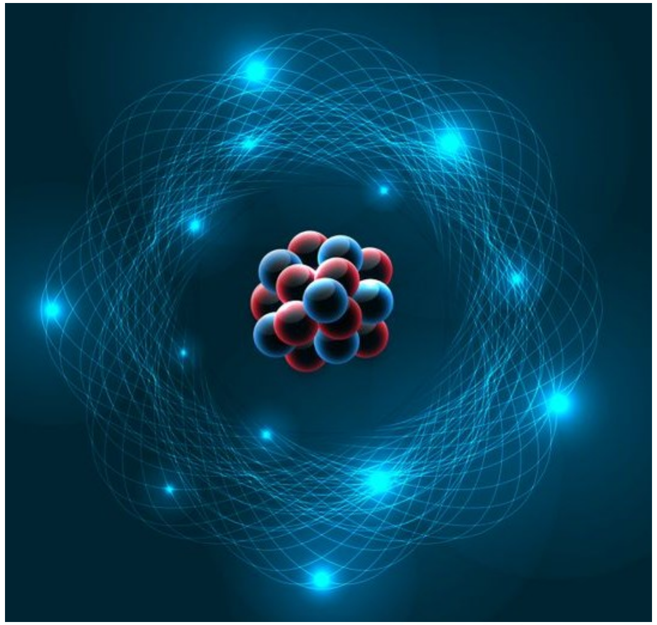
Physics 111: Mechanics
Chapter_06



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THE EDGE IN KNOWLEDGE

Work and Kinetic Energy

- To understand and calculate the *work done by a force*
- To understand the meaning of *kinetic energy*
- To learn how *work changes the kinetic energy* of a body and how to use this principle
- To relate work and kinetic energy when the forces are not constant or the body follows a curved path
- To solve problems involving *power*

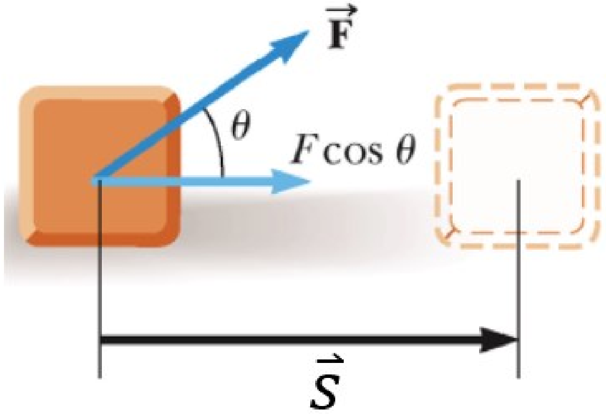
Definition of work W

- The work, W, done by a constant force on an object is defined as the scalar (dot) product of the component of the force along the direction of the displacement and the magnitude of the displacement: $\boldsymbol{W \equiv \vec{F} \cdot \vec{s} = (F\cos\theta)s}$

✓ F is the magnitude of the force

✓ s is the object's magnitude of displacement

✓ θ is the angle between F and s.



- SI unit: N×m = J, J = (kg×m/s^2)×m

Review scalar product

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$
$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\vec{A}\cdot\vec{B} = A_x\hat{i}\cdot B_x\hat{i} + A_y\hat{j}\cdot B_y\hat{j} + A_z\hat{k}\cdot B_z\hat{k}$$
$$= A_xB_x + A_yB_y + A_zB_z$$

$$\hat{i}\cdot\hat{j} = 0;\ \hat{i}\cdot\hat{k} = 0;\ \hat{j}\cdot\hat{k} = 0$$
$$\hat{i}\cdot\hat{i} = 1;\ \hat{j}\cdot\hat{j} = 1;\ \hat{k}\cdot\hat{k} = 1$$

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

$$\vec{F}\cdot\vec{s} = F_xx + F_yy + F_zz$$

Example

A loaded grocery cart is rolling across a parking lot in a strong wind. You apply a constant force $\vec{F} = (30\text{ N})\vec{\boldsymbol{\imath}} - (40\text{ N})\vec{\boldsymbol{\jmath}}$ to the cart as it undergoes a displacement from position $\vec{r}1 = (0\text{ m})\vec{\boldsymbol{\imath}} + (6.0\text{ m})\vec{\boldsymbol{\jmath}}$ to position $\vec{r}2 = (-9\text{ m})\vec{\boldsymbol{\imath}} + (3.0\text{ m})\vec{\boldsymbol{\jmath}}$. How much work does the force you apply do on the grocery cart?

$$\vec{F} \cdot \vec{s} = F_x x + F_y y + F_z z$$

$$\vec{F} = (30\text{ N})\vec{\boldsymbol{\imath}} - (40\text{ N})\vec{\boldsymbol{\jmath}}$$

$$\vec{s} = \overrightarrow{r2} - \overrightarrow{r1} = (-9\text{ m})\vec{\imath} - (3.0\text{ m})\vec{\jmath}$$

$$\vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{s}} = \left[(30\text{ N})\hat{\boldsymbol{i}} - (40\text{ N})\hat{\boldsymbol{j}}\right] \cdot \left[(-9.0\text{ m})\hat{\boldsymbol{i}} - (3.0\text{ m})\hat{\boldsymbol{j}}\right]$$

$$\vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{s}} = (30\text{ N})(-9.0\text{ m}) + (-40\text{ N})(-3.0\text{ m}) = -270\text{ N}\cdot\text{m} + 120\text{ N}\cdot\text{m} = -150\text{ J.}$$

Positive, negative, and zero work

$$W \equiv \vec{F} \cdot \vec{s} = (F cos\phi)s$$

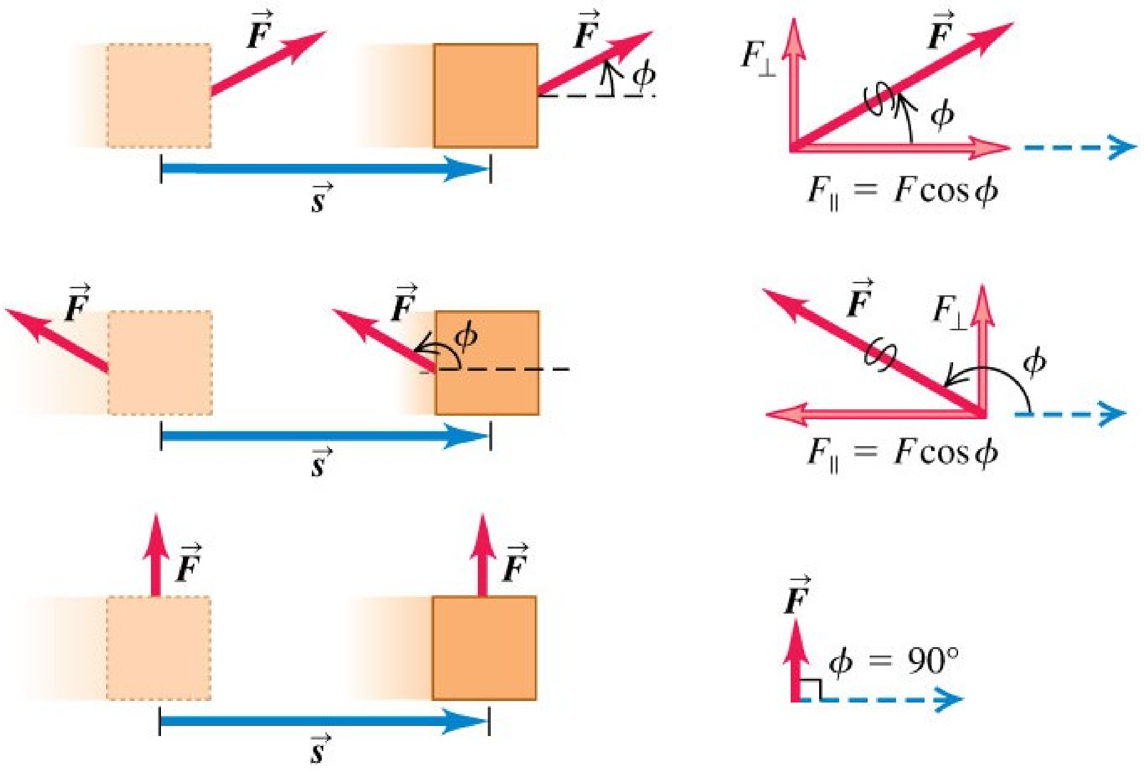
- A force can do positive, negative, or zero work depending on the angle between the force and the displacement.

Direction of Force (or Force Component) | **Situation** | **Force Diagram**

(a) **Force $\vec{F}$ has a component in direction of displacement:**
$W = F_{\parallel}s = (F\cos\phi)s$
Work is *positive.*

(b) **Force $\vec{F}$ has a component opposite to direction of displacement:**
$W = F_{\parallel}s = (F\cos\phi)s$
Work is *negative* (because $F\cos\phi$ is negative for $90° < \phi < 180°$).

(c) **Force $\vec{F}$ (or force component $F_{\perp}$) is perpendicular to direction of displacement:** The force (or force component) does *no* work on the object.



Example:

When you lift a box, is the work positive or negative?

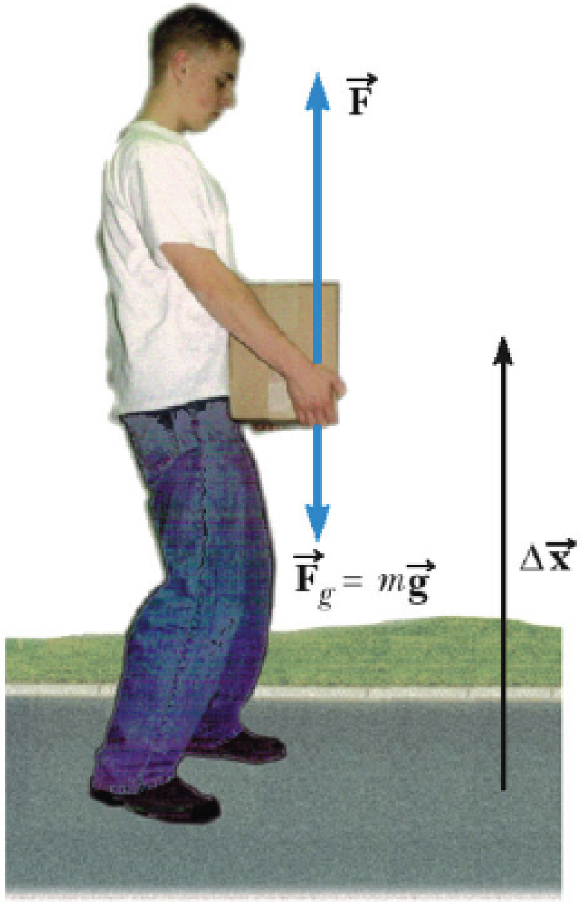
(A)Positive ✓

(B)Negative

(C)Zero

When you lower the box?

Negative



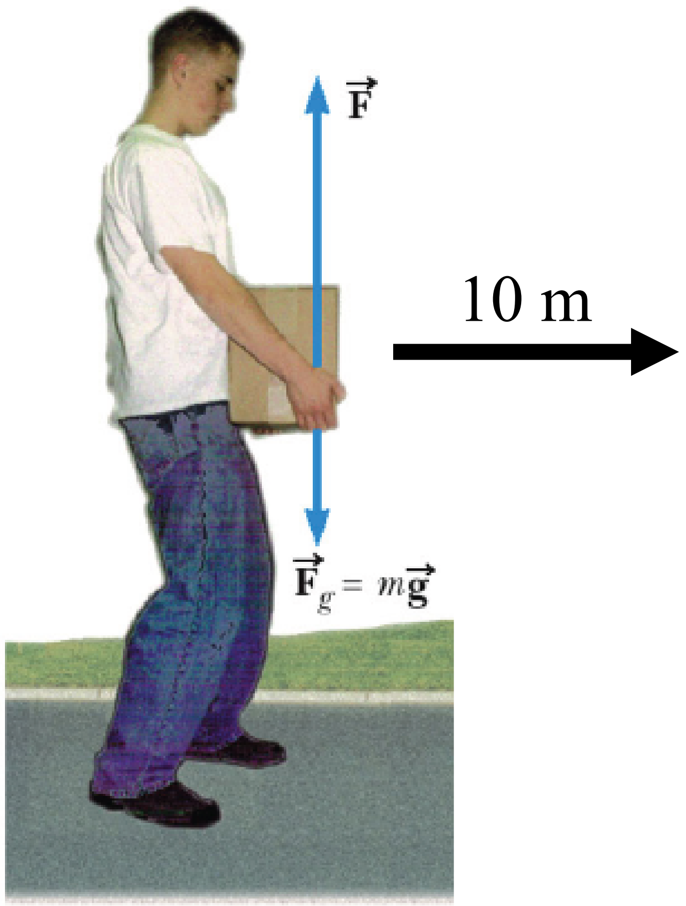
Example: When you move box horizontally 10 m with a constant velocity, what is the work done by you?

Positive

Negative

Zero ✓ $W \equiv \vec{F} \cdot \vec{s} = Fs(cos90)$

It depends on the mass of the box



Example:

A 60-kg skydiver descends at a constant velocity of 15 m/s thanks to her parachute. What is the work (in kJ) is done by the parachute on the skydiver as she descends 1.0 km?

$$W \equiv \vec{F} \cdot \vec{s} = (F\cos\phi)s$$

$$F = mg = 60 \times 9.8 = 588\ N$$

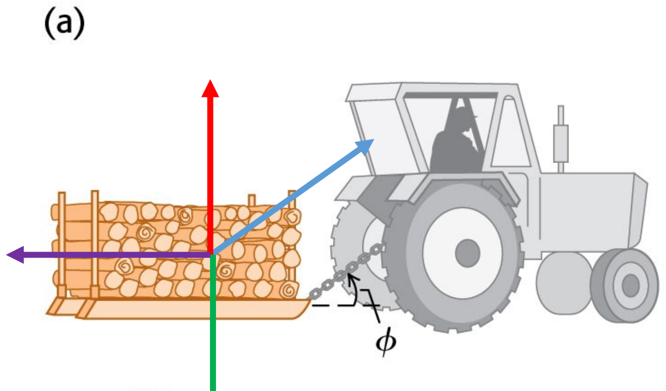
What is the angle between the displacement and force?

$$\phi = 180^{\circ},\ \cos\phi = -1$$

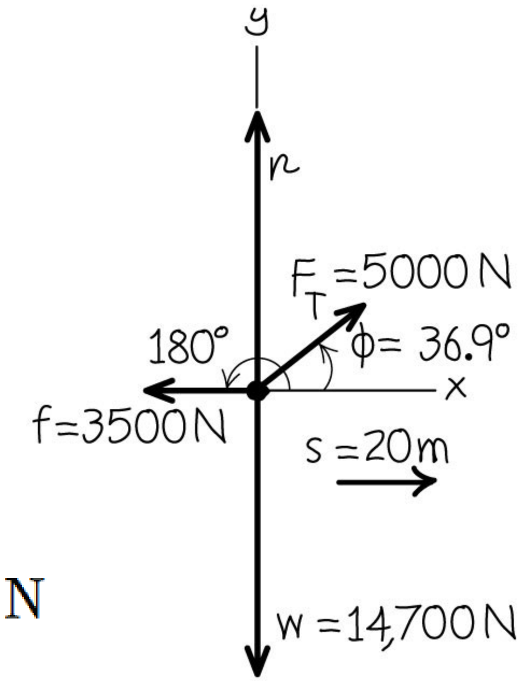
$$W = -588 \times 1000 = -588\ kJ$$

Work done by several forces

The total weight of the sled and the load is 14,700 N. The tractor exerts a constant 5000-N force at an angle of 36.9º above the horizontal. Displacement is 20 m. A 3500-N friction force opposes the sled's motion. Find the total work done by all forces.



(b) Free-body diagram for sled



Step 1: Equation for the total work done by all forces?

$$\vec{F} \cdot \vec{s} = F_x x + F_y y + F_z z$$

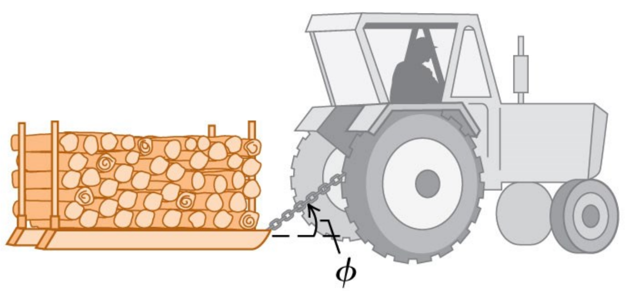
Step 2: We need to know F_x, and F_y, decompose.

$$\begin{aligned} \sum F_x &= F_T \cos\phi + (-f) = (5000\,\mathrm{N})\ \cos 36.9^\circ - 3500\,\mathrm{N} \\ &= 500\,\mathrm{N} \end{aligned}$$

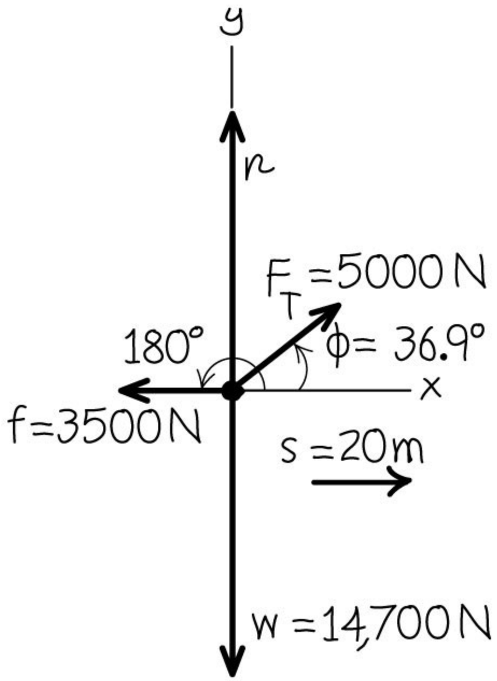
$$\sum F_y = F_T \sin\phi + n + (-w) = (5000\,\mathrm{N})\ \sin 36.9^\circ + n - 14,700\,\mathrm{N}$$

Work done by several forces

(a)



(b) Free-body diagram for sled



The total weight of the sled and the load is 14,700 N. The tractor exerts a constant 5000-N force at an angle of 36.9º above the horizontal. Displacement is 20 m. A 3500-N friction force opposes the sled's motion. Find the total work done by all forces.

$$\vec{F}\cdot\vec{s} = F_x x + F_y y + F_z z$$

Step 3: We need to know *x*, and *y*.

x = 20 m, and y = 0 m. $\sum F_y \bullet y = 0$

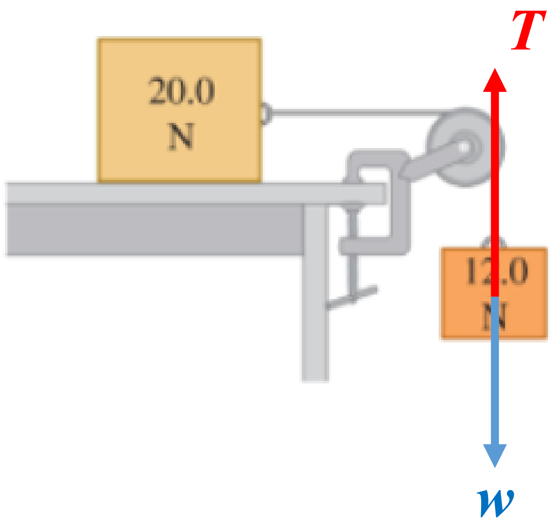
$$\begin{aligned}\sum F_x &= F_T \cos\phi + (-f) = (5000\,\text{N})\ \cos 36.9^\circ - 3500\,\text{N} \\ &= 500\,\text{N}\end{aligned}$$

$$W_{\text{tot}} = \left(\sum \vec{F}\right)\cdot\vec{s} = \left(\sum F_x\right) s = (500\ \text{N})(20\ \text{m}) = 10,000\ \text{J}$$

Example: two objects

Two blocks are connected by a very light string passing over a massless and frictionless pulley. Traveling at **constant speed**, the **20 N** block moves **75 cm** to the right and the **12 N** block moves **75 cm** downward.

(A) How much work is done on the **12 N block** by **(i) gravity** and **(ii) the tension** in the string?



Q1: Which object should we focus on? **12 N block**

Q2: For gravity, what is the angle between the displacement and force?

$$\phi = 0° \text{ and } W = (12.0 \text{ N})(0.750 \text{ m})\cos 0° = 9.00 \text{ J}.$$

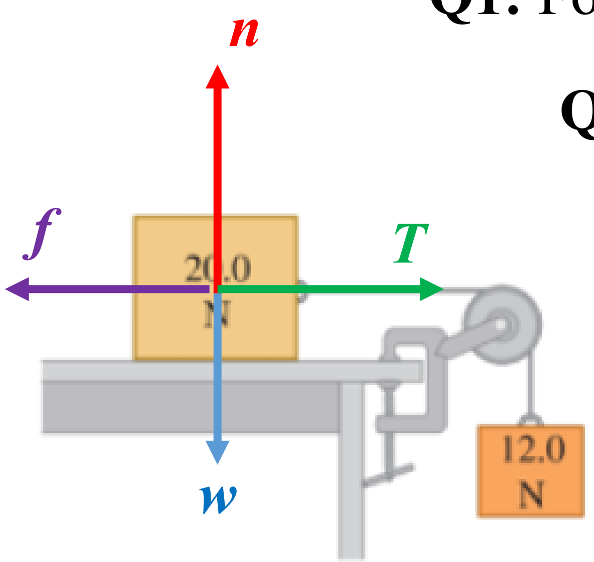
Q3: For tension, what is the angle between the displacement and force?

$$\phi = 180° \text{ and } W = (12.0 \text{ N})(0.750 \text{ m})\cos 180° = -9.00 \text{ J}.$$

Q4: What is the total work done on this 12-N block? $W_{tot} = 0$

Example: two objects

Two blocks are connected by a light string passing over a massless/frictionless pulley. Traveling at **constant speed**, the **20 N** block moves **75 cm** to the right and the **12 N** block moves **75 cm** downward. (B) How much work is done on the **20 N block** by (i) gravity, (ii) the tension in the string, (iii) friction, and (iv) the normal force?



Q1: For gravity, angle between the displacement and force? (i) $\phi = 90°$ and $W = 0$.

Q2: For tension, angle between the displacement and force?

(ii) $\phi = 0°$ and $W = (12.0 \text{ N})(0.750 \text{ m})\cos 0° = 9.00 \text{ J}$.

Q3: For friction, angle between the displacement and force?

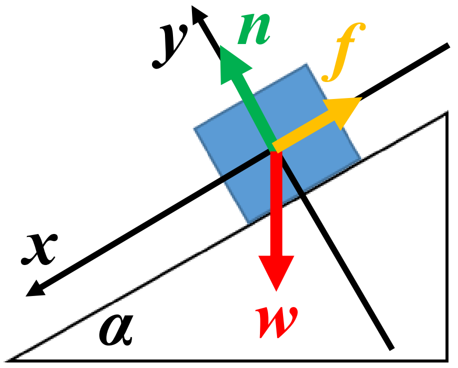
$\phi = 180°$ and $W = (12.0 \text{ N})(0.750 \text{ m})\cos 180° = -9.00 \text{ J}$.

Q4: For normal force, angle between the displacement and force?

(iv) $\phi = 90°$ and $W = 0$. $W_{tot} = 0$ for each block.

Example: inclined surface

A block of mass ***m*** is released from rest at the top of an incline that makes an angle ***α*** with the horizontal. The coefficient of kinetic friction between the block and incline is $\boldsymbol{\mu_k}$. The top of the incline is a distant ***h*** above the bottom of the incline. Derive an expression for the work $\boldsymbol{W_f}$ done on the block by **friction** as it travels from the top of the incline to the bottom.



Q1: For friction, angle between the displacement and force?

$\phi = 180°$ $W_f = fs \cos 180° = -fs$

Q2: Magnitude of friction force?

$f = \mu_k n$ $n - mg\cos\alpha = 0$ $n = mg\cos\alpha$

$= \mu_k mg\cos\alpha$

Q3: Magnitude of displacement? $s = h/\sin\alpha$

$$W_f = -\mu_k mg\cos\alpha\left(\frac{h}{\sin\alpha}\right) = -\frac{\mu_k mgh}{\tan\alpha}.$$

Example: inclined surface

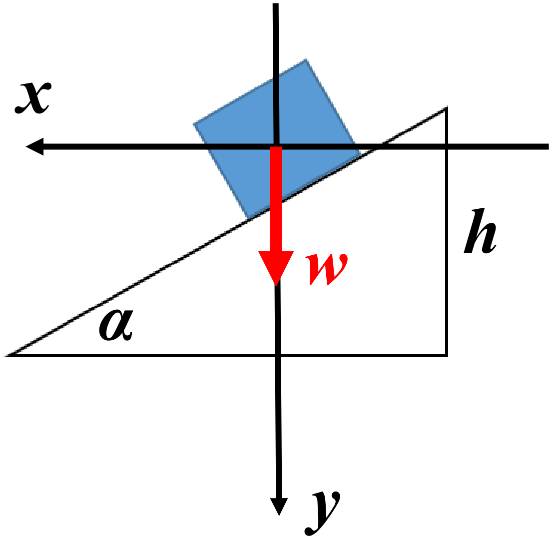
A block of mass ***m*** is released from rest at the top of an incline that makes an angle ***α*** with the horizontal. The coefficient of kinetic friction between the block and incline is μ_k. The top of the incline is a distant ***h*** above the bottom of the incline. Derive an expression for the work W_w done on the block by **gravity** as it travels from the top of the incline to the bottom.

Q1: What is the displacement along *y* axis? $(y_2 - y_1) = h$

Q2: What is the displacement along *x* axis? $(x_2 - x_1) = \frac{h}{\tan\alpha}$

Q3: What is the component of gravity along *y* axis? mg

Q4: What is the component of gravity along *x* axis? $F_x = 0$

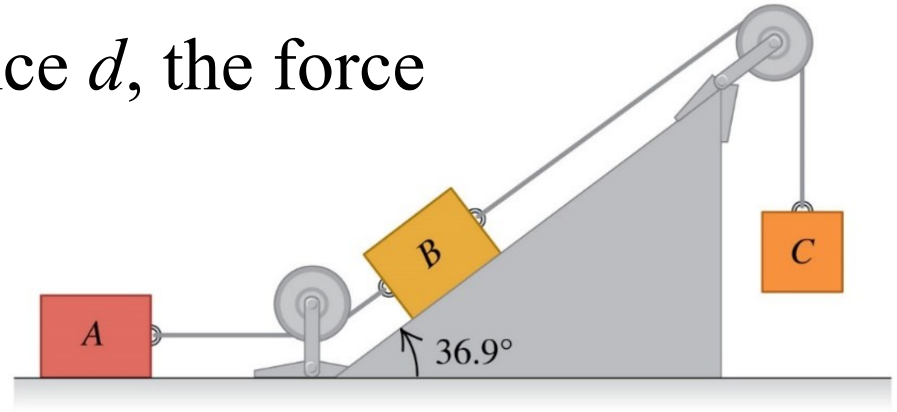


$$\vec{F}\cdot\vec{s} = F_x x + F_y y + F_z z \qquad W_{mg} = mg(y_2 - y_1) = mgh$$

Example:

Three blocks are connected as shown. The ropes and pulleys are of negligible mass. When released, *C* moves downward, *B* moves up the ramp, and *A* moves to the right.

After each block has moved a distance *d*, the force of gravity has done



(A). positive work on *A*, *B*, and *C*.

(B). zero work on *A*, positive work on *B*, and negative work on *C*.

(C). zero work on *A*, negative work on *B*, and positive work on *C*.

(D). none of these

Kinetic Energy

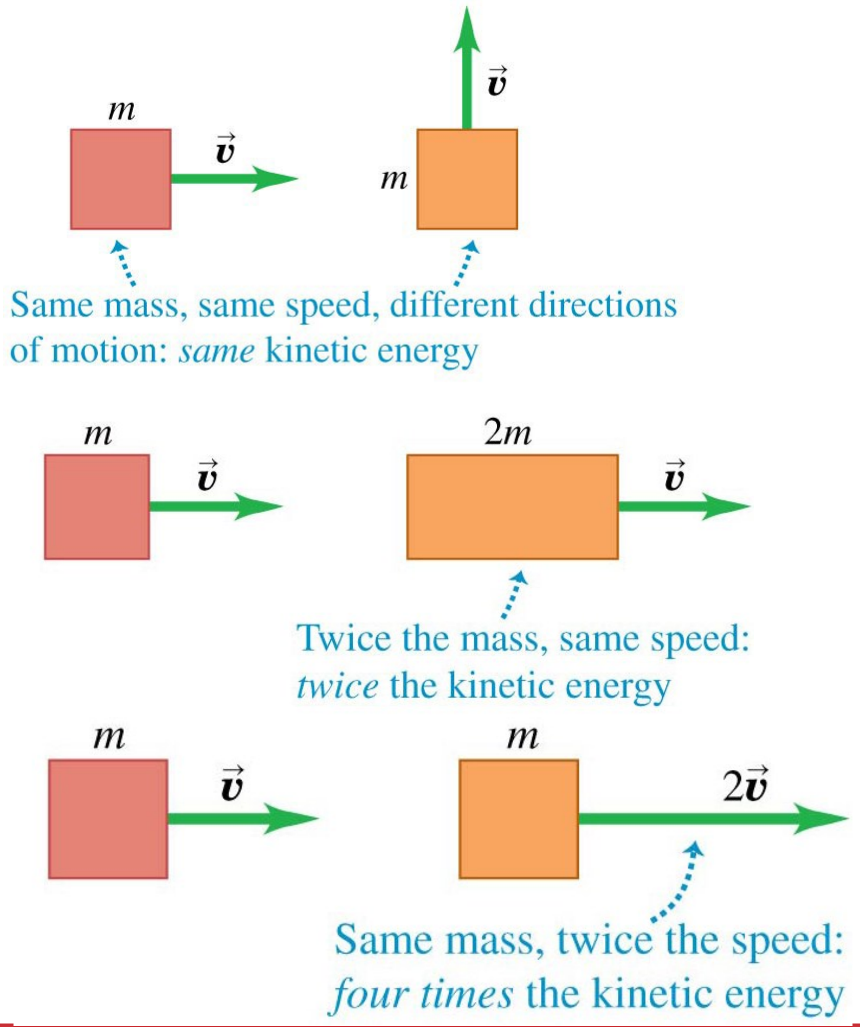
- For an object *m* moving with a speed of *v*,

Kinetic energy of a particle

$$K = \frac{1}{2}mv^2$$

Mass of particle (m); Speed of particle (v)

- Like work, the kinetic energy is a scalar; it depends on only the object's mass and speed, not its direction of motion.
- Kinetic energy is energy associated with the state of motion. **Can be zero, but never be negative**.
- SI unit: joule (J) $1\ \text{J} = 1\ \text{kg m}^2/\text{s}^2$



Example: Kinetic Energy

A tiger has a mass of about **70 kg** and has been clocked to run at up to **32 m**/s.

(A) How many joules of kinetic energy does such a tiger has?

(B) By what factor would its kinetic energy change if its speed were doubled?

(a) $K = \frac{1}{2}mv^2 = \frac{1}{2}(70 \text{ kg})(32 \text{ m/s})^2 = 3.6 \times 10^4 \text{ J}.$

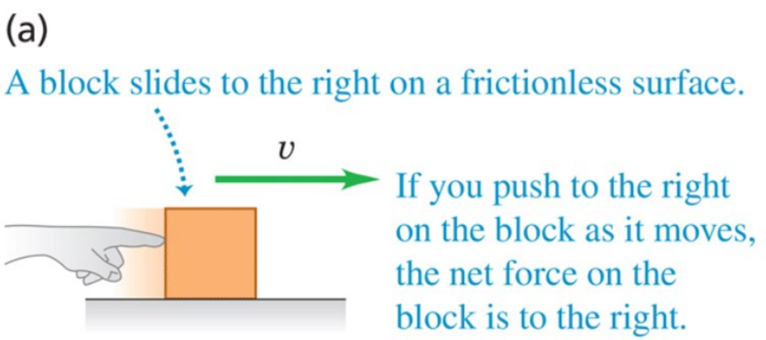
(b) K is proportional to v^2, so K increases by a factor of 4 when v doubles.

Work and Kinetic Energy

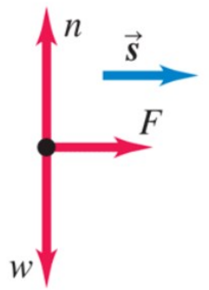
$$W \equiv (F\cos\phi)s = \vec{F}\cdot\vec{s}$$

$$\vec{F}\cdot\vec{s} = F_x x + F_y y + F_z z$$

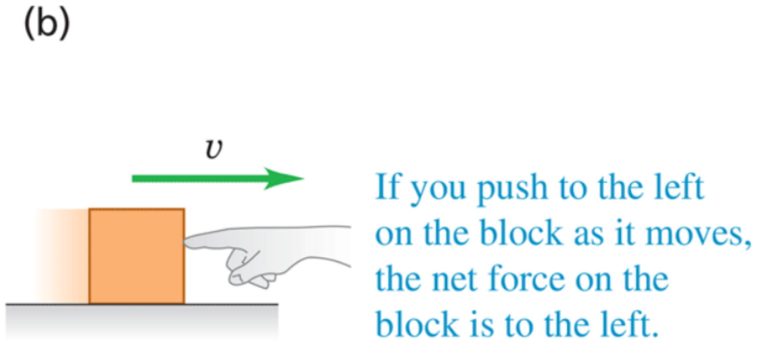
$$K = \frac{1}{2}mv^2$$



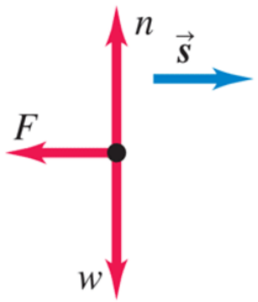
$\phi = 0°, \cos\phi = 1$



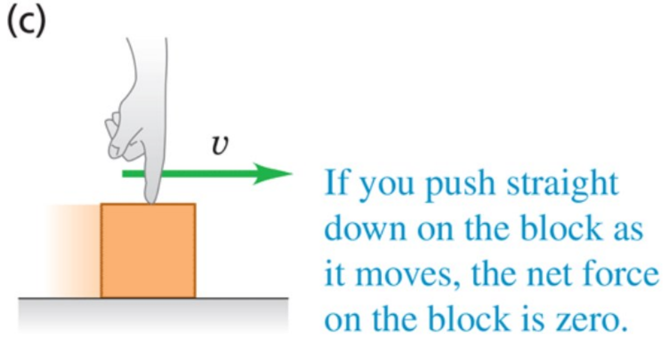
- The total work done on the block during a displacement $\vec{s}$ is positive: $W_{tot} > 0$.
- The block speeds up.



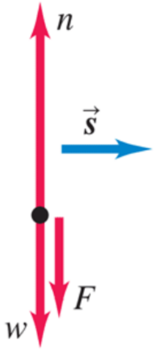
$\phi = 180°, \cos\phi = -1$



- The total work done on the block during a displacement $\vec{s}$ is negative: $W_{tot} < 0$.
- The block slows down.



$\phi = 90°, \cos\phi = 0$



- The total work done on the block during a displacement $\vec{s}$ is zero: $W_{tot} = 0$.
- The block's speed stays the same.

Kinetic Energy and Newton's laws

Initial point: $K_0 = \frac{1}{2}mv_0^2$

Final point: $K = \frac{1}{2}mv^2$

Chang of kinetic energy →

$$\Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$
$$= \frac{1}{2}m(v^2 - v_0^2)$$

Newton's second law: $(v^2 - v_0^2) = 2a(x - x_0)$ Constant acceleration!

So: $\Delta K = \frac{1}{2}m(v^2 - v_0^2) = \frac{1}{2}m2a(x - x_0) = ma(x - x_0) = F\Delta x = W$

This is the Work-Energy Theorem! $\vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{s}} = W = \Delta K$

Example: Work-Energy Theorem

A loaded grocery cart (m = 20 kg) is rolling across a parking lot. You apply a constant force $\vec{F} = (30\text{ N})\vec{\imath} - (40\text{ N})\vec{\jmath}$ to the cart as it undergoes a displacement from position $\overrightarrow{r1} = (0\text{ m})\vec{\imath} + (1.0\text{ m})\vec{\jmath}$ to position $\overrightarrow{r2} = (8\text{ m})\vec{\imath} + (3.0\text{ m})\vec{\jmath}$. The cart is initially at rest, what is its speed at position $\overrightarrow{r2}$?

$$\vec{s} = \vec{r}2 - \vec{r}1$$

$$\vec{F} \cdot \vec{s} = F_x x + F_y y + F_z z$$

$$\vec{F} \cdot \vec{s} = \left[(30\text{ N})\hat{\imath} - (40\text{ N})\hat{\jmath}\right] \cdot \left[(8.0\text{ m})\hat{\imath} + (2.0\text{ m})\hat{\jmath}\right]$$

$$\vec{F} \cdot \vec{s} = (30\text{ N})(8.0\text{ m}) + (-40\text{ N})(2.0\text{ m}) = 240\text{ N}\cdot\text{m} - 80\text{ N}\cdot\text{m} = 160\text{ J.}$$

$$\vec{F} \cdot \vec{s} = 160\text{ J} = \Delta K = \frac{1}{2}m(v^2 - v_0^2)$$

$$160\text{ J} = \frac{1}{2}(20\ kg)v^2 \qquad v = 4\text{ m/s}$$