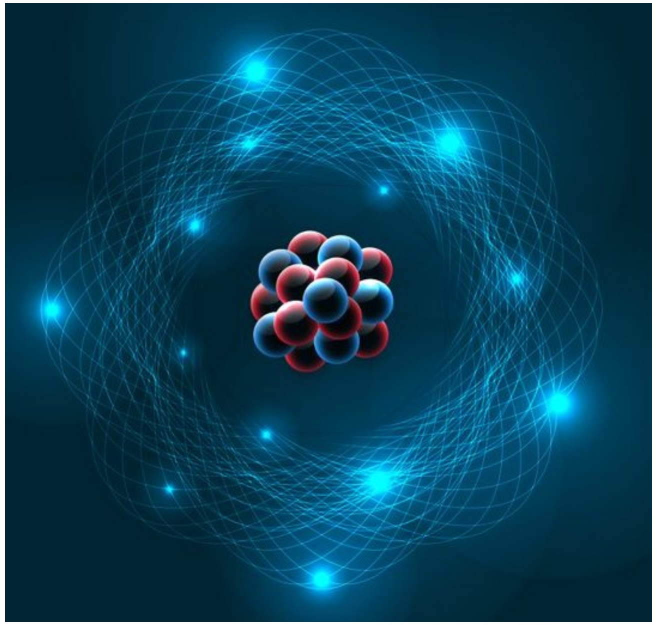
Quiz

$$W = F\Delta x = \Delta K = \frac{1}{2}m\left(v^2 - v_0^2\right)$$

$$W = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$$

Physics 111: Mechanics
Chapter07



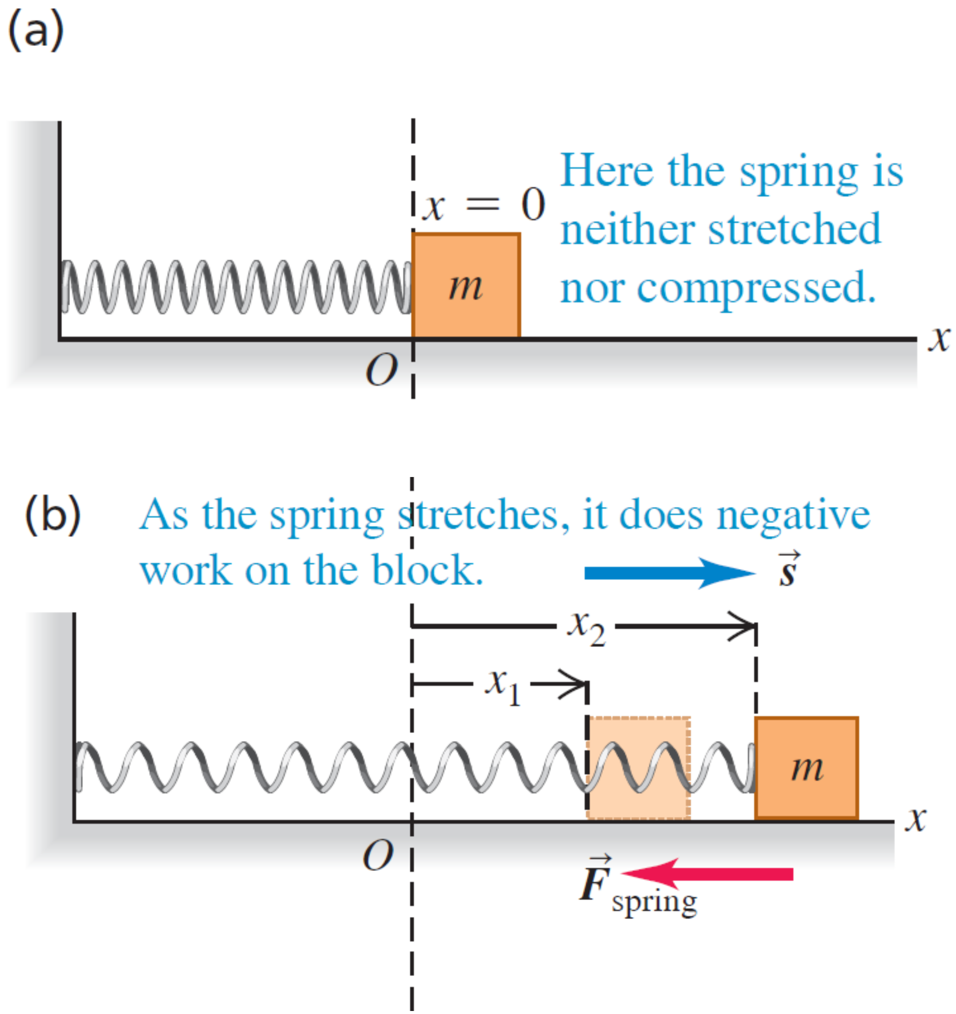
Junjie Yang

Department of Physics, NJIT

Potential Energy and Energy Conservation

- To use *elastic potential energy* for a body attached to a spring
- To use *gravitational potential energy* in vertical motion
- To solve problems involving *conservative* and *nonconservative* forces
- To determine the properties of a conservative force from the corresponding *potential-energy function*

Work done by a spring



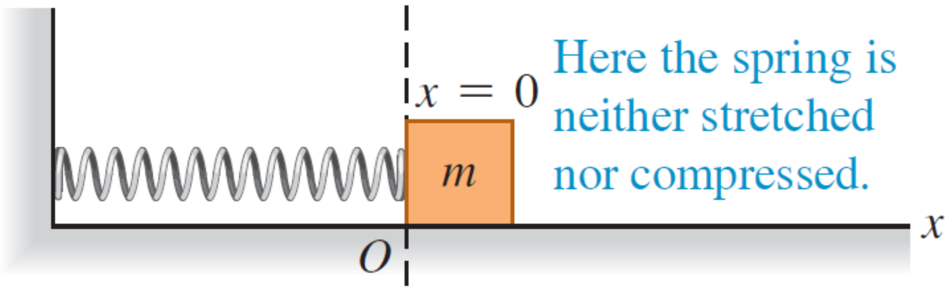
In figure (b), the kinetic energy of the block decreases as the spring is stretched.

Q: Where does the kinetic energy go? Disappear?

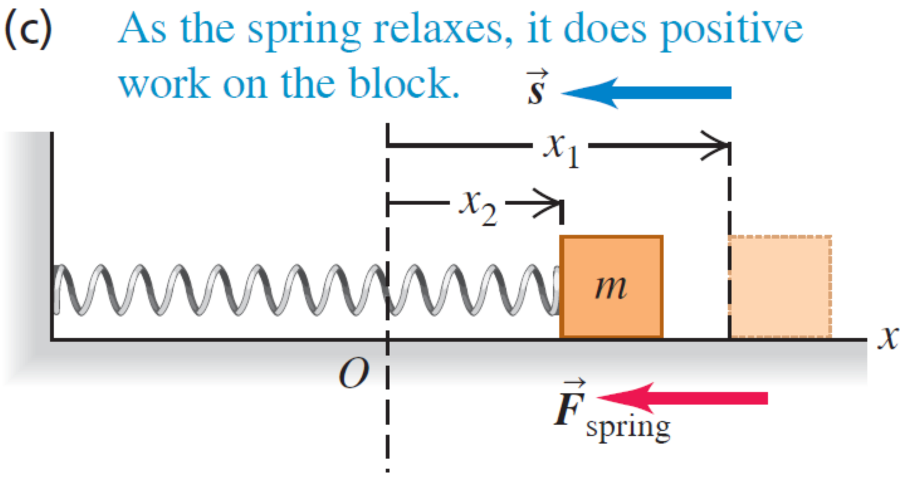
The spring is stretched and becomes energized. It stores some energy!

Work done by a spring

(a)



(c)

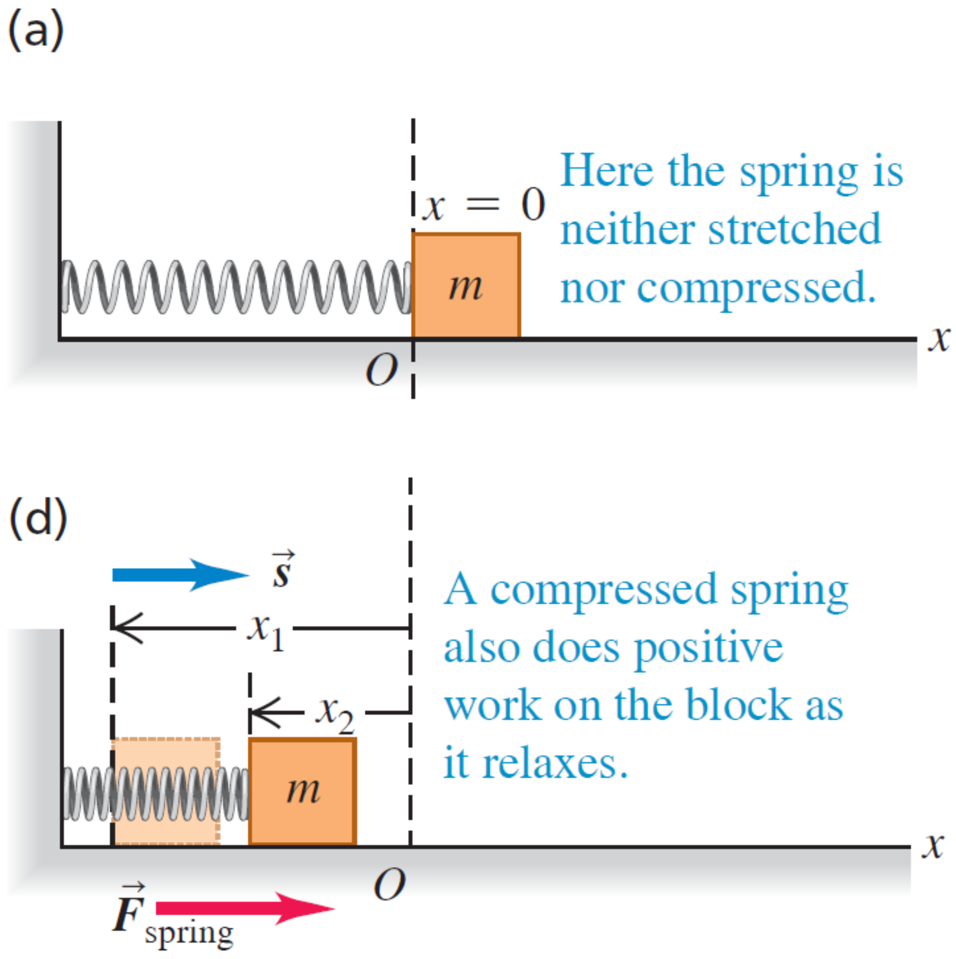


In figure (c), the kinetic energy of the block increases as the spring is relaxed.

Q: Where does the kinetic energy come from?

Stretched spring stores some energy.
It releases the energy when it is relaxed.

Work done by a spring



In figure (d), the kinetic energy of the block increases as the spring is relaxed from compressed state.

Q: Where does the kinetic energy come from?

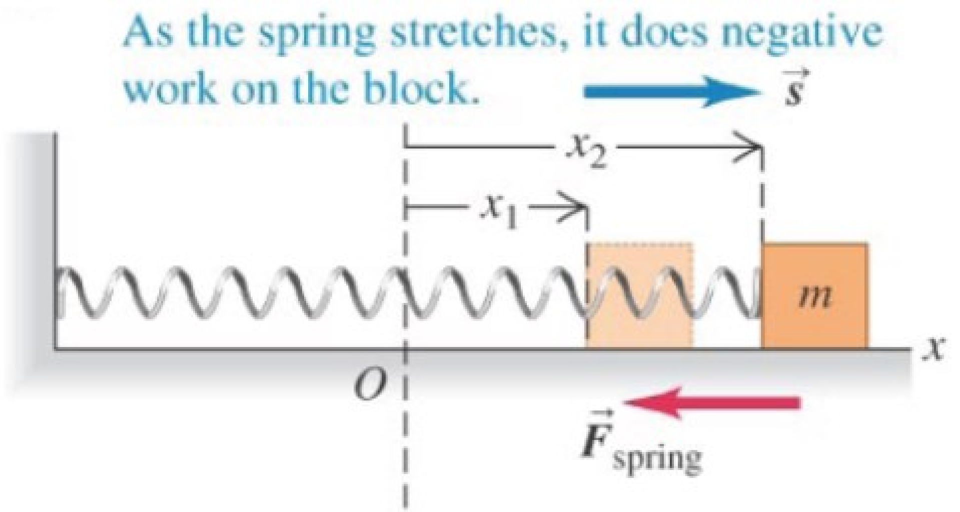
Compressed spring also stores some energy.
It releases the energy when it is relaxed.

Potential energy in spring!

Potential energy in spring

- Work done **by** the spring on the block from x_1 to x_2

$$W_s = \int_{x_1}^{x_2} F_s dx = \int_{x_1}^{x_2} -kx dx = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$$



- Work done **on** the spring (increase its energy)

$$W = \int_{x_1}^{x_2} F_x dx = \int_{x_1}^{x_2} kx dx = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$$

Final Initial

- **Elastic Potential energy**

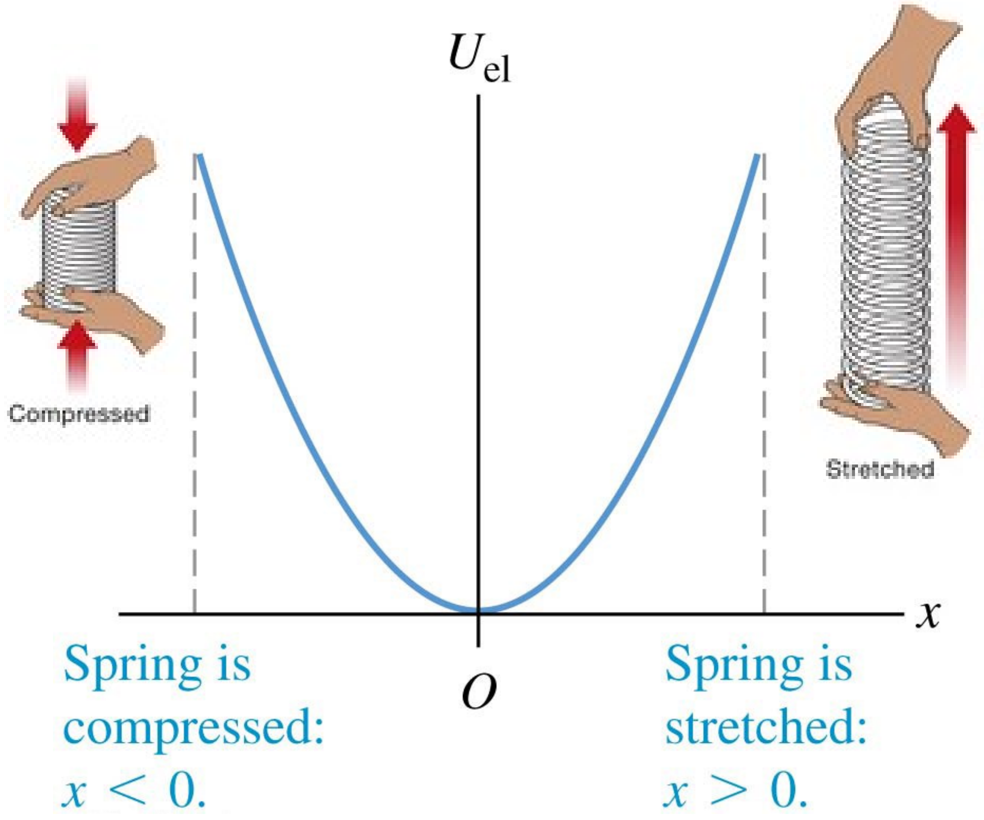
Elastic potential energy stored in a spring → $U_{el} = \frac{1}{2}kx^2$

Force constant of spring (k)

Elongation of spring (x) ($x > 0$ if stretched, $x < 0$ if compressed)

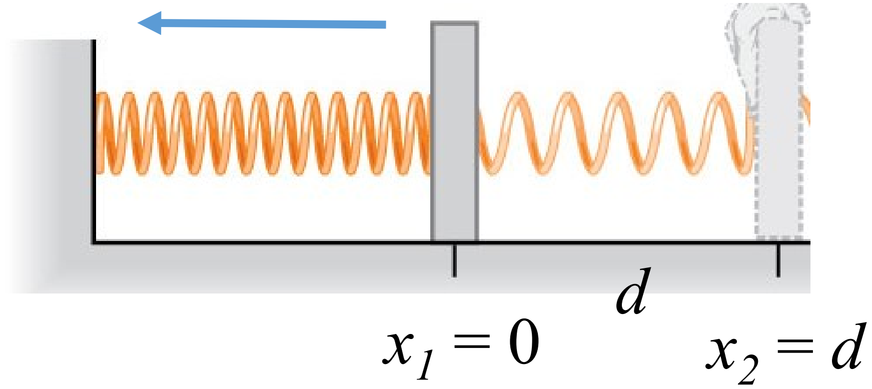
Potential energy in spring

- Elastic Potential energy: $U_{el} = \frac{1}{2}kx^2$
- The graph of elastic potential energy for an ideal spring is a parabola.
- x is the extension or compression of the spring.
- Elastic potential energy is never negative.



Example: potential energy in spring

A glider of mass **0.1 kg** is attached to the end of a horizontal air track by a spring with force constant **20.0 N/m**. The spring is pulled a distance ***d* = 0.106 m** out of equilibrium. Find the **potential energy** stored in the spring.

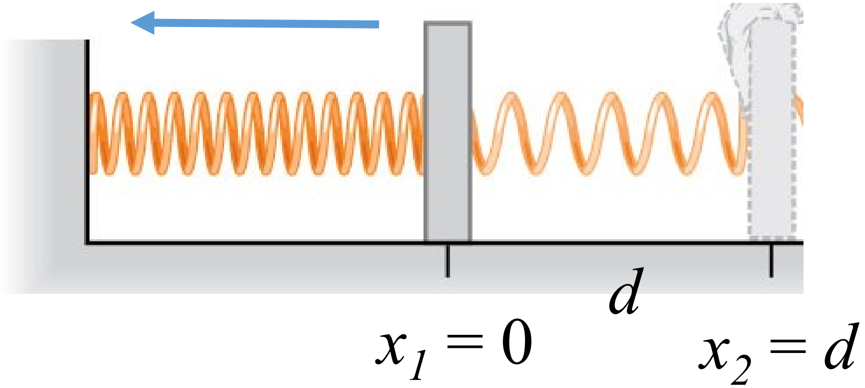


$$U_{el} = \frac{1}{2}kx^2 = \frac{1}{2}k(x_2 - x_1)^2$$

$$= \frac{1}{2}(20\,N/m)(0.106\,m)^2$$
$$= 0.112\,J$$

Example: potential energy in spring

A glider of mass **0.1 kg** is attached to the end of a horizontal air track by a spring with force constant **20.0 N/m**. The length of the spring is 2 m at equilibrium. If you wanted to **store 10.0 J** of potential energy in this spring, what would be the ***total* length** of the spring?



$$U_{el} = \frac{1}{2}kx^2$$
$$= \frac{1}{2}(20\,N/m)x^2$$
$$= 10\,J$$

$$x = \pm 1\text{ m}$$

$$L = 2 \pm 1\text{ m}$$

$$L = 1\,m\ or\ 3\,m$$

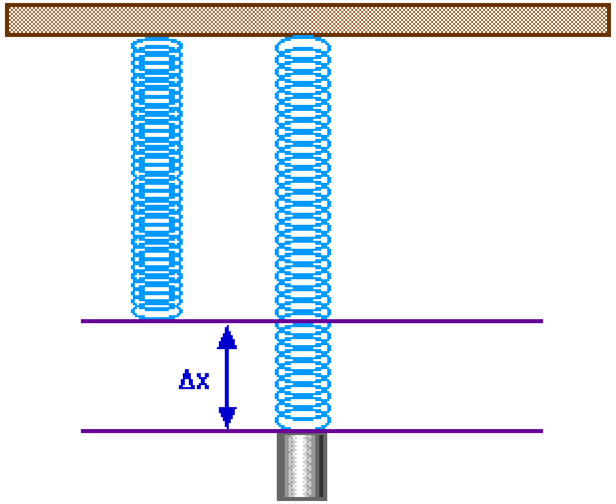
Example: Potential energy in spring

An ideal spring of negligible mass is **12.00 cm long when nothing is attached** to it. When you hang a **3.15 kg weight** from it, you measure its length to be **13.40 cm**. If you wanted to **store 10.0 J** of potential energy in this spring, what would be its ***total* length**? Assume that it continues to obey Hooke's law.

$U_{el} = \frac{1}{2}kx^2$ But k is unknown, need to find k first.

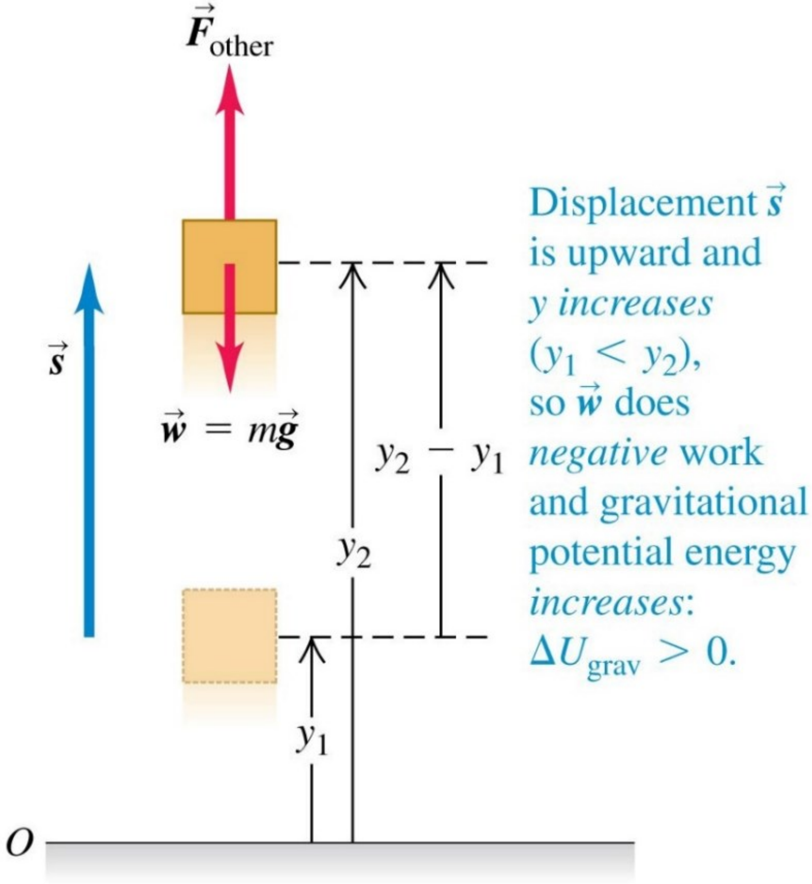
$F = kx$ to find k.
$$k = \frac{F}{x} = \frac{mg}{x} = \frac{(3.15\ \text{kg})(9.80\ \text{m/s}^2)}{0.1340\ \text{m} - 0.1200\ \text{m}} = 2205\ \text{N/m}.$$

$$x = \pm\sqrt{\frac{2U_{el}}{k}} = \pm\sqrt{\frac{2(10.0\ \text{J})}{2205\ \text{N/m}}} = \pm 0.0952\ \text{m} = \pm 9.52\ \text{cm}.$$



The spring could be either stretched 9.52 cm or compressed 9.52 cm.
If it is stretched, the total length of the spring would be 12.00 cm + 9.52 cm = 21.52 cm.
If it is compressed, the total length would be 12.00 cm – 9.52 cm = 2.48 cm.

Gravitational Potential Energy

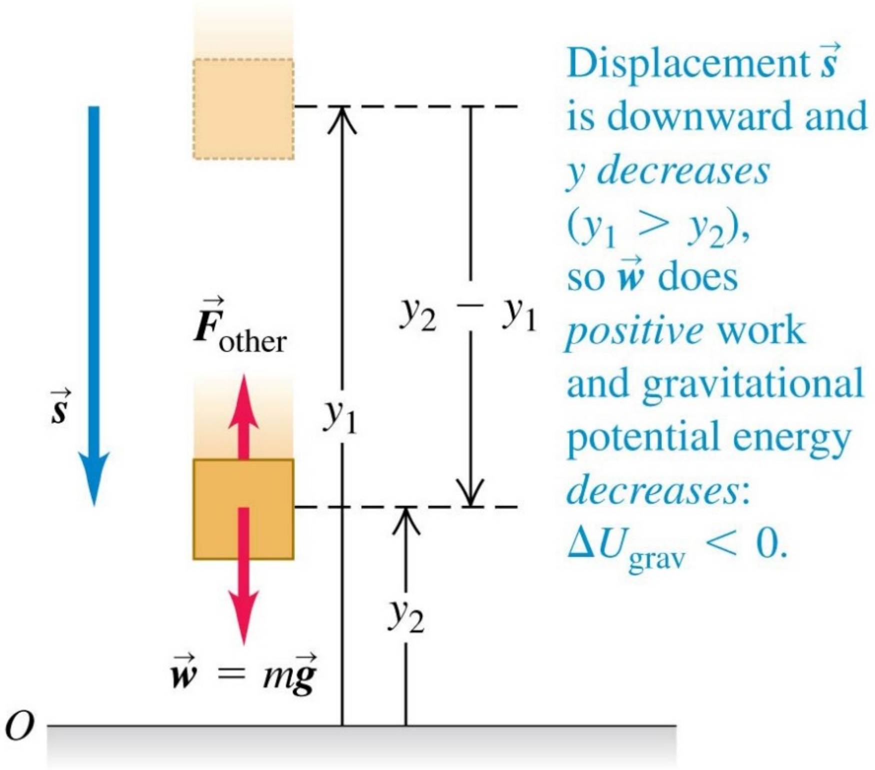


We throw an object upward. Its kinetic energy decreases as it travels upward.

Q: Where does the kinetic energy go? Disappear?

The kinetic energy is transferred to “the interaction between the object and earth”. The “object-earth system ”stores some energy!

Gravitational Potential Energy



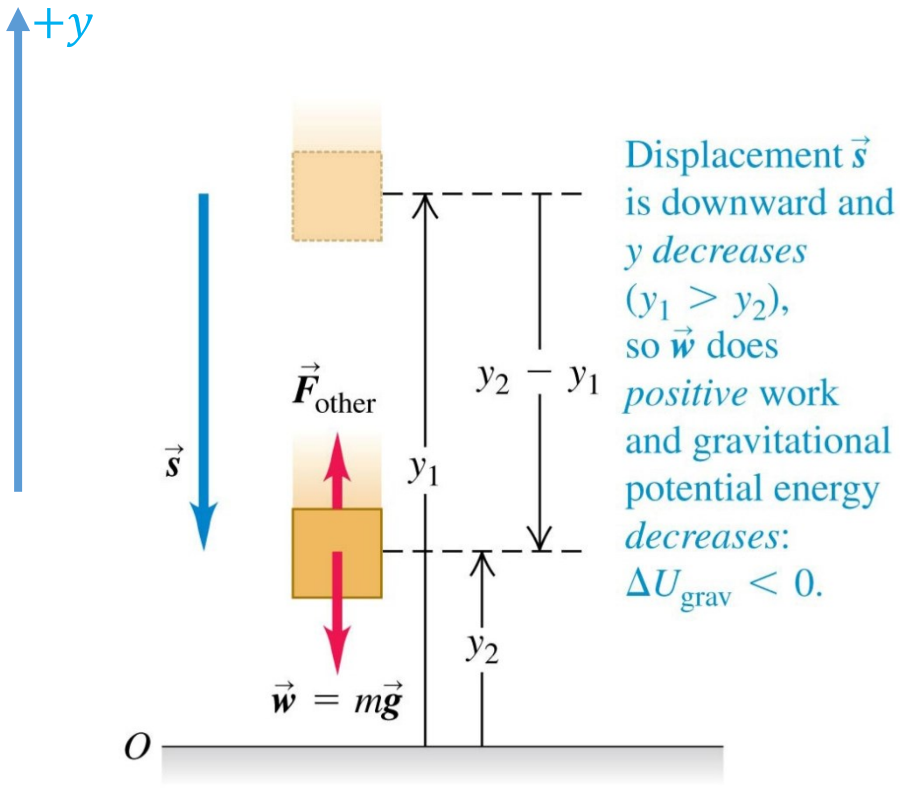
For a falling object, its kinetic energy increases as it is falling down.

Q: Where does the kinetic energy come from?

The "earth-object system" releases some energy! This energy is converted to the kinetic energy!

Gravitational Potential Energy!

Gravitational Potential Energy



Work done **by** the gravity on the block, from y_1 to y_2

$$W_{grav} = Fs = (-w)(y_2 - y_1)$$
$$= mgy_1 - mgy_2$$

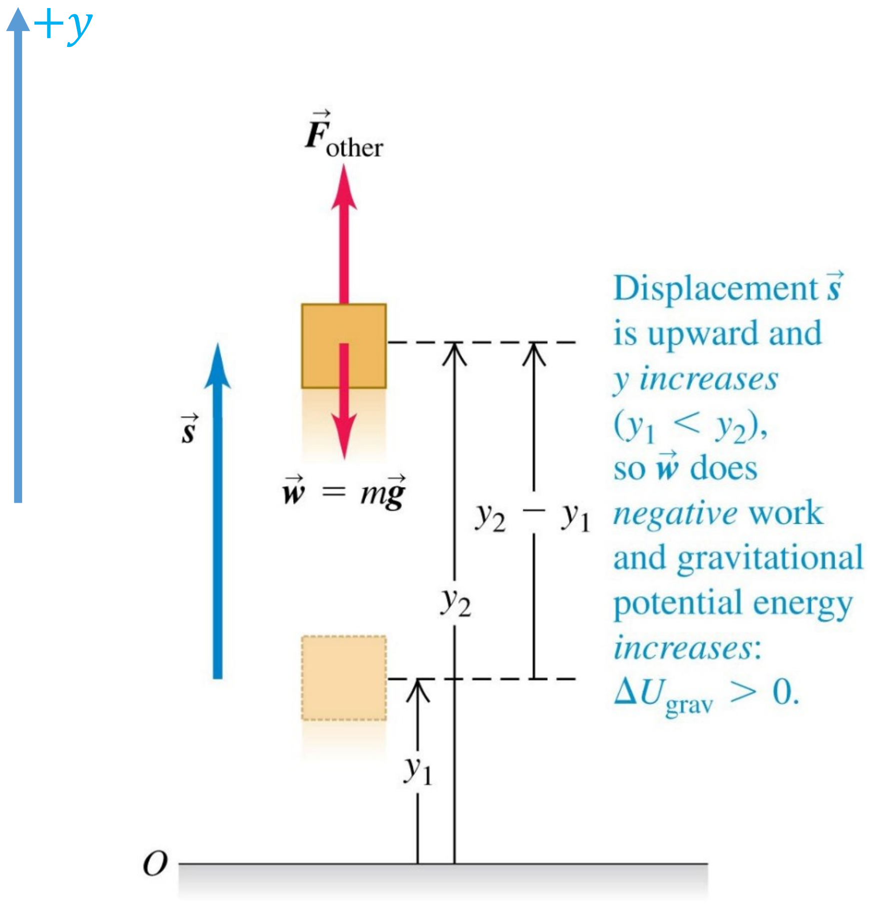
$$U_{grav} = mgy$$

$$W_{grav} = U_{grav,1} - U_{grav,2}$$
$$= -(U_{grav,2} - U_{grav,1}) = -\Delta U_{grav}$$

$$W_{grav} = -\Delta U_{grav}$$

The work done by gravity is equal to the negative change in gravitational potential energy.

Gravitational Potential Energy



Work done **by** the gravity on the block, from y_1 to y_2

$$W_{grav} = (-w)(y_2 - y_1)$$
$$= mgy_1 - mgy_2$$
$$= -\left(U_{grav,2} - U_{grav,1}\right)$$
$$= -\Delta U_{grav}$$

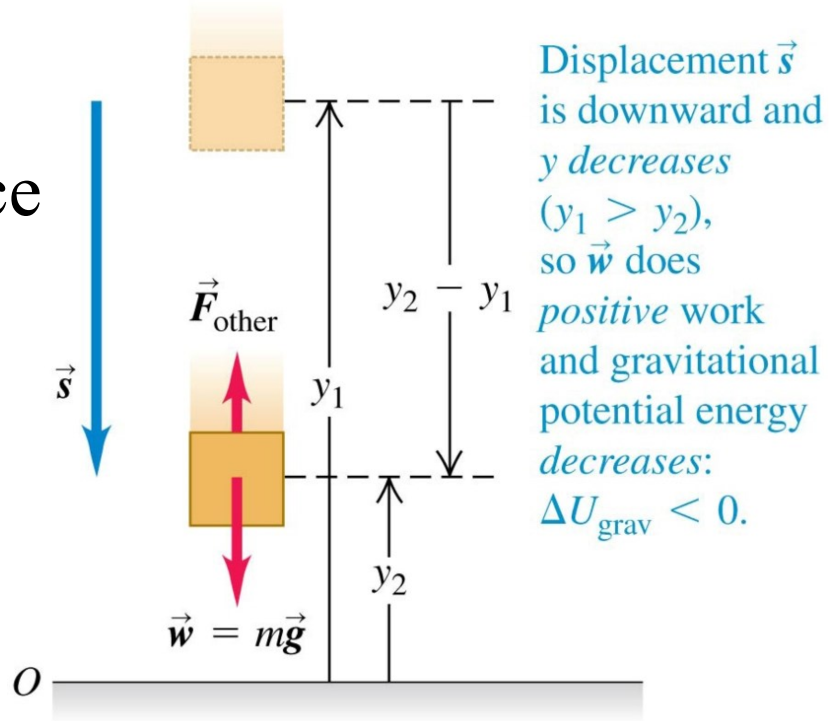
$$U_{grav} = mgy$$

$$W_{grav} = -\Delta U_{grav}$$

The work done by gravity is equal to the negative change in gravitational potential energy.

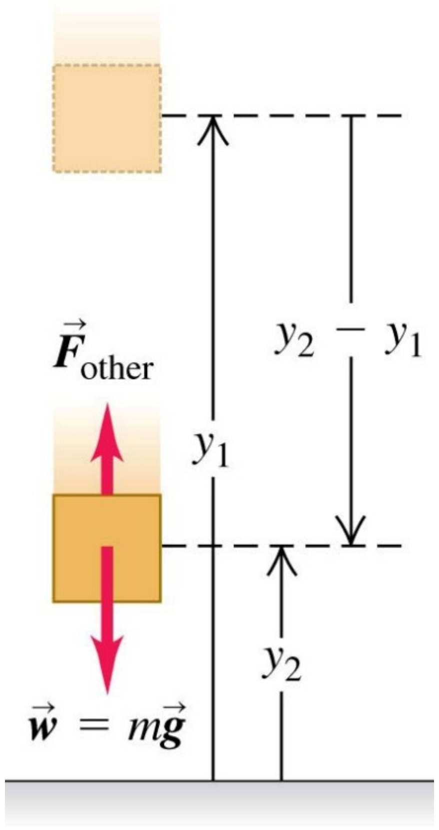
Potential Energy

- The gravitational potential energy: $U_{grav} = mgy$
  m is the mass of an object
  g is the acceleration of gravity
  y is the vertical position relative to the Earth surface
- SI unit: joule (J)
- Gravitational Potential Energy is the energy associated with the relative position of an object in space near the Earth's surface
- **Shared by both the object and Earth**



Reference Level

- A **location** where the gravitational **potential energy is zero** must be chosen for each problem
- The choice is **arbitrary**…
- So choose a convenient location for the zero-reference height: Often the Earths' surface;
  May be some other point suggested by the problem
- Once the position is chosen, it must remain fixed for the entire problem.
- The gravitational potential energy of an object can be zero, negative or positive.



Example: A **50.0 kg** marathon runner runs up the stairs to the top of **443-m**-tall Tower. To lift herself to the top in **15.0 minutes**, what must be her **average power** output?

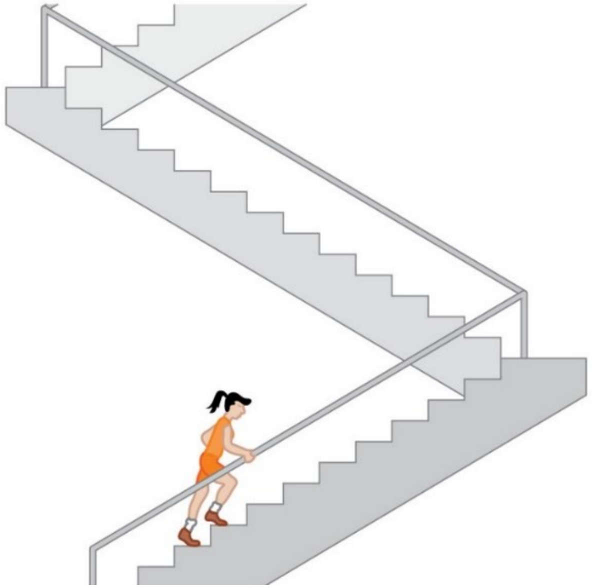
$$P_{av} = \frac{\Delta W}{\Delta t} \qquad W_{grav} = -\Delta U_{grav}$$

(A). 241 kW.

(B). 0.217 kW.

(C). 217 kW.

(D). 0.241 kW. ✓



$$W_{runner} = \Delta U_{grav} = mgh$$
$$= (50.0\ \text{kg})\,(9.8\ \text{m/s}^2)\,(443\ \text{m})$$
$$= 2.17\times10^5\ \text{J}$$

15 min = 900 s

$$P_{av} = \frac{2.17\times105}{900\ s} = 241\ \text{W}$$
$$= 0.241\ \text{kW} = 0.323\ \text{hp}$$

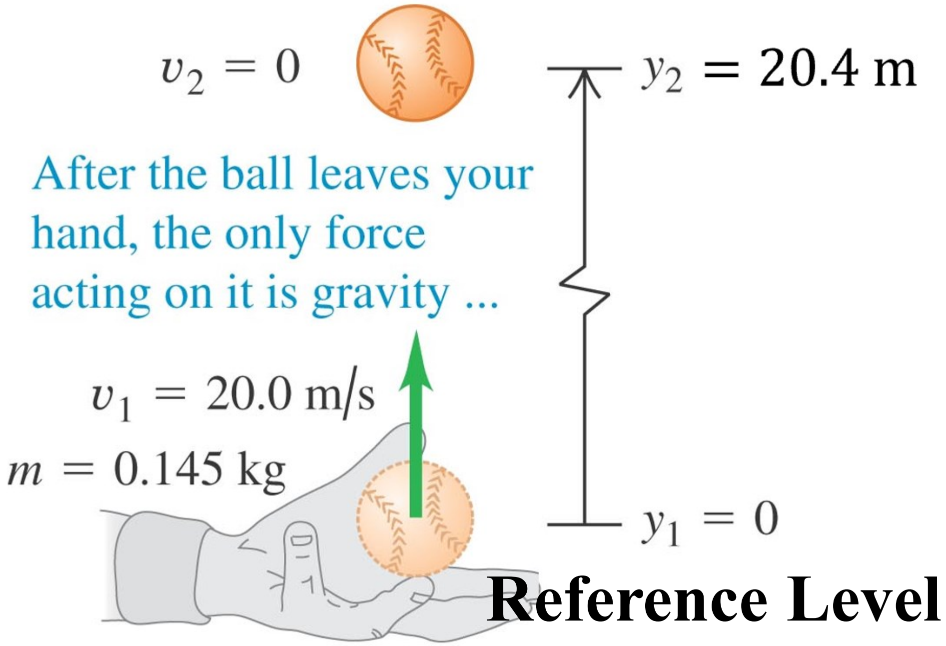
Example: Gravitational Potential Energy

You throw a **0.145-kg** baseball straight up, giving it an initial velocity of magnitude **20.0 m/s**, it flies up 20.4 m and reaches the highest point. What is the change in its gravitational potential energy?

$$\Delta U_{grav} = mgy_2 - mgy_1$$

$$= (0.145\ kg)(9.8)(20.4\ m) - 0 = 28.99\ J$$

What happened to the kinetic energy?



Reference Level

Extended work-energy theorem

- The work-kinetic energy theorem can be extended to include gravitational potential energy:

$$W_{\text{tot}} = K_2 - K_1 = \Delta K$$

$$W_{\text{grav}} = U_{\text{grav},1} - U_{\text{grav},2} = -\left(U_{\text{grav},2} - U_{\text{grav},1}\right) = -\Delta U_{\text{grav}}$$

- If we only have gravitational force and all work done by all rest forces are zero, then

$$W_{tot} = W_{grav} \qquad K_2 - K_1 = U_{\text{grav},1} - U_{\text{grav},2}$$

$$K_1 + U_{\text{grav},1} = K_2 + U_{\text{grav},2}$$

$K + U$ does not change

Conservation of Mechanical Energy

- $K + U$ is called the total mechanical energy: $K_1 + U_{grav,1} = K_2 + U_{grav,2}$

$$E = K + U_{grav}$$

- Since $K_1 + U_{grav,1} = K_2 + U_{grav,2}$
- So $E = K + U_{grav} = \text{constant}$

A quantity that always has the same value is called a *conserved* quantity.

- The total mechanical energy is conserved and remains the same at all times

$$\tfrac{1}{2}mv_1^{\,2} + mgy_1 = \tfrac{1}{2}mv_2^{\,2} + mgy_2$$

(If only gravity does work)

Example: Conservation of Mechanical Energy

You throw a **0.145-kg** baseball straight up, giving it an initial velocity of magnitude **20.0 m/s**. Find how high it goes, ignoring air resistance.

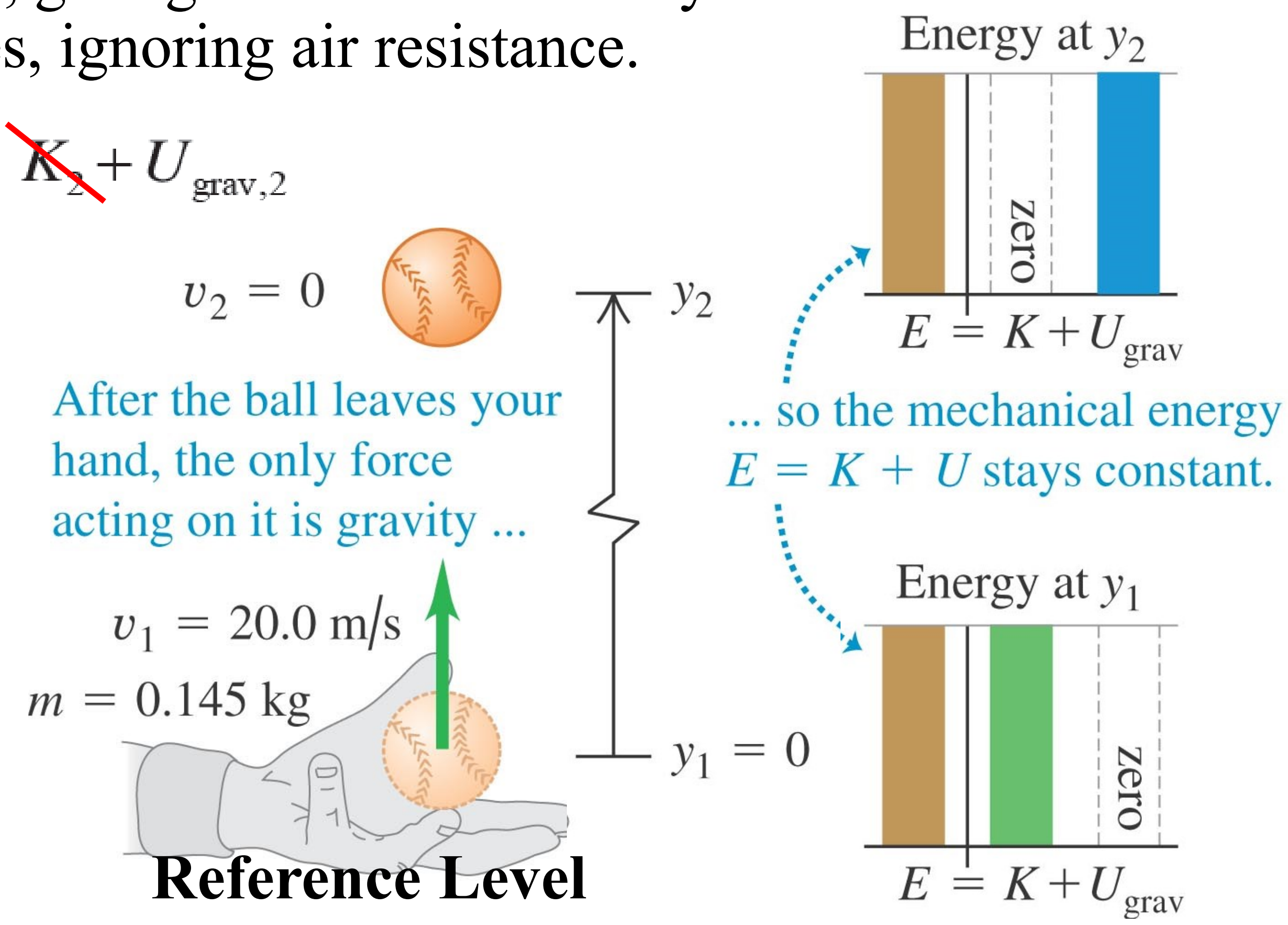
$$E = K + U_{grav} = \text{constant} \qquad K_1 + U_{grav,1} = K_2 + U_{grav,2}$$

We have $y_1 = 0, U_{grav,1} = 0 = mgy_1 = 0$

$$K_2 = \frac{1}{2}mv_2^2 = 0 \qquad K_1 = U_{grav,2}$$

$$\frac{1}{2}mv_1^2 = mgy_2$$

$$y_2 = \frac{v_1^2}{2g} = \frac{(20.0\,m/s)^2}{2(9.8\,m/s^2)} = 20.4\,m$$



Example:

You toss a **0.150-kg** baseball straight upward so that it leaves your hand moving at **20.0 m/s**. The ball reaches a maximum height $\boldsymbol{y_2}$. What is the speed of the ball when it is at a height of $\boldsymbol{y_2/2}$? Ignore air resistance.

(A), 10.0 m/s

(B), less than 10.0 m/s but > 0.

(C), greater than 10.0 m/s

(D), zero

(E), Not enough information.

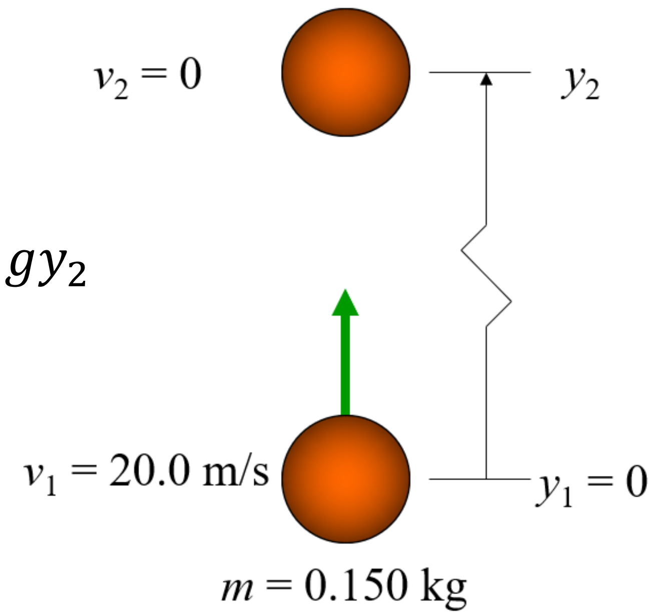
$$mgy_1 + \frac{1}{2}mv_1^2 = mgy_2 + \frac{1}{2}mv_2^2$$

$$0 + \frac{1}{2}mv_1^2 = mgy_2 + 0 \qquad \frac{1}{2}v_1^2 = gy_2$$

$$mgy_1 + \frac{1}{2}mv_1^2 = m\frac{gy_2}{2} + \frac{1}{2}mv_x^2$$

$$0 + \frac{1}{2}mv_1^2 = m\frac{1}{4}v_1^2 + \frac{1}{2}mv_x^2$$

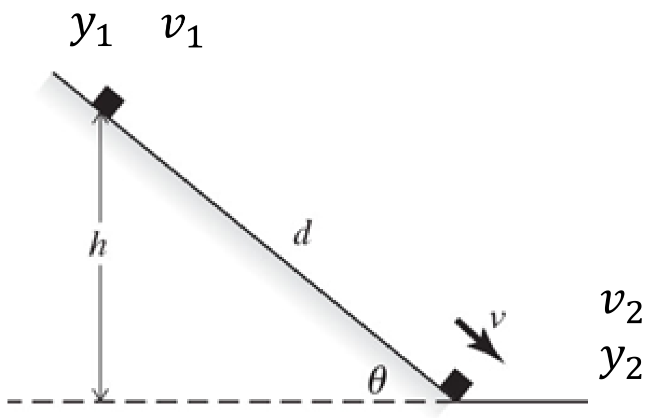
$$\frac{1}{2}v_1^2 = v_x^2 \qquad v_x = \frac{20}{\sqrt{2}}$$



Reference Level

Example: inclined surface

A **5.00 kg** block is released from rest on a ramp that is inclined at an angle of **60.0°** below the horizontal. The initial position of the block is a vertical distance of **2.00 m** above the bottom of the ramp. What is the speed of the block when it reaches the bottom? Ignore friction force.



No friction, only gravity does work,
K + *U* does not change

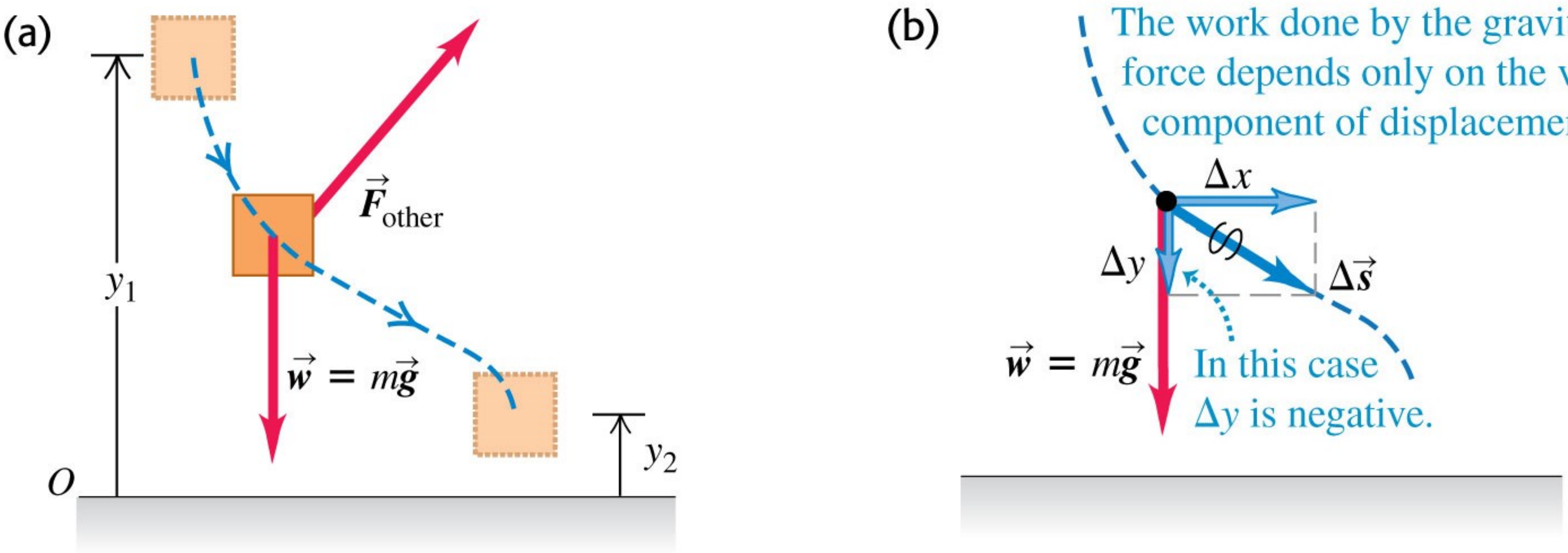
$$mgy_1 + \frac{1}{2}mv_1^2 = mgy_2 + \frac{1}{2}mv_2^2$$

$$(5\,kg)(9.8)(2\,m) + \frac{1}{2}m(0)^2 = mg(0) + \frac{1}{2}(5\,kg)v_2^2$$

$$v_2 = 6.26\,m/s$$

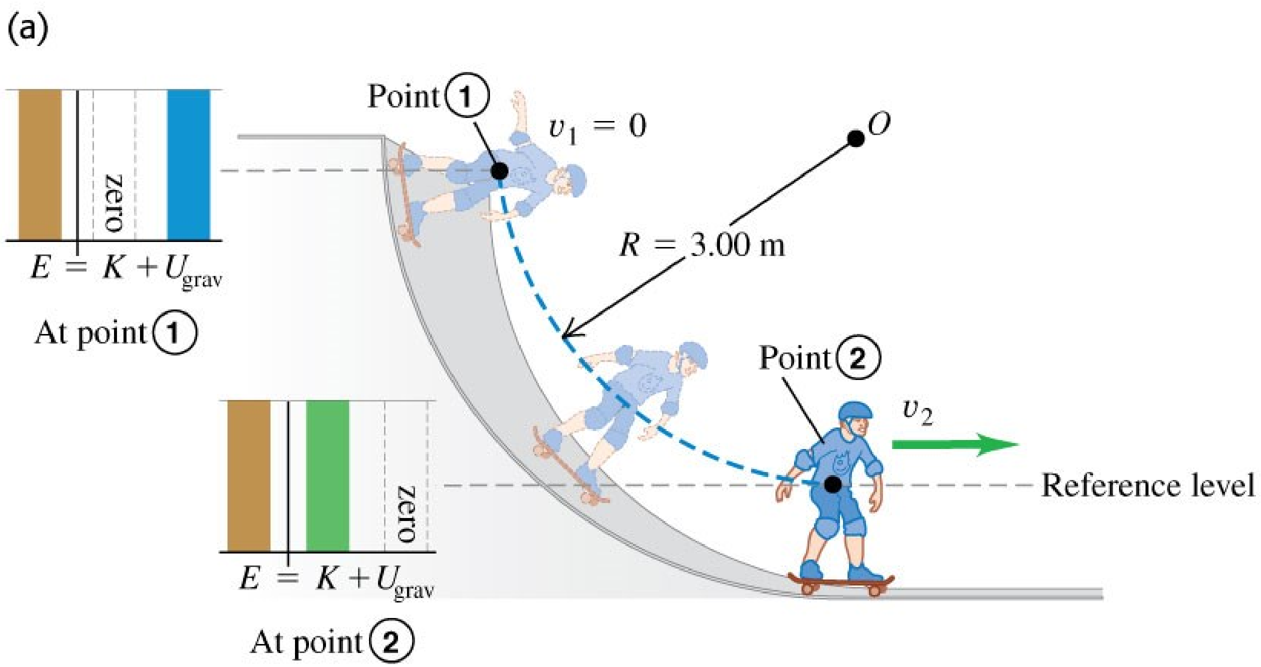
Work and energy along a curved path

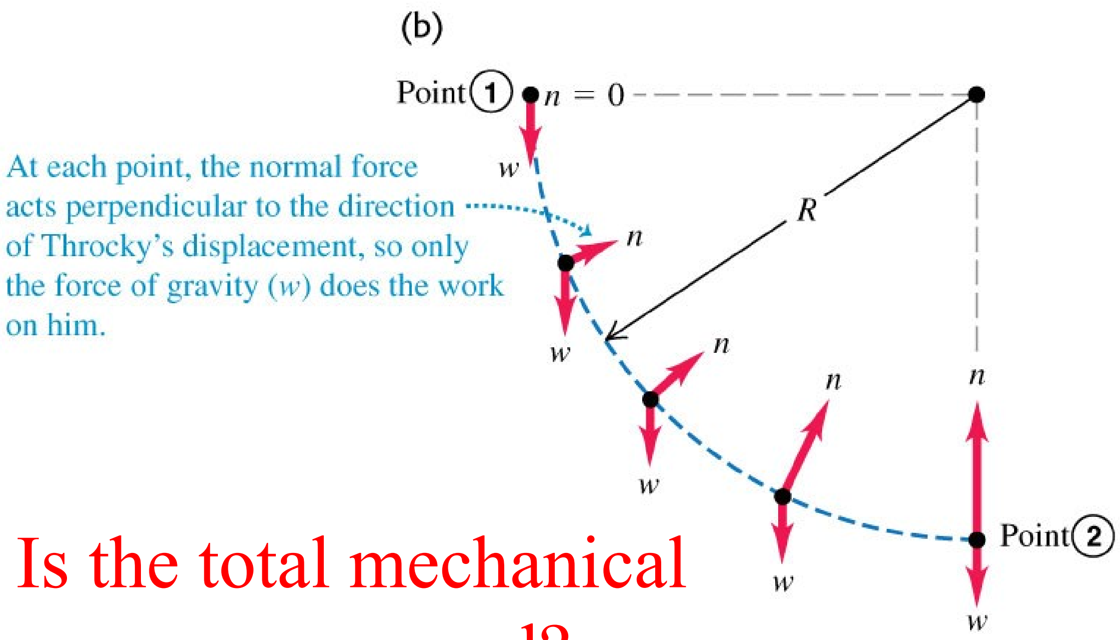
- We can use the same expression for gravitational potential energy whether the body's path is curved or straight.



Example: Motion in a vertical circle without friction

A person+skateboard (total mass **25.0 kg**) from rest down a curved, **no friction**. If we treat them as a particle, they moves through a quarter-circle with radius ***R* = 3.00 m**. Find the **speed at the bottom** of the ramp.



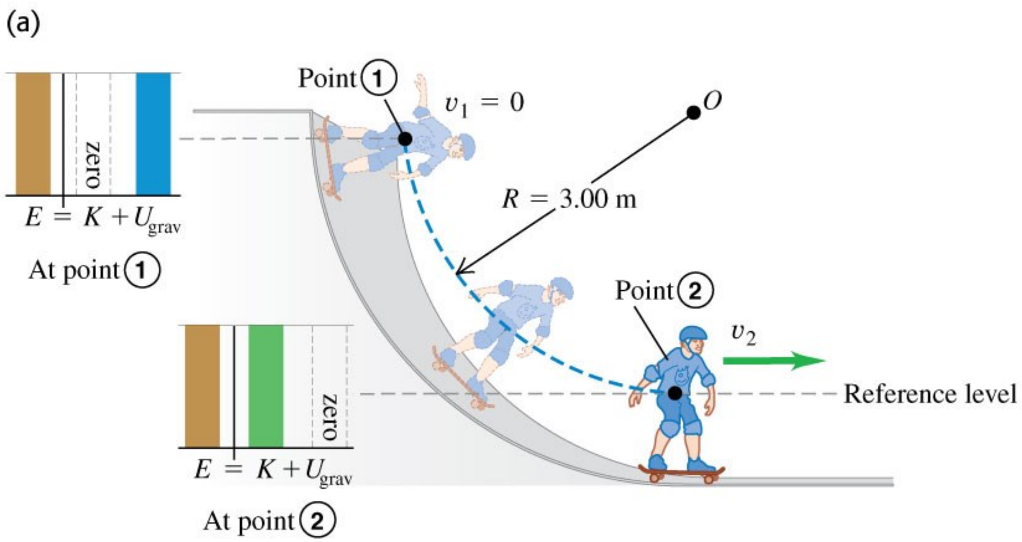


Is the total mechanical energy conserved?

Example: Motion in a vertical circle without friction

A person+skateboard (total mass **25.0 kg**) from rest down a curved, **no friction**. If we treat them as a particle, they moves through a quarter-circle with radius ***R* = 3.00 m**. Find the speed at the bottom of the ramp.

$$E = K + U_{grav} = \text{constant}$$



$$K_1 = 0 \qquad U_{grav,1} = mgR$$

$$K_2 = \frac{1}{2} m v_2^2 \qquad U_{grav,2} = 0$$

For conservation of mechanical energy,

$$K_1 + U_{grav,1} = K_2 + U_{grav,2}$$

$$0 + mgR = \frac{1}{2} m v_2^2 + 0$$

$$v_2 = \sqrt{2gR} = \sqrt{2(9.80\, m/s^2)(3.00\, m)} = 7.67\, m/s$$