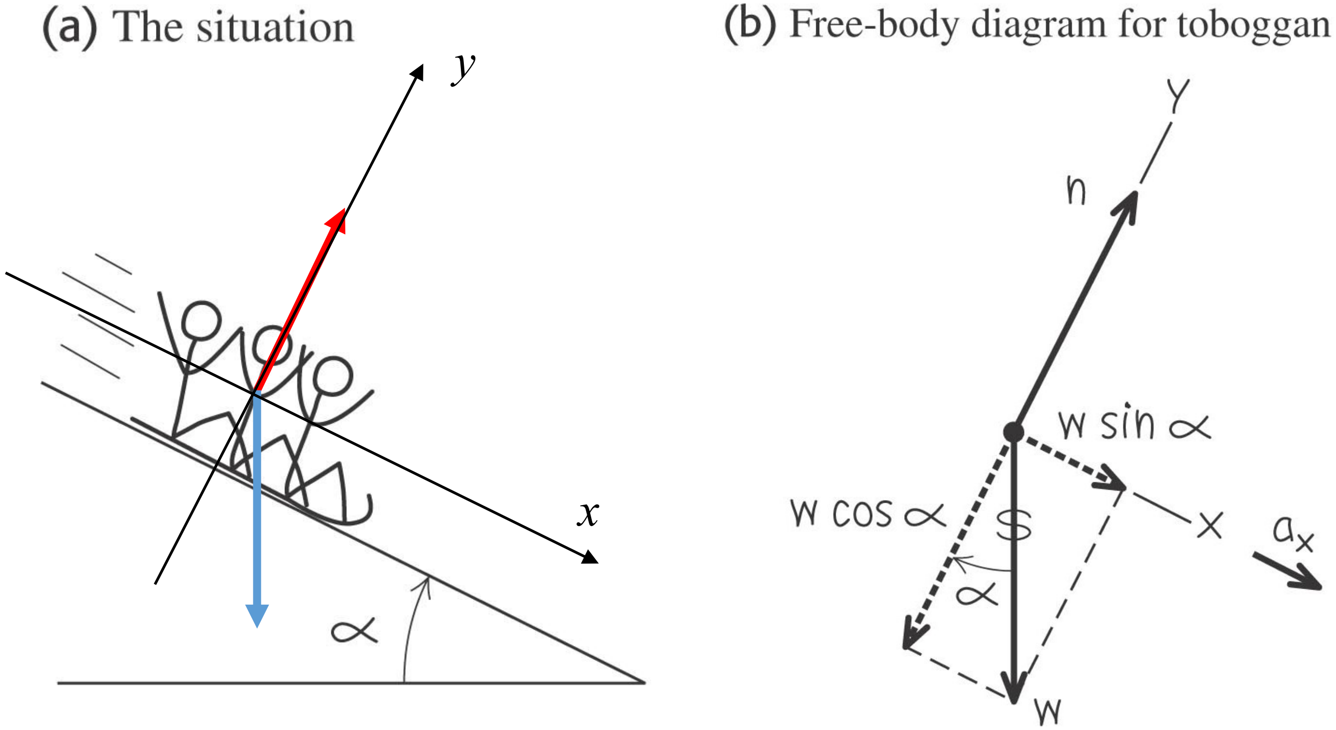
- The normal force must be perpendicular to the surface.
- There is no “*ma* force.”





Example: Acceleration down the hill

- Consider the toboggan on a steeper hill with no friction. It is accelerating. What is the acceleration if $\alpha = 30$ degrees? (Mass is missing?)



$$\sum F_x = w\ sin\alpha = mg\ sin\alpha = ma_x$$

$$a_x = g\ sin\alpha$$

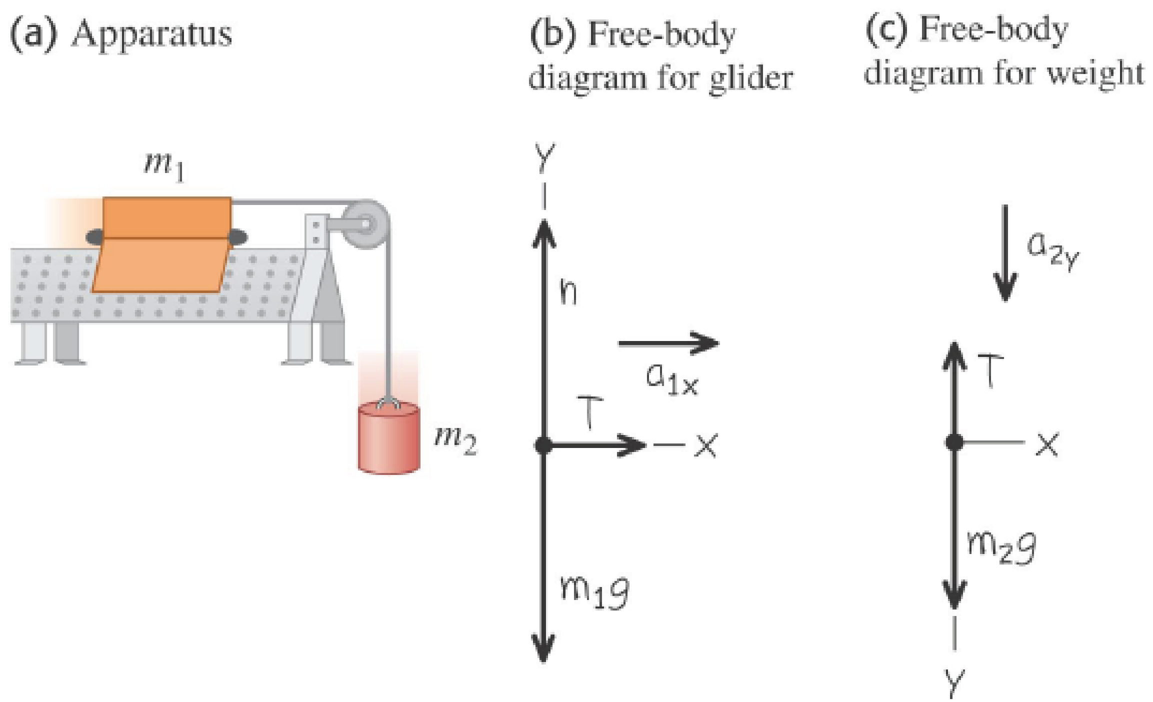
$$a_x = (9.8)\sin(30°) = 4.9$$

$$\sum F_y = n - w\ cos\alpha = ma_y = 0$$

$$n = w\ cos\alpha = mg\ cos\alpha$$

Two Connected Objects

The glider (m_1 = 1 kg) on the air track and the falling weight (m_2 = 1 kg) move in different direction, but their accelerations have the same magnitude. Find the acceleration and tension in the string.



Glider: $\sum F_x = T = m_1 a_{1x} = m_1 a$

Glider: $\sum F_y = n + (-m_1 g) = m_1 a_{1y} = 0$

Lab weight: $\sum F_y = m_2 g + (-T) = m_2 a_{2y} = m_2 a$

Glider: $T = m_1 a$

Lab weight: $m_2 g - T = m_2 a$

Acceleration

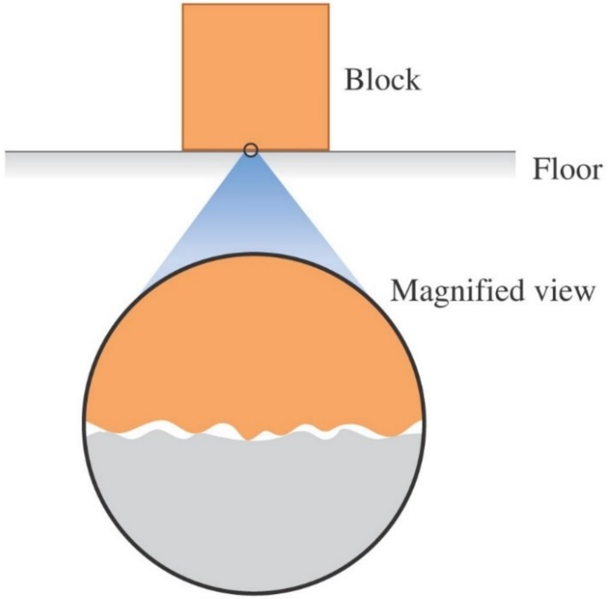
$$a = \frac{m_2}{m_1 + m_2} g$$

Tension

$$T = \frac{m_1 m_2}{m_1 + m_2} g$$

Frictional forces

- A body rests or slides on a surface: the *friction force* is parallel to the surface.
- Friction between two surfaces arises from interactions between molecules on the surfaces.

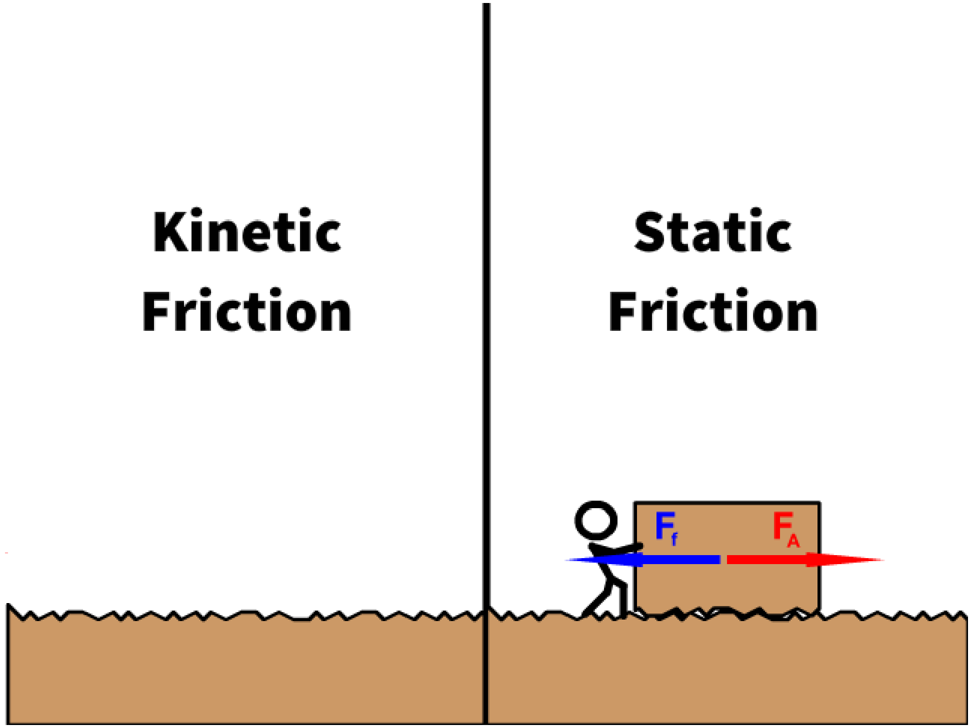


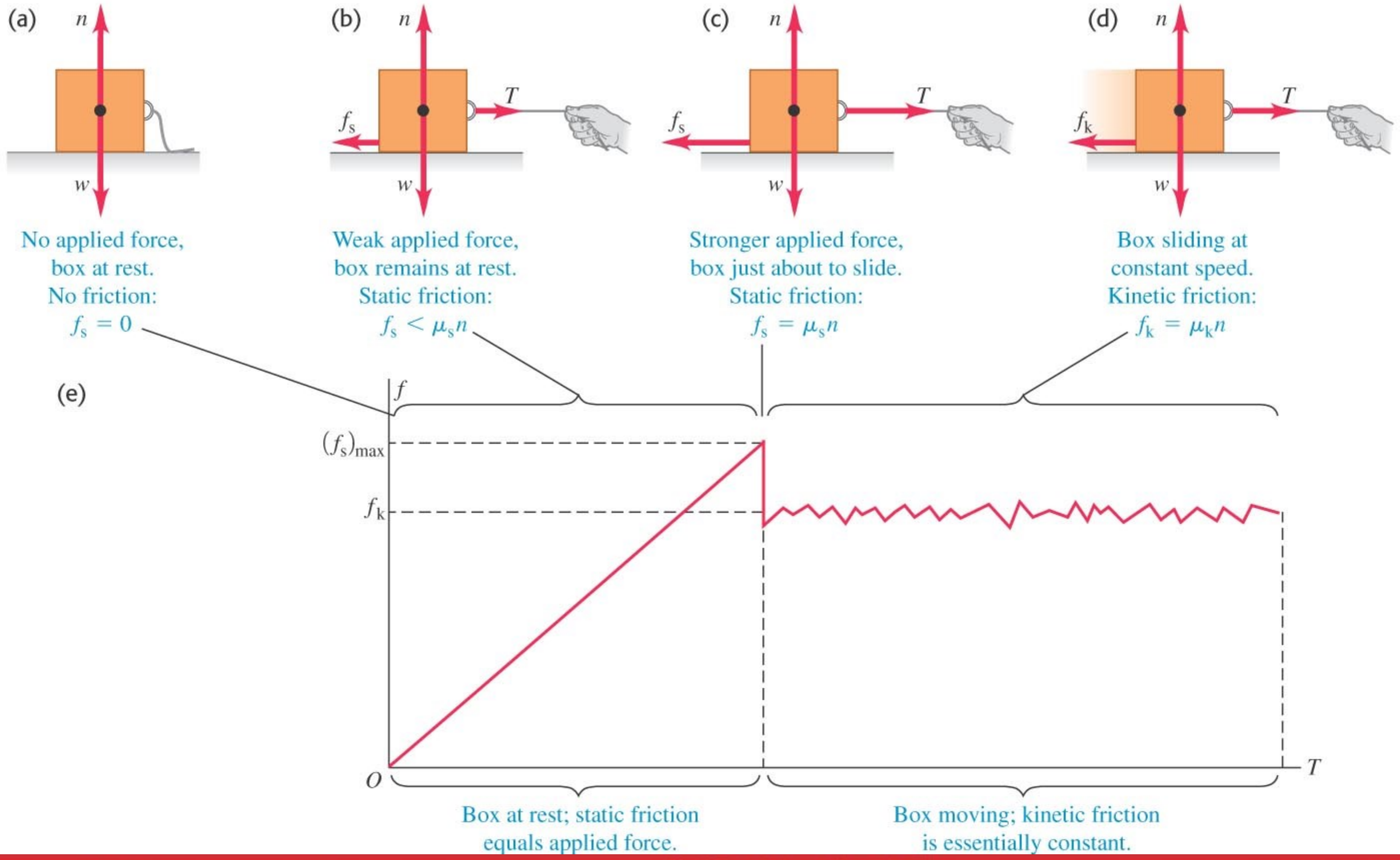
On a microscopic level, even smooth surfaces are rough; they tend to catch and cling.

Kinetic and static friction

- *Kinetic friction* acts when a body slides over a surface.
- The *kinetic friction force* is $f_k = \mu_k \boldsymbol{n}$.

- *Static friction* acts when there is no relative motion between bodies.
- The *static friction force* can **vary** between zero and its maximum value: $f_s \leq \mu_s \boldsymbol{n}$.





Before the box slides, static friction acts.

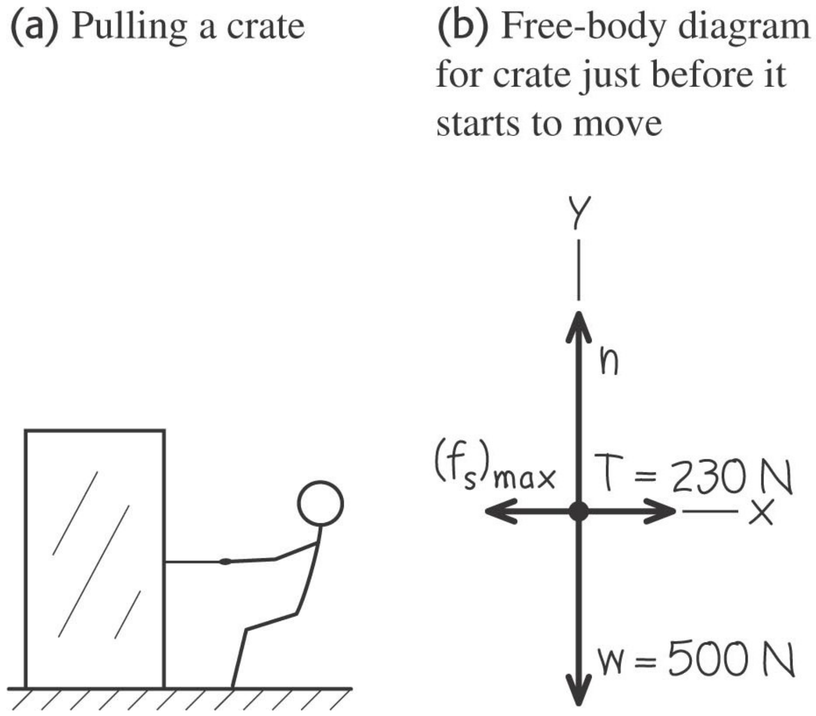
$$f_s \leq \mu_s \boldsymbol{n}$$

But once it starts to slide, kinetic friction acts.

$$f_k = \mu_k \boldsymbol{n}.$$

Example: Friction in horizontal motion

You want to move a **500 N** crate. To start the crate moving, you have to pull with a **230 N** horizontal force. What are the **coefficients** of **static** friction?



Just before the crate starts to move, we have

$$\sum F_x = T + (-(f_s)_{max}) = 0$$

So $(f_s)_{max} = T = 230$ N

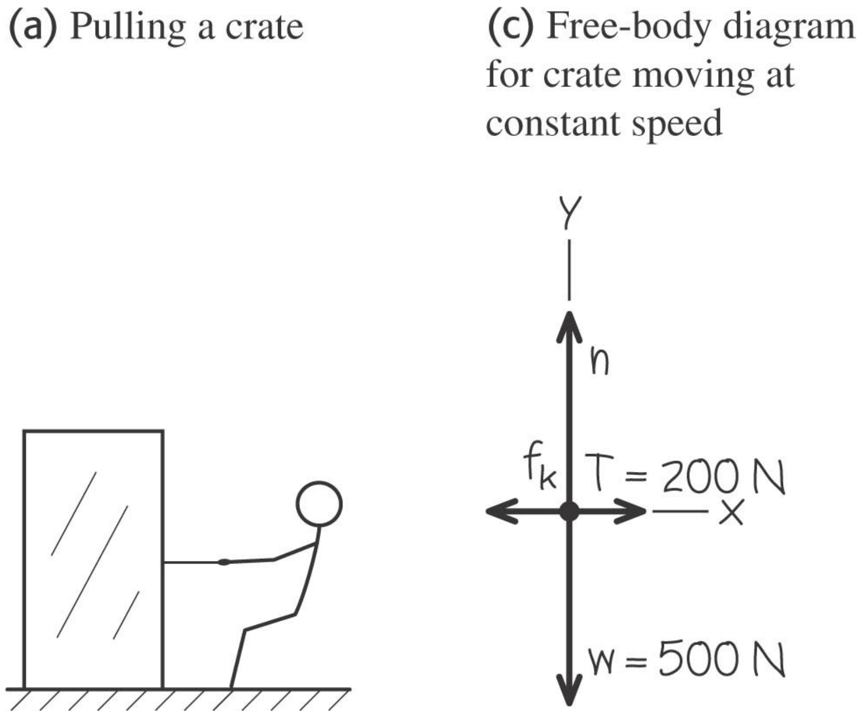
$$\sum F_y = n + (-w) = 0$$

So $n = w = 500$ N

$(f_s)_{max} = \mu_s n$, so $\mu_s = (f_s)_{max}/n = 230\ N/500\ N = 0.46$

Example: Friction in horizontal motion

You want to move a **500 N** crate. Once the crate starts to move, you can keep it moving at constant velocity with only **200 N**. What are the **coefficients** of **kinetic** friction?



After the crate starts to move, we have

$$\sum F_x = T + (-f_k) = 0$$

So $f_k = T = 200$ N

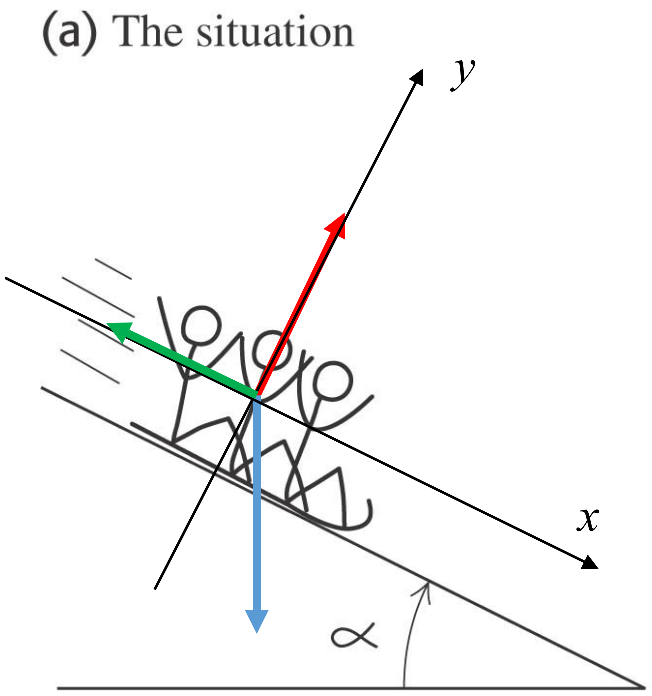
$$\sum F_y = n + (-w) = 0$$

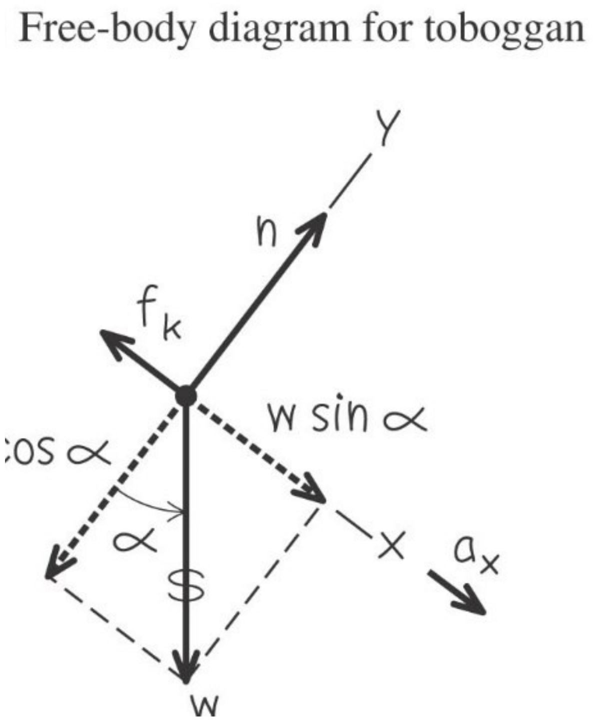
So $n = w = 500$ N

Since $f_k = \mu_k n$, so $\mu_k = f_k / n = 200N / 500\ N = 0.4$

Example: slope has friction

Consider the toboggan on a steeper hill with **friction**. It is **accelerating**. What is the **acceleration** if α=45 degrees and μ_k = 0.5?





$$\sum F_x = mg\ sin\alpha + (-f_k) = ma_x$$

$$\sum F_y = n + (-mg\ cos\alpha) = 0$$

$$n = mg\ cos\alpha \qquad f_k = \mu_k n = \mu_k mg\ cos\alpha$$

$$mg\ sin\alpha + (-\mu_k mg\ cos\alpha) = ma_x$$

$$a_x = g(sin\alpha - \mu_k\ cos\alpha)$$

$$a_x = (9.8)(\sqrt{2}/2 - 0.5\sqrt{2}/2)$$

Example:

The system shown is on verge of starting moving up the incline. Each block weighs 20 N and an incline angle is 28º. The coefficient of static friction between the block M and the incline is closest to which value?

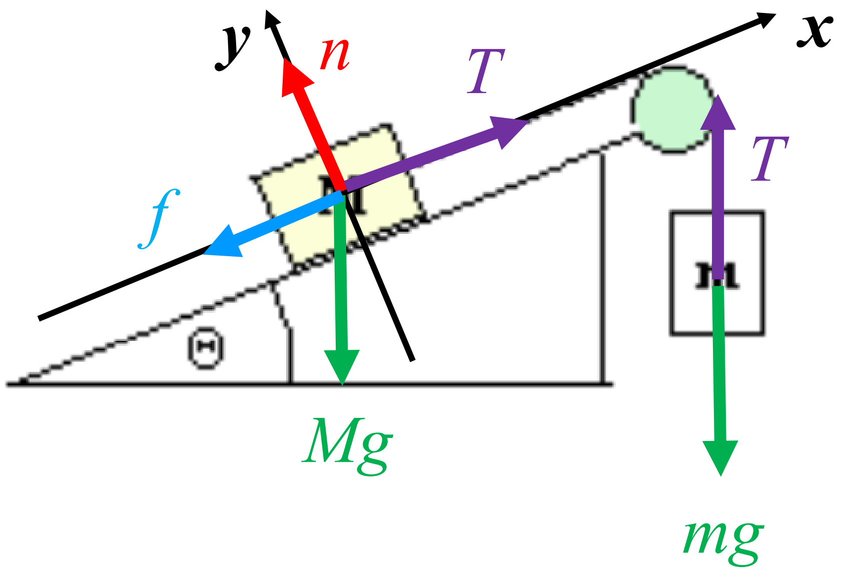
$$\sum F_x = (-Mgsin\theta) + (-f_s) + T = Ma_x = 0$$

$f_s = \mu_s n = T - Mgsin\theta$ $\qquad$ $T = mg = 20\ N$

$$f_s = \mu_s n = 20\ N - 20sin28° = 10.6\ N$$

$$\sum F_y = n + (-Mgcos\theta) = Ma_y = 0$$

$n = Mgcos\theta = 17.7\ N$ $\qquad$ $17.7\mu_s = 10.6$ $\qquad$ $\mu_s = 0.6$

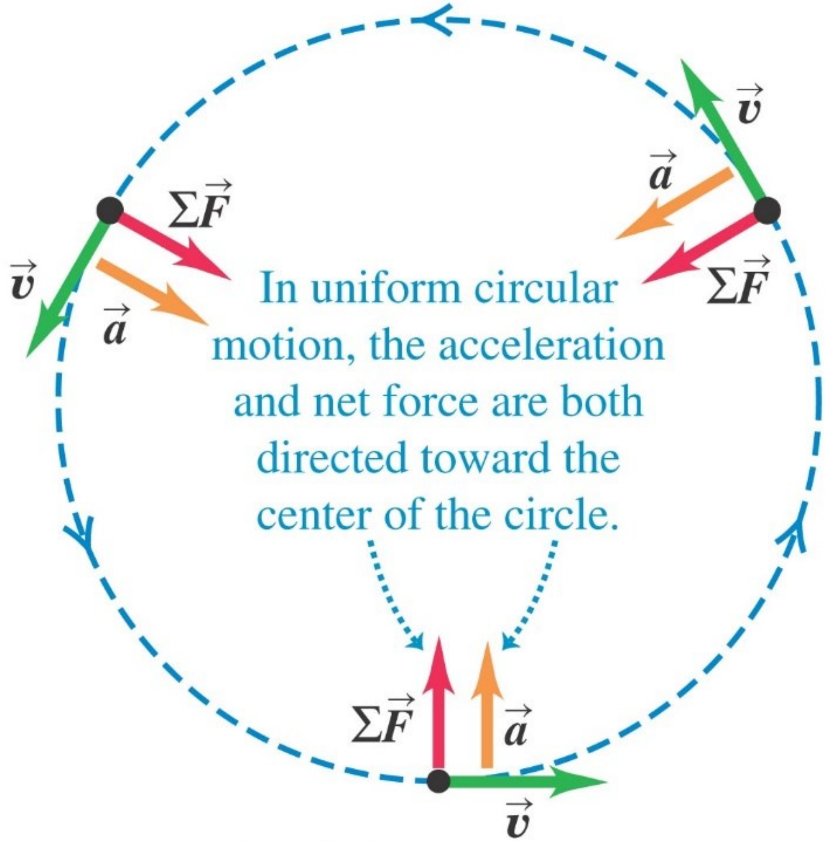


Dynamics of circular motion

- If a particle is in uniform circular motion, both its acceleration and the **net force** on it are directed **toward the center** of the circle.

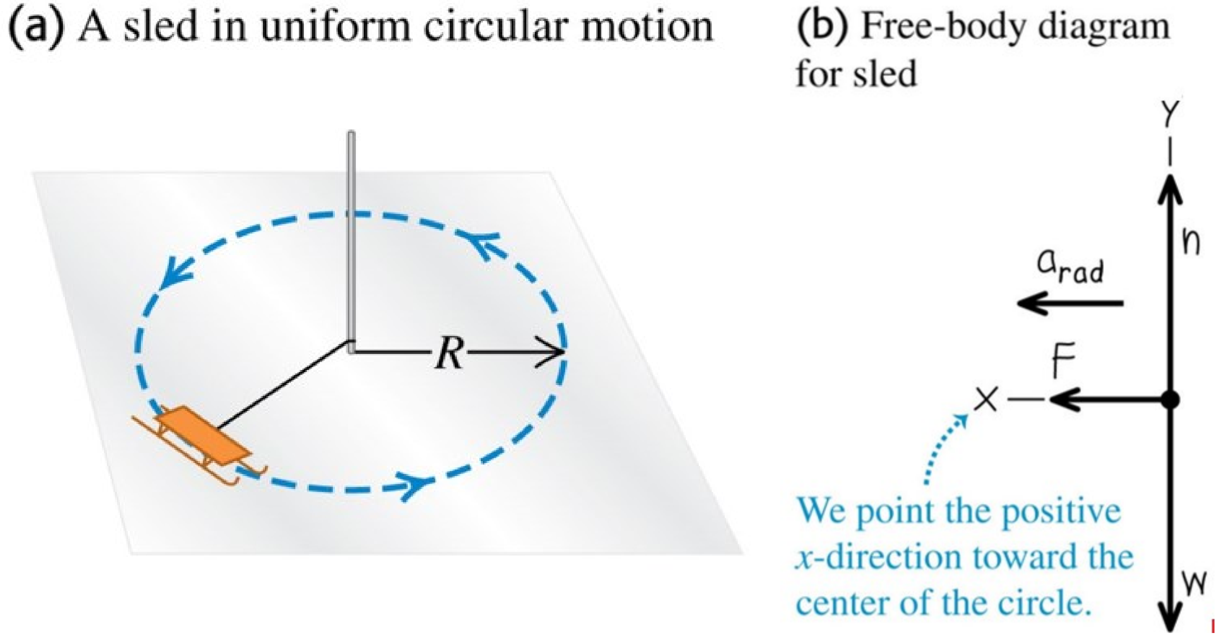
- The net force on the particle is $F_{net} = mv^2/R$.
- You know v and R, so you can get period T

$$a_{rad} = \frac{v^2}{R} = \frac{4\pi^2 R}{T^2} \qquad T = \frac{2\pi R}{v}$$



Example: circular motion

A sled with a mass of 25 kg rests on a horizontal sheet of **frictionless** ice. It is attached by a **5 m** rope to a post set in the ice. Once given a push, the sled **revolves uniformly** in a **circle** around the post. If the sled makes **five complete revolutions every minute**, find the force ***F*** exerted on it by the rope.



$$\sum F_x = F = ma_{rad}$$

$$a_{rad} = \frac{4\pi^2 R}{T^2} = \frac{4\pi^2(5.00\ m)}{(12.0\ s)^2} = 1.37\ m/s^2$$

$$F = ma_{rad} = (25.0\ kg)\left(1.37\frac{m}{s^2}\right) = 34.3\ \text{N}$$