

Example: Work and Kinetic Energy

A soccer ball with mass **0.420 kg** is initially moving with speed **2.00 m/s**. A soccer player kicks the ball, exerting a **constant force** of magnitude **40.0 N** in the same direction as the ball's motion. Over what **distance** must the player's foot be in contact with the ball to increase the ball's speed to **6.00 m/s**?

$$W_F = (F \cos \phi)s$$

$$W_{\text{tot}} = K_2 - K_1$$

$$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(0.420 \text{ kg})(2.00 \text{ m/s})^2 = 0.84 \text{ J} \quad K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(0.420 \text{ kg})(6.00 \text{ m/s})^2 = 7.56 \text{ J}$$

$$W_{\text{tot}} = K_2 - K_1 = 7.56 \text{ J} - 0.84 \text{ J} = 6.72 \text{ J}$$

$$W_F = (F \cos \phi)s$$

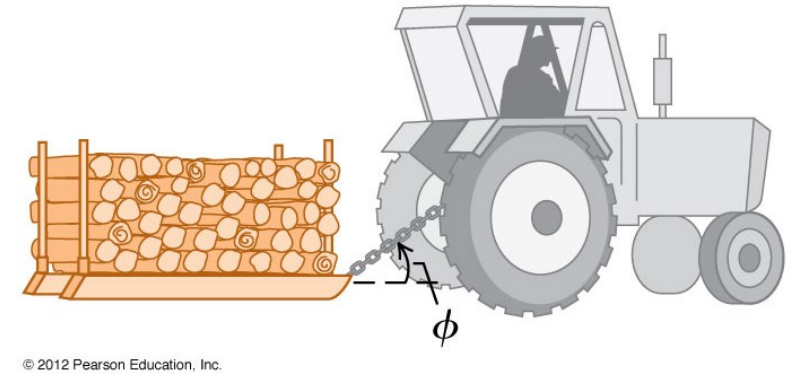
$$s = \frac{W}{F \cos \phi} = \frac{6.72 \text{ J}}{(40.0 \text{ N})(\cos 0)} = 0.168 \text{ m.}$$

Example:

A tractor driving at a **constant speed** pulls a sled loaded with firewood. There is friction between the sled and the road.

The **total work** done on the sled after it has moved a distance of 20 m is

- (A). positive.
- (B). negative.
- (C). zero.
- (D). not enough information given to decide



Example:

$$W = F\Delta x = \Delta K = \frac{1}{2}m(v^2 - v_0^2)$$

A **1.5 kg** book is sliding along a **rough** horizontal surface. At point A it is moving at **3.21 m/s**, and at point B it has slowed to **1.25 m/s**.

(A) How much total work was done on the book between A and B?

(B) If **-0.75 J** of total work is done on the book from B to C, how fast is it moving at point C?

$$W_{\text{net}} = K_B - K_A = \frac{1}{2}(1.50 \text{ kg})[(1.25 \text{ m/s})^2 - (3.21 \text{ m/s})^2] = -6.56 \text{ J.}$$

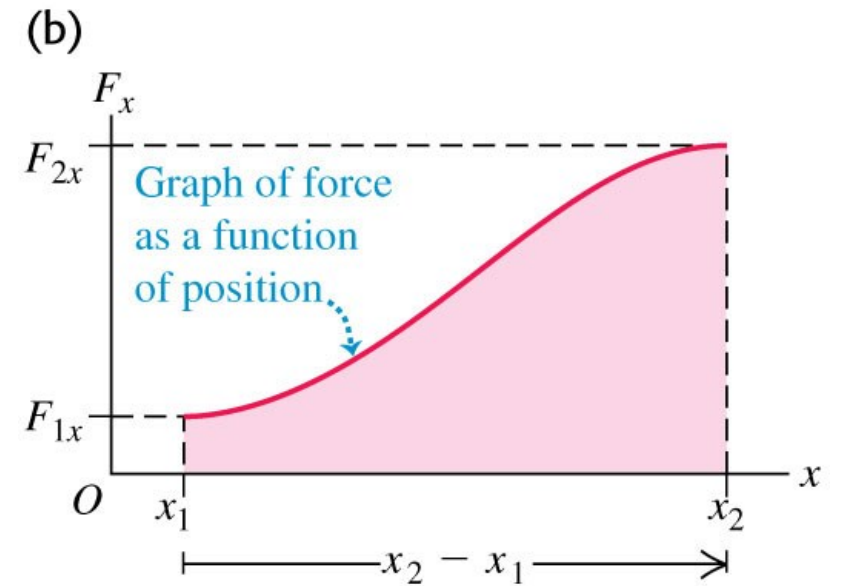
$$W_{\text{net}} = K_C - K_B \quad K_C = K_B + W_{\text{net}} = \frac{1}{2}(1.50 \text{ kg})(1.25 \text{ m/s})^2 - 0.750 \text{ J} = +0.422 \text{ J.}$$

$$K_C = \frac{1}{2}mv_C^2 \quad v_C = \sqrt{2K_C / m} = 0.750 \text{ m/s.}$$

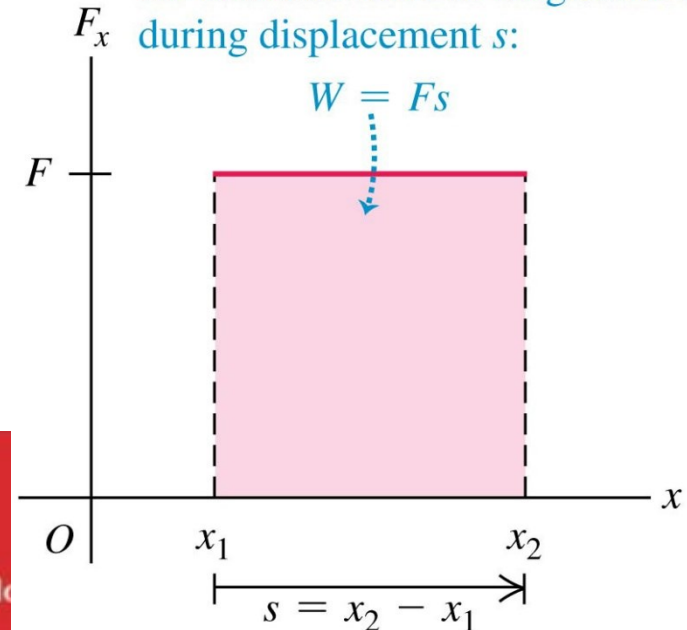
Work with Varying Forces

- Many forces are not constant.
- Work done by a varying force?
- Dividing the total displacement into many small segments.

(a) Particle moving from x_1 to x_2 in response to a changing force in the x -direction



The rectangular area under the graph represents the work done by the constant force of magnitude F during displacement s :

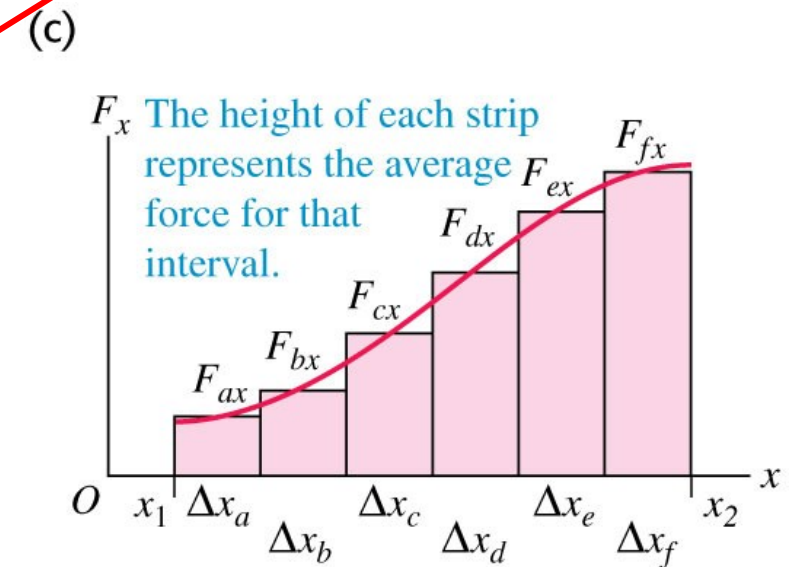
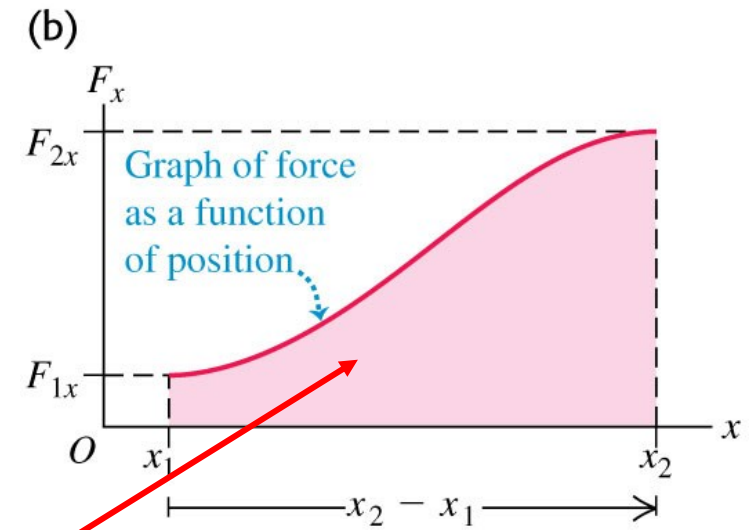


Work with Varying Forces

- On a graph of force as a function of position, the total work done by the force is represented by the area under the curve between the initial and the final position
- Straight-line motion:

$$W = F_{ax} \Delta x_a + F_{bx} \Delta x_b + \dots$$

$$W = \int_{x_1}^{x_2} F_x dx$$



Work-Energy theorem: Varying Forces

Work-energy theorem holds for varying forces as well as for constant ones.

$$W_{tot} = \int_{x_1}^{x_2} F_x dx = \int_{x_1}^{x_2} ma_x dx = \int_{x_1}^{x_2} mv_x \frac{dv_x}{dx} dx$$

$$W_{tot} = \int_{v_1}^{v_2} mv_x dv_x$$

$$W_{tot} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = \Delta K$$

1D motion

$$a_x = \frac{dv_x}{dt} = \frac{dv_x}{dx} \frac{dx}{dt} = v_x \frac{dv_x}{dx}$$

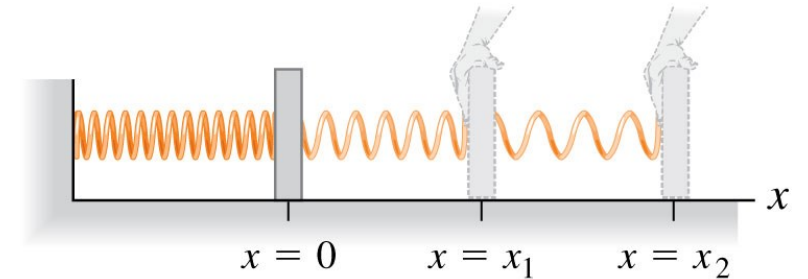
Calculate W : $\left\{ \begin{array}{l} W_{tot} = \int_{x_1}^{x_2} F_x dx \\ W_{tot} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = \Delta K \end{array} \right.$

Varying force: Stretching a spring

- The force required to stretch a spring a distance x is proportional to x : $F_x = kx$. (Hooke's law) k is the *spring constant*.
- To stretch a spring, we must do work
- We apply equal and opposite forces to the ends of the spring and gradually increase the forces
- The work we must do to stretch the spring from x_1 to x_2

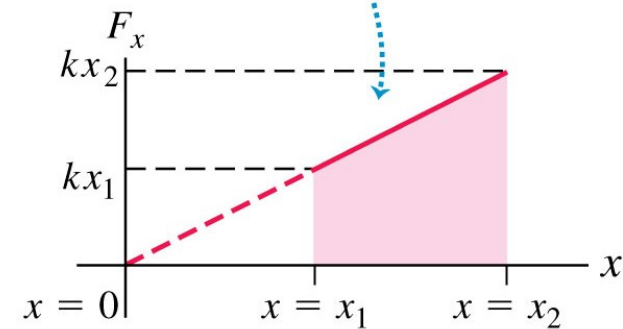
$$W = \int_{x_1}^{x_2} F_x dx = \int_{x_1}^{x_2} kx dx = \frac{1}{2} kx_2^2 - \frac{1}{2} kx_1^2$$

(a) Stretching a spring from elongation x_1 to elongation x_2



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The trapezoidal area under the graph represents the work done on the spring to stretch it from $x = x_1$ to $x = x_2$: $W = \frac{1}{2} kx_2^2 - \frac{1}{2} kx_1^2$

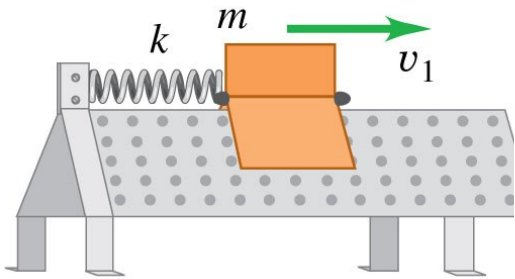


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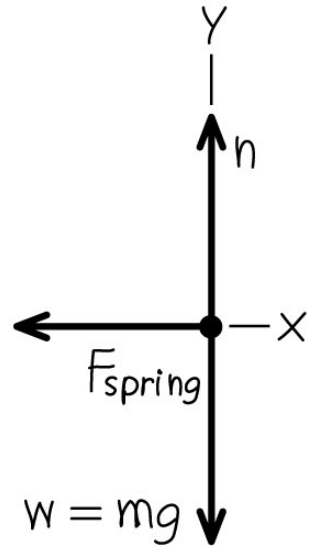
Example: Work done by spring

A glider of mass **0.1 kg** is attached to the end of a horizontal air track by a **spring** with force constant **20.0 N/m**. **Initially** the spring is **unstretched**, and the glider is moving at **1.50 m/s** to the right. Find the maximum distance ***d*** that the glider moves to the right (no friction).

(a)



(b) No friction



Q1: Initially unstretched spring, $x_1 = ?$ $x_1 = 0$

Q2: Finally, displacement is d , stretched spring, $x_2 = ?$
 $x_2 = d$

Q3: From $x_1 = 0$ to $x_2 = d$, what work is done by the glider?

To stretch the spring, the glider does positive work on spring

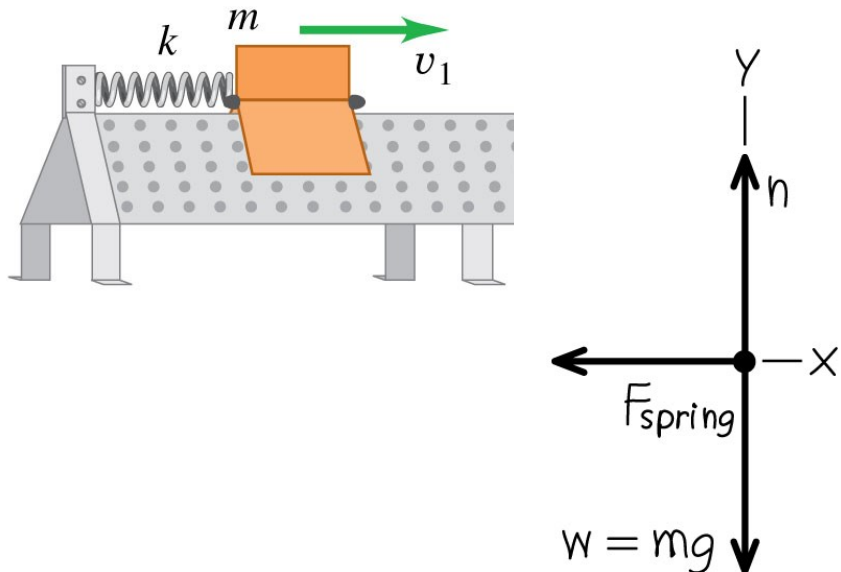
$$W = \frac{1}{2} kx_2^2 - \frac{1}{2} kx_1^2 = \frac{1}{2} kd^2 - \frac{1}{2} k0^2 = \frac{1}{2} kd^2$$

for the glider, the work is negative $W = -\frac{1}{2} kd^2$

Example: Work done by spring

A glider of mass **0.1 kg** is attached to the end of a horizontal air track by a **spring** with force constant **20.0 N/m**. **Initially** the spring is **unstretched**, and the glider is moving at **1.50 m/s** to the right. Find the maximum distance d that the glider moves to the right (no friction).

(a) (b) No friction



For the glider, the work is negative $W = -\frac{1}{2}kd^2$

Q4: What is the effect on the glider of this negative work?

Decrease the kinetic energy of glider to zero

$$W = -\frac{1}{2}kd^2 = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$d = 0.106 \text{ m}$$

$$v_2 = 0$$

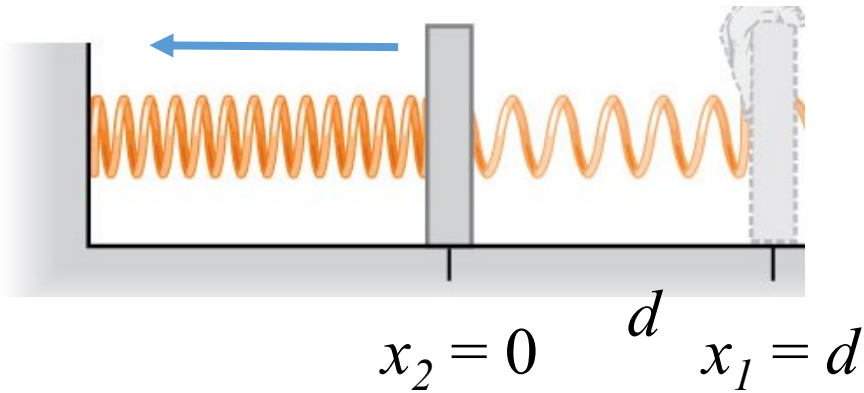
$$v_1 = 1.5 \text{ m/s}$$

$$m = 0.1 \text{ kg}$$

$$k = 20 \text{ N/m}$$

Example: Work done by spring

A glider of mass **0.1 kg** is attached to the end of a horizontal air track by a spring with force constant **20.0 N/m**. The spring is pulled a distance $d = 0.106$ m out of equilibrium and released. Find the **maximum speed (in m/s)** achieved by the glider.



Q1: From $x_1 = d$ to $x_2 = 0$, what is sign of the work done by spring on the block?

Displacement and force parallel, positive, the kinetic energy of block will increase.

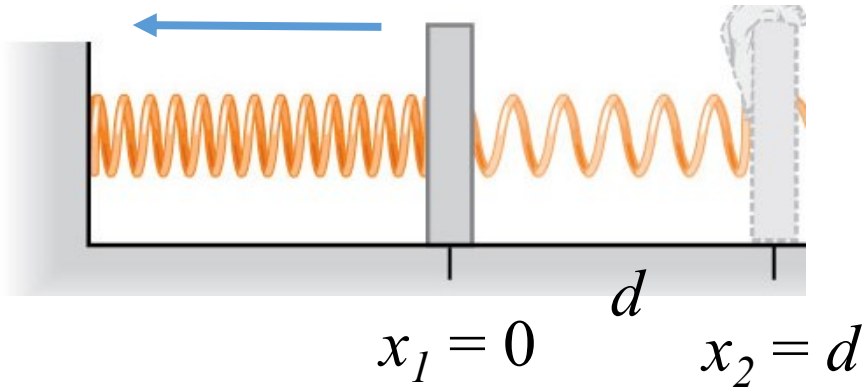
Q2: What will happen if the block moves to the left of $x_2 = 0$?

Displacement and force opposite, negative work, the kinetic energy of block will increase.

So maximum speed is at the $x_2 = 0$

Example: Work done by spring

A glider of mass **0.1 kg** is attached to the end of a horizontal air track by a spring with force constant **20.0 N/m**. The spring is pulled a distance $d = 0.106$ m out of equilibrium and released. Find the **maximum speed (in m/s)** achieved by the glider.



Q3: From $x_1 = d$ to $x_2 = 0$, what is work done by spring?

$$W = \frac{1}{2}kd^2$$

According to work-kinetic energy theorem,

$$W = \frac{1}{2}kd^2 = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

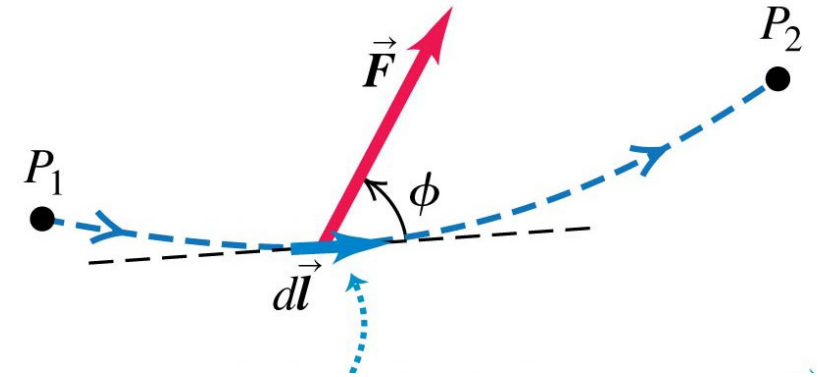
$$v_2 = 1.5 \text{ m/s}$$

$$v_1 = 0 \quad m = 0.1 \text{ kg}$$

$$k = 20 \text{ N/m} \quad d = 0.106 \text{ m}$$

Work–Energy Theorem for Motion Along a Curve

- A particle moves along a curved path from point P_1 to P_2 , acted on by a force that varies in magnitude and direction.
- The work can be found using a **line integral**:



During an infinitesimal displacement $d\vec{l}$, the force \vec{F} does work dW on the particle:

$$dW = \vec{F} \cdot d\vec{l} = F \cos \phi \, dl$$

$$W = \int_{x_1}^{x_2} F_x \, dx$$

Upper limit = final position P_2

Scalar product (dot product) of \vec{F} and displacement $d\vec{l}$

Work done on a particle by a varying force \vec{F} along a curved path

$$W = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{l} = \int_{P_1}^{P_2} F \cos \phi \, dl = \int_{P_1}^{P_2} F_{\parallel} \, dl$$

Lower limit = initial position P_1

Angle between \vec{F} and $d\vec{l}$

Component of \vec{F} parallel to $d\vec{l}$

Power

- The **rate** at which energy is transferred is important in the design and use of practical device
- The **time rate** of energy transfer is called **power**
- The **average power** is given by

$$\text{Average power during time interval } \Delta t \rightarrow P_{\text{av}} = \frac{\Delta W}{\Delta t}$$

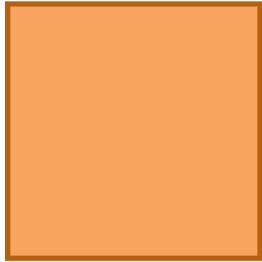
Work done during time interval

Duration of time interval

- Power is the time rate of energy transfer. Power is valid for any means of energy transfer.

Example: Lifting a Box

$$\Delta t = 5 \text{ s}$$

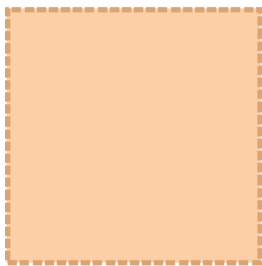


Work you do on the box
to lift it in $\Delta t = 5 \text{ s}$:

$$\Delta W = 100 \text{ J}$$

Your average power output:

$$P_{\text{av}} = \frac{\Delta W}{\Delta t} = \frac{100 \text{ J}}{5 \text{ s}} = 20 \text{ W}$$



$$t = 0$$

$$\Delta t = 1 \text{ s}$$



Work you do on the same
box to lift it the same
distance in $\Delta t = 1 \text{ s}$:

$$\Delta W = 100 \text{ J}$$

Your average power output:

$$P_{\text{av}} = \frac{\Delta W}{\Delta t} = \frac{100 \text{ J}}{1 \text{ s}} = 100 \text{ W}$$



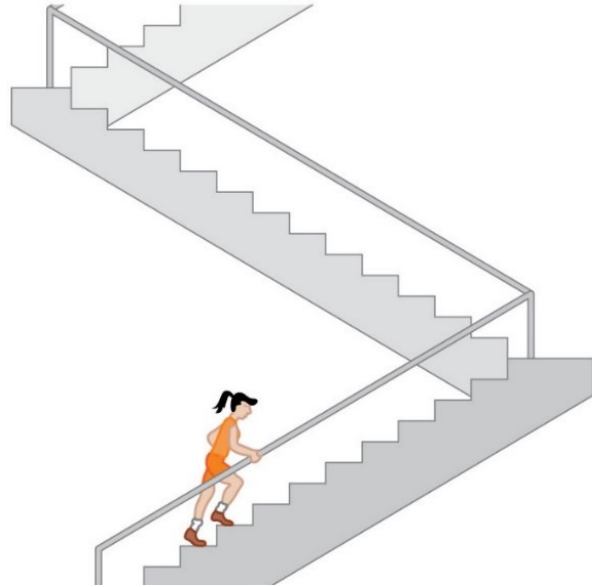
$$t = 0$$

Example:

A **50.0 kg** marathon runner runs up the stairs to the top of **443-m-tall** Tower. To lift herself to the top in **15.0 minutes**, what must be her **average power** output?

$$P_{av} = \frac{\Delta W}{\Delta t}$$

- (A). 241 kW.
- (B). 0.217 kW.
- (C). 217 kW.
- (D). 0.241 kW.



$$\begin{aligned} W &= mgh \\ &= (50.0 \text{ kg}) (9.8 \text{ m/s}^2) (443 \text{ m}) \\ &= 2.17 \times 10^5 \text{ J} \end{aligned}$$

$$15 \text{ min} = 900 \text{ s}$$

$$\begin{aligned} P_{av} &= \frac{2.17 \times 10^5}{900 \text{ s}} = 241 \text{ W} \\ &= 0.241 \text{ kW} = 0.323 \text{ hp} \end{aligned}$$

Instantaneous Power and Units

- Definition of instantaneous power

$$P = \lim_{\Delta t \rightarrow 0} \frac{W}{\Delta t} = \frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$$

$$\bar{P} = \frac{W}{\Delta t} = \frac{F\Delta x}{\Delta t} = F\bar{v}$$

$$P = \vec{F} \cdot \vec{v} = Fv \cos \theta$$

- The SI unit of power is called the **watt**
1 watt = 1 joule / second = 1 kg · m² / s³
- A unit of power in the US Customary system is **horsepower**
1 hp = 550 ft · lb/s = 746 W
- Units of power can also be used to express units of work or energy
1 kWh = (1000 W)(3600 s) = 3.6 × 10⁶ J

Example: You are applying a constant horizontal force $F = 3.0 \mathbf{i} - 2.0 \mathbf{j}$ (N) to a crate that is sliding on a factory floor. At the instant that the velocity of the crate is $\mathbf{v} = 2.0 \mathbf{i} + 2.0 \mathbf{j}$ (m/s), what is the **instantaneous power** (P) supplied by this force?

(A). 10.0 watt.

(B). 2.0 watt.

(C). 6.0 watt.

(D). -4.0 watt.

$$P = \lim_{\Delta t \rightarrow 0} \frac{W}{\Delta t} = \frac{dW}{dt}$$

$$P = \vec{F} \cdot \vec{v} = Fv \cos \theta$$

$$F \cdot v = (3.0\mathbf{i} - 2.0\mathbf{j}) \cdot (2.0\mathbf{i} + 2.0\mathbf{j})$$

Example: average power

A **50.0-kg** marathon runner runs up the stairs to the top of the Chicago's **443-m-**tall Willis Tower in 15 minutes. The stairs make **30°** with horizontal. How is her **average speed** (m/s) along the stair direction?

Q1: Which equation or knowledge can we use to find average speed?

$$\bar{P} = \frac{W}{\Delta t} = \frac{F\Delta x}{\Delta t} = F\bar{v}$$

Q2: What is W and time Δt ? $\Delta t = 900$ S

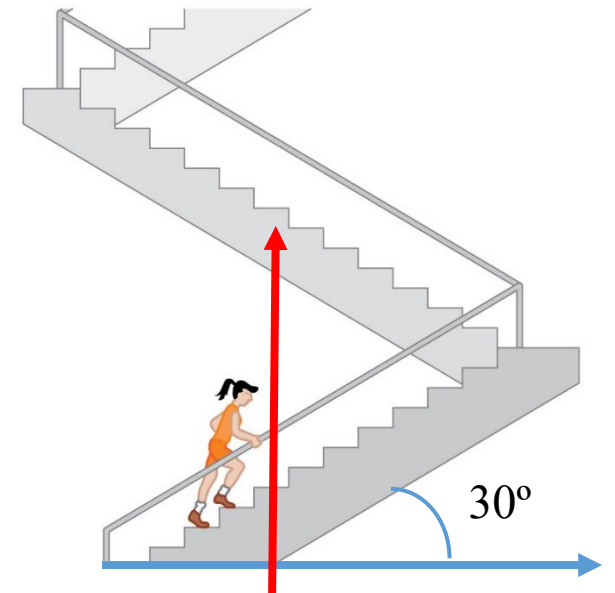
$$W = mgh = (50.0 \text{ kg}) (9.8 \text{ m/s}^2) (443 \text{ m}) = 2.17 \times 10^5 \text{ J}$$

$$\bar{P} = \frac{W}{\Delta t} = \frac{2.17 \times 10^5 \text{ J}}{900 \text{ s}} = 241 \text{ W} = F\bar{v} = mg\bar{v} \quad \bar{v} = 0.492 \text{ m/s} ?$$

$$\bar{v}_{\perp} = 0.492 \text{ m/s}$$

$$\bar{v}_{\perp} = v_{\text{average}} \cdot \sin 30^{\circ}$$

$$v_{\text{average}} = 0.984 \text{ m/s}$$



Example:

A **20.0 kg** rock is sliding on a rough, horizontal surface at **8.00 m/s** and eventually stops due to friction. The coefficient of **kinetic friction** between the rock and the surface is **0.200**. What **average power** is produced by friction as the rock stops?

$$P = F \bar{v} = ?$$

Q1: What is the \bar{v} ?

$$\bar{v} = \frac{(0 + 8 \text{ m/s})}{2} = (4 \text{ m/s})$$

(A) ✓ 156.8 watt.

(B). 640.0 watt.

Q2: What is the F ?

$$F = \mu_k n = (\mu_k m g) = (0.2 \cdot 20 \text{ kg} \cdot 9.8)$$

(C). 0.0 watt.

$$P = F \bar{v} = (0.2 \cdot 20 \text{ kg} \cdot 9.8) \cdot (4 \text{ m/s}) = 156.8 \text{ watt}$$

(D). Depends on the speed of the rock.

Example: inclined surface

A **5.00 kg** block is released from rest on a ramp that is inclined at an angle of **60.0°** below the horizontal. The initial position of the block is a vertical distance of **2.00 m** above the bottom of the ramp. If the speed of the block is **5.00 m/s** when it reaches the bottom, what was the **average power of friction force**?

$$P = f \bar{v} = (-15.37 \text{ N})(0 + 5 \text{ m/s})/2 = -38.4 \text{ watt}$$

Q1: What is the \bar{v} ? $\bar{v} = (0 + 5 \text{ m/s})/2 = (2.5 \text{ m/s})$

Q2: What is the f ? $W_{total} = W_f + W_g$ $W_f = fd$

$$W_{tot} = K_2 - K_1, \text{ where } K_1 = 0, K_2 = \frac{1}{2}mv^2 \quad W_f = K_2 - W_g = \frac{1}{2}mv^2 - mgh$$

$$W_f = \frac{1}{2}(5.00 \text{ kg})(5.00 \text{ m/s})^2 - (5.00 \text{ kg})(9.80 \text{ m/s}^2)(2.00 \text{ m}) = -35.5 \text{ J} = fd, \text{ so } f = -15.37 \text{ N}$$

