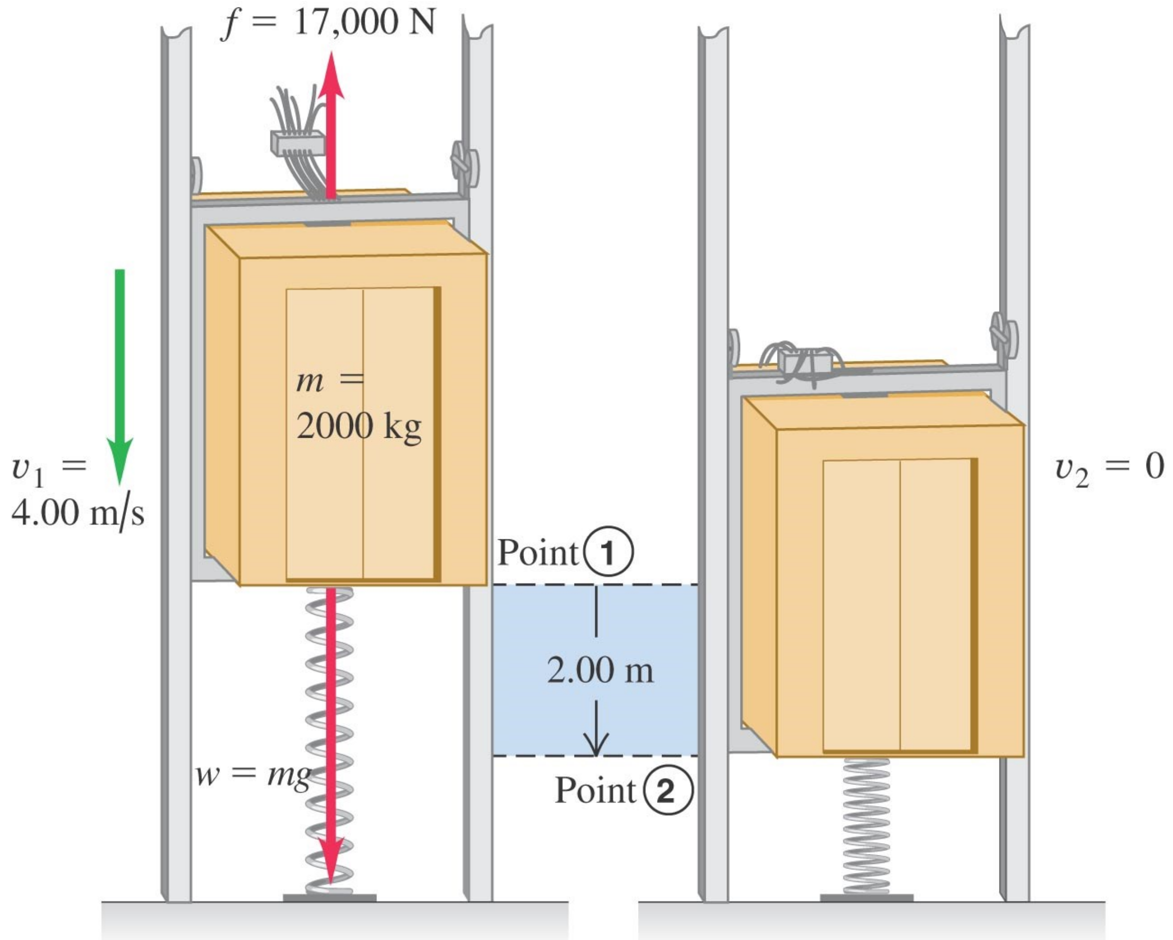
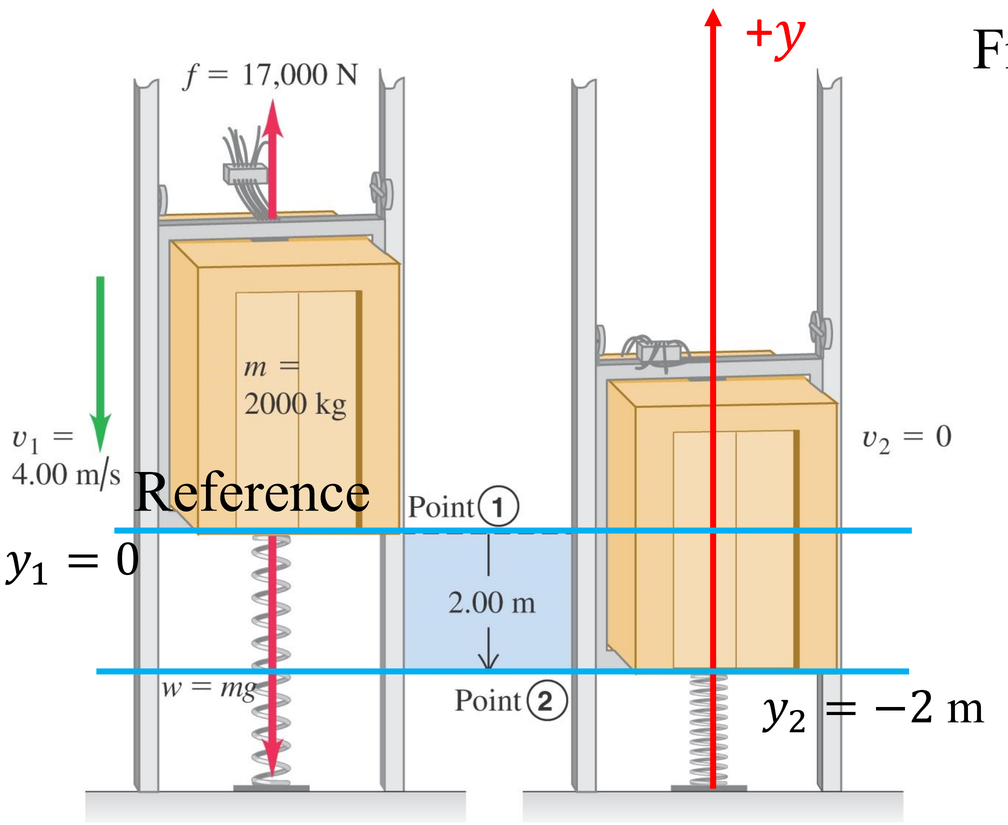
Review: A system having two potential energies and friction

A 2000-kg (19,600-N) elevator with broken cables in a test rig is falling at 4.00 m/s when it contacts a cushioning spring at the bottom of the shaft. The spring is intended to stop the elevator, compressing 2.0 m as it does so. During the motion a safety clamp applies a constant 17,000-N frictional force to the elevator. What is the necessary force constant ***k*** for the spring?



Review: A system having two potential energies and friction



Friction $K_1 + \cancel{U_1} + W_{other} = \cancel{K_2} + U_2$

$U_1 = U_{grav} + U_{el} = 0$

$K_2 = 0$

$$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(2000\,kg)(4.0\,m/s)^2 = 16{,}000\,J$$

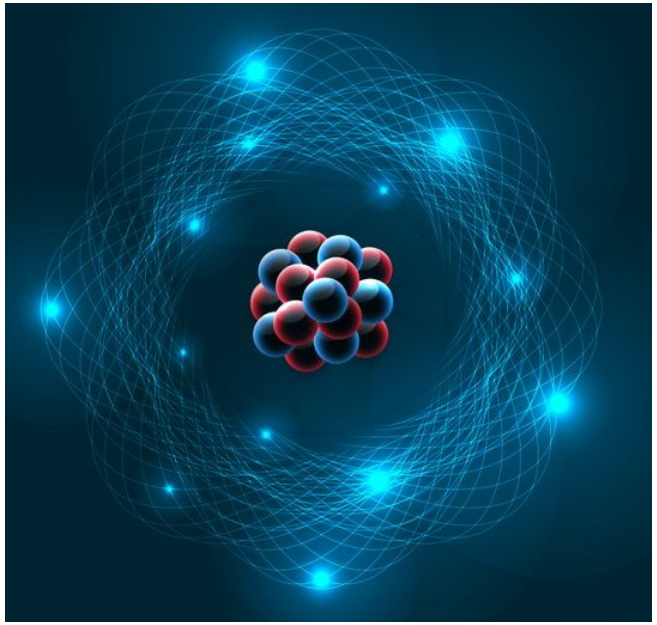
$$W_{other} = W_f = Fd = -(17{,}000\,N)(2.0\text{ m}) = \text{-34,000 J}$$

$$U_2 = mgy_2 + \frac{1}{2}ky_2^2$$

$$mgy_2 = (2000\,kg)(9.8\,m/s^2)(-2.0\text{ m}) = \text{-39,200 J}$$

$$16{,}000 + 0 + (-34{,}000) = 0 + (-39{,}200) + \frac{1}{2}k(-2.0\,m)^2$$

$$k = \frac{2[16{,}000\text{ J} + (-34{,}000\text{ J}) - (-39{,}200\,J)]}{(-2.00\,m)^2} = 1.06 \times 10^4\text{ N/m}$$



Physics 111: Mechanics Chapter08

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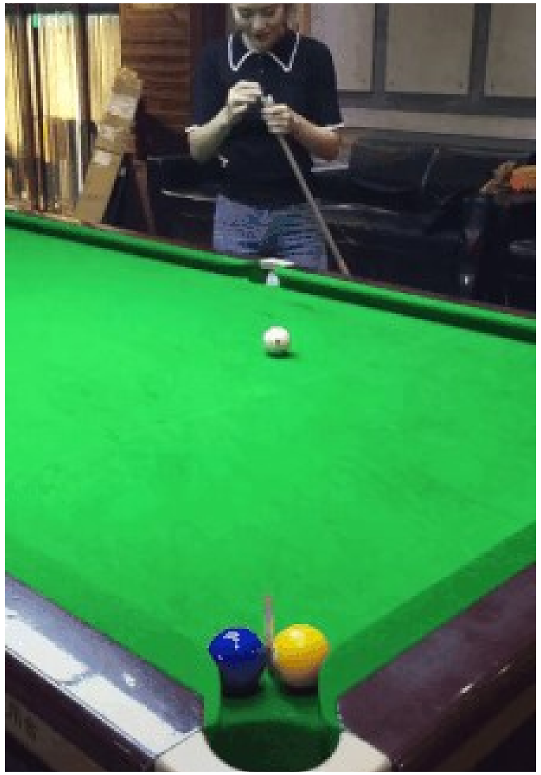
Momentum, Impulse, and Collisions

- To learn the meaning of the *momentum* of a particle and how an *impulse* causes it to change
- To learn how to use the *conservation of momentum*
- To learn how to solve problems involving *collisions*
- To learn the definition of the center of mass of a system and what determines how it moves



Rediscover Newton's 2nd Law

- Newton's 2nd law: $\vec{F}_{net} = \sum \vec{F} = m\vec{a}$

$$\vec{F}_{net} = m\vec{a} = m d\vec{v} / dt = d(m\vec{v}) / dt$$

- Acceleration: $\vec{a} = d\vec{v} / dt$
- $\boldsymbol{m\vec{v}}$ is called "**momentum**": $\vec{\boldsymbol{p}} = \boldsymbol{m\vec{v}}$

Newton's second law in terms of momentum: The net external force acting on a particle ... $\sum \vec{F} = \dfrac{d\vec{p}}{dt}$ ... equals the rate of change of the particle's momentum.

$$\sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t}$$

- A new **vector**: It depends on the mass and velocity of an object.
- The terms momentum and linear momentum will be used interchangeably in the text.

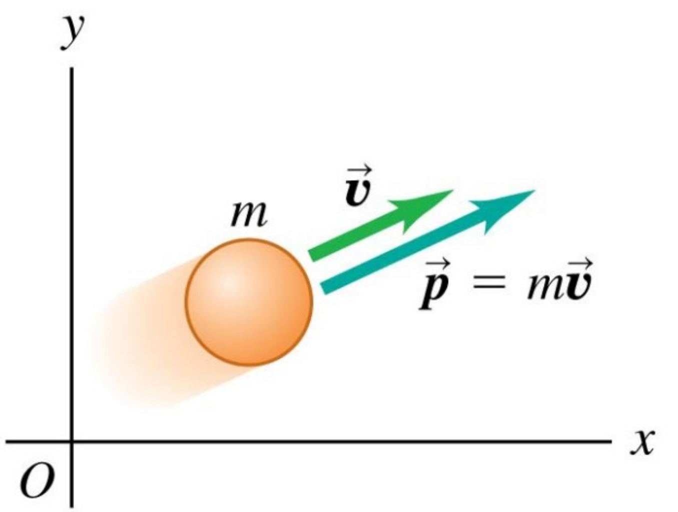
Linear Momentum

- Linear momentum: $\vec{\boldsymbol{p}} = \boldsymbol{m}\vec{\boldsymbol{v}}$
- What is the direction of momentum?
- Its direction is the same as the direction of the velocity

- Momentum can be expressed in its components

$$p_x = mv_x, p_y = mv_y, p_z = mv_z$$

- The SI units of momentum are kg. m/s.



Momentum $\vec{p}$ is a vector quantity; a particle's momentum has the same direction as its velocity $\vec{v}$.

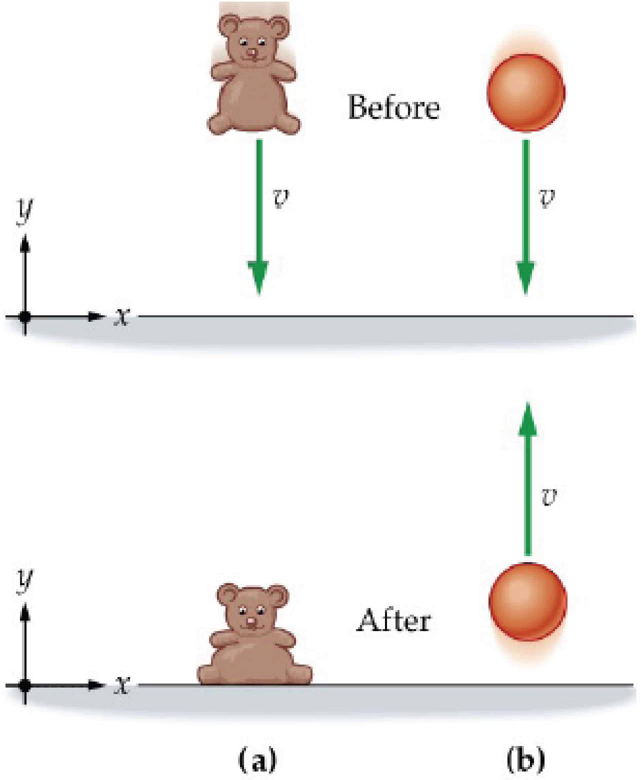
Example: Calculate the change in momentum

A teddy bear and a ball are both dropped vertically downward with speed $\boldsymbol{v}$ and hit the ground. The teddy bear stops after hitting the ground, while the ball bounces back upward with the same speed $\boldsymbol{v}$. **Q:** What is the change in momentum for the teddy bear and for the bouncing ball?

Momentum change: $\Delta\vec{p} = \vec{p}_2 - \vec{p}_1 = m\vec{v}_2 - m\vec{v}_1 = m(\vec{v}_2 - \vec{v}_1)$

For the teddy bear $\Delta p = m[0 - (-v)] = mv$

For the bouncing ball $\Delta p = m[v - (-v)] = 2mv$



More generally, in 2D or 3D space, we have to consider the vector nature of momentum.

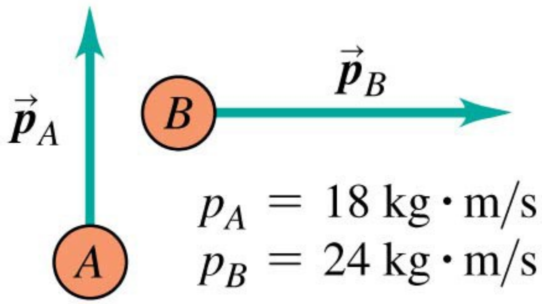
Momentum is a vector

- Use vector addition to add momenta

$$\vec{P} = \vec{p}_A + \vec{p}_B + \cdots = m_A\vec{v}_A + m_B\vec{v}_B + \cdots$$

- Components of the momenta are also conserved.

$$P_x = p_{Ax} + p_{Bx} + \cdots$$
$$P_y = p_{Ay} + p_{By} + \cdots$$
$$P_z = p_{Az} + p_{Bz} + \cdots$$

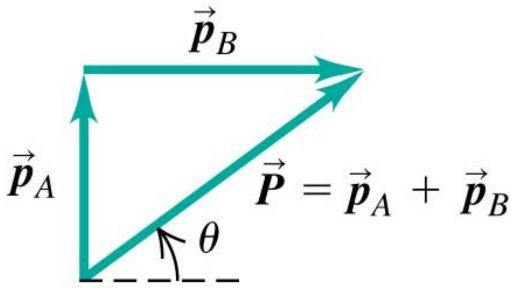


A system of two particles with momenta in different directions

What is the total momenta?

$$p_A + p_B = 18 + 24 = 42?$$

Instead, use vector addition:



◀ **RIGHT!**

$$P = |\vec{p}_A + \vec{p}_B| = 30\ \mathrm{kg \cdot m/s}\ \text{at}\ \theta = 37^\circ$$

You CANNOT find the magnitude of the total momentum by adding the magnitudes of the individual momenta!

$$P = p_A + p_B = 42\ \mathrm{kg \cdot m/s}$$ ◀ **WRONG**

Impulse

$$\sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t} \qquad \Delta\vec{p} = \vec{F}\Delta t = \vec{F}(t_2 - t_1) = \vec{J} \qquad \Rightarrow \vec{J} = \Delta\vec{p}$$

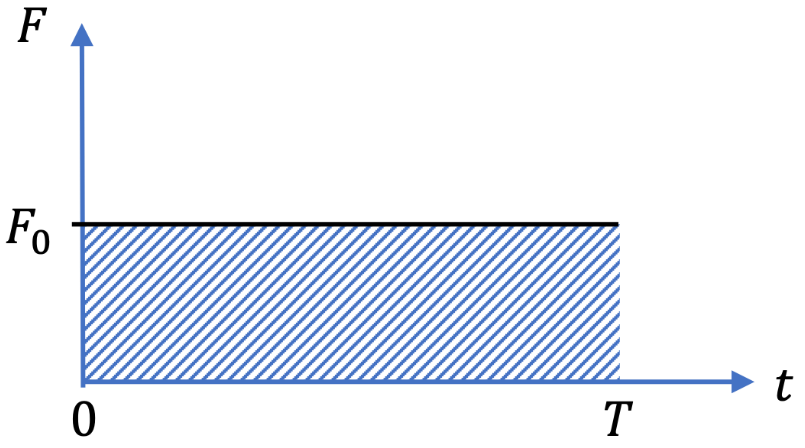
- The ***impulse*** of a force is the product of the force and the time interval during which it acts.

Impulse of a **constant net external force** → $\vec{J} = \sum\vec{F}(t_2 - t_1) = \sum\vec{F}\Delta t$

Constant net external force

Time interval over which net external force acts

On a graph of ΣF_x versus time, the impulse is equal to the area under the curve, as shown in Figure below.



The effect of impulse is changing the momentum of an object.

Impulse-Momentum Theorem

- The *theorem* states that the impulse acting on a system is equal to the change in the momentum of the system

$$\vec{J} = \Delta\vec{p} = m\vec{v}_f - m\vec{v}_i = \vec{p}_f - \vec{p}_i$$

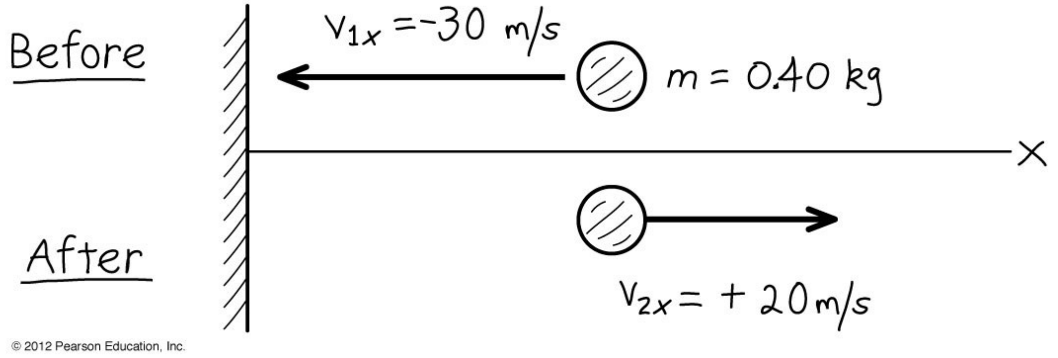
Component form

$$J_x = p_{fx} - p_{ix} = mv_{fx} - mv_{ix}$$

$$J_y = p_{fy} - p_{iy} = mv_{fy} - mv_{iy}$$

Example:

A ball (mass **0.40 kg**) is initially moving to the left at **30 m/s**. After hitting the wall, the ball is moving to the right at **20 m/s**. What is the **impulse of the net force** on the ball?



$$\vec{J} = \Delta\vec{p} = m\vec{v}_f - m\vec{v}_i$$

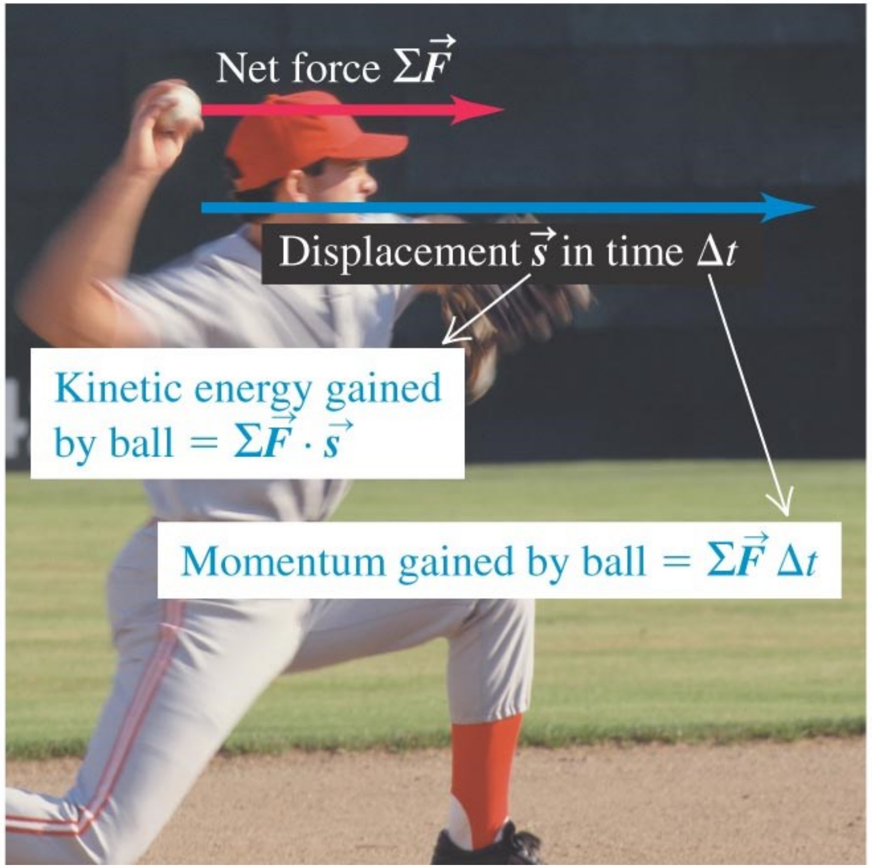
$\Delta\vec{p}$ = (0.4 kg) • [+20 m/s – (-30 m/s)]
= (0.4 kg) • (+50 m/s) = 20 kg • m/s

Compare momentum and kinetic energy

- Changes in momentum depend on the *time* over which the net force acts, but changes in kinetic energy depend on the *distance* over which the net force acts.

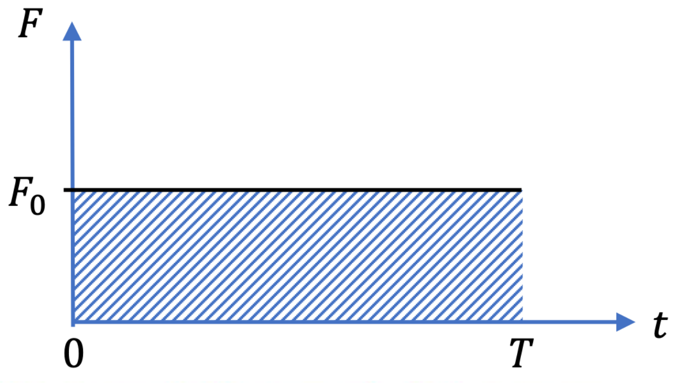
$$\vec{J} = \vec{F}\Delta t = \Delta\vec{p} = m\vec{v}_f - m\vec{v}_i$$

$$W = \int_{x_1}^{x_2} F_x dx = W_{tot} = K_2 - K_1 = \Delta K = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$



Impulse and Average Force

- For a constant force, the impulse is equal to the area under the curve, as shown in Figure below.



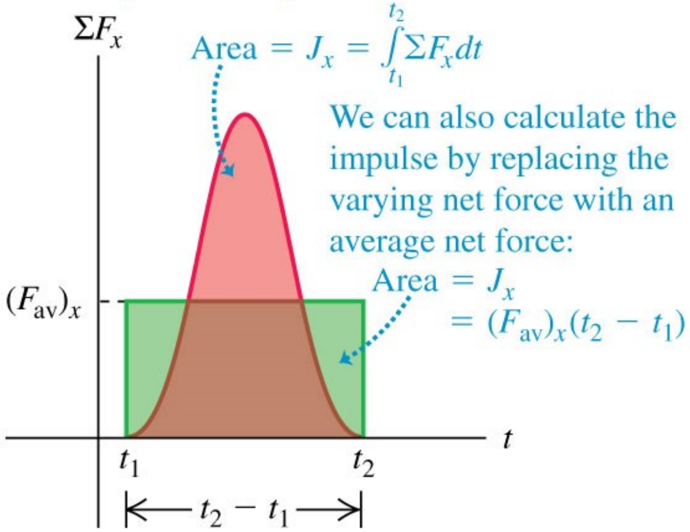
$$\vec{J} = \Delta\vec{p} = \vec{F}\Delta t \qquad \Rightarrow \vec{F} = \frac{\vec{J}}{\Delta t}$$

Q: How about a varying force? How to calculate the impulse and how to find out the force?

Impulse of a **general net external force** (either constant or varying) $\vec{J} = \int_{t_1}^{t_2} \Sigma\vec{F}dt$

- Upper limit = final time
- Time integral of net external force
- Lower limit = initial time

The area under the curve of net force versus time equals the impulse of the net force:

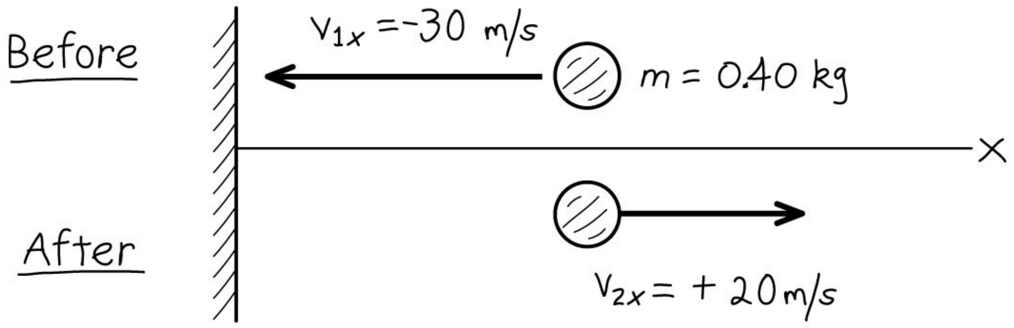


We can define "**Average force**"

$$\int_{t_1}^{t_2} \sum \vec{F}\, dt = \vec{J} = \vec{F}_{avg}(t_2 - t_1) = \Delta\vec{p} \qquad \Rightarrow \vec{F}_{avg} = \frac{\Delta\vec{p}}{\Delta t}$$

Example:

A ball (mass **0.40 kg**) is initially moving to the left at **30 m/s**. After hitting the wall, the ball is moving to the right at **20 m**/s. The collision takes **2.0 s**. What is the **average force** on the ball during its collision with the wall?



$$\vec{F}_{avg} = \frac{\Delta\vec{p}}{\Delta t}$$

$$\vec{J} = \Delta\vec{p} = m\vec{v}_f - m\vec{v}_i$$

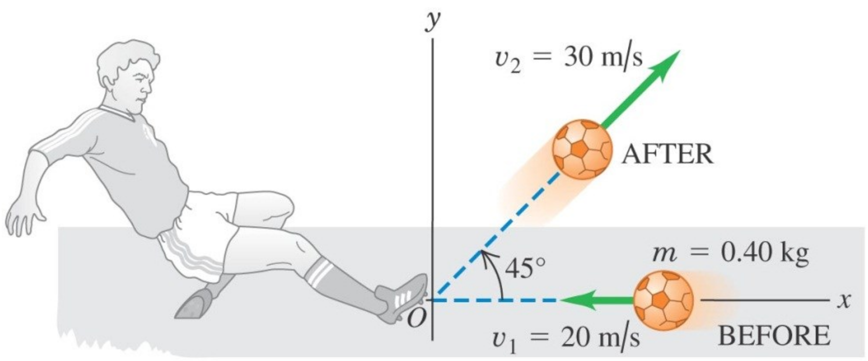
$$\vec{J} = \Delta\vec{p} = (0.4\text{ kg}) \bullet [+20\text{ m/s} - (-30\text{ m/s})] = (0.4\text{ kg}) \bullet (+50\text{ m/s}) = 20\text{ kg} \bullet \text{m/s}$$

$$\vec{F}_{avg} = \frac{\Delta\vec{p}}{\Delta t} = \frac{20\text{ kg} \bullet \text{m/s}}{2\text{ s}} = 10\text{ N}$$

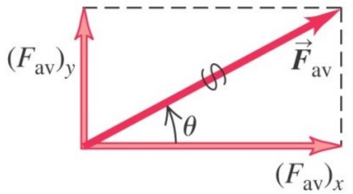
Example: Impulse and average force in 2D

A soccer ball has a mass of **0.4 kg**. Initially it is moving to the left at **20 m/s**, but then it is kicked. After the kick it is moving at **45º** upward and to the right with speed **30 m/s**. Find the *impulse* of the net force and the *average net force*, assuming a collision time ***Δt* = 0.01 s**.

(a) Before-and-after diagram



(b) Average force on the ball



$$\vec{F}_{avg} = \frac{\Delta\vec{p}}{\Delta t}$$

Q: Which quantity do we need to find first?

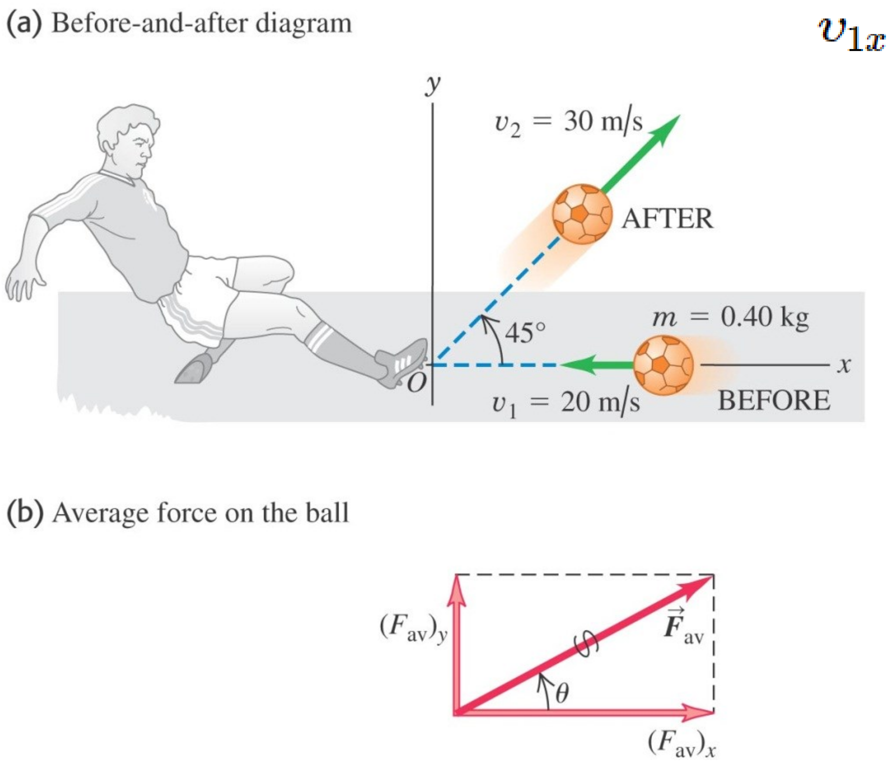
We need to find $\Delta\vec{p}$

In 2D, we need to find Δp_x and Δp_y

So, we need to find $m\Delta v_x$ and $m\Delta v_y$

Example: Impulse and average force in 2D

A soccer ball has a mass of **0.4 kg**. Initially it is moving to the left at **20 m/s**, but then it is kicked. After the kick it is moving at **45º** upward and to the right with speed **30 m/s**. Find the *impulse* of the net force and the *average net force*, assuming a collision time **Δt = 0.01 s**.



$$v_{1x} = -20\ \mathrm{m/s} \quad v_{1y} = 0$$

Using $\cos 45^\circ = \sin 45^\circ = 0.707$

$$v_{2x} = v_{2y} = (30\,\mathrm{m/s})(0.707) = 21.2\,\mathrm{m/s}$$

$$\begin{aligned} J_x &= p_{2x} - p_{1x} = m\,(v_{2x} - v_{1x}) \\ &= (0.40\,\mathrm{kg})[21.2\,\mathrm{m/s} - (-20\,\mathrm{m/s})] = 16.5\,\mathrm{kg\cdot m/s} \\ J_y &= p_{2y} - p_{1y} = m(v_{2y} - v_{1y}) \\ &= (0.40\,\mathrm{kg})(21.2\,\mathrm{m/s} - 0) = 8.5\,\mathrm{kg\cdot m/s} \end{aligned}$$

$$(F_{av})_x = \frac{J_x}{\Delta t} = 1650\ \mathrm{N} \quad (F_{av})_y = \frac{J_y}{\Delta t} = 850\ \mathrm{N}$$

$$F_{av} = \sqrt{(1650\ \mathrm{N})^2 + (850\ \mathrm{N})^2} = 1.9 \times 10^3\ \mathrm{N} \qquad \theta = \arctan\frac{850\,\mathrm{N}}{1650\,\mathrm{N}} = 27^\circ$$

Total Momentum of Colliding Balls

Start from impulse-momentum theorem

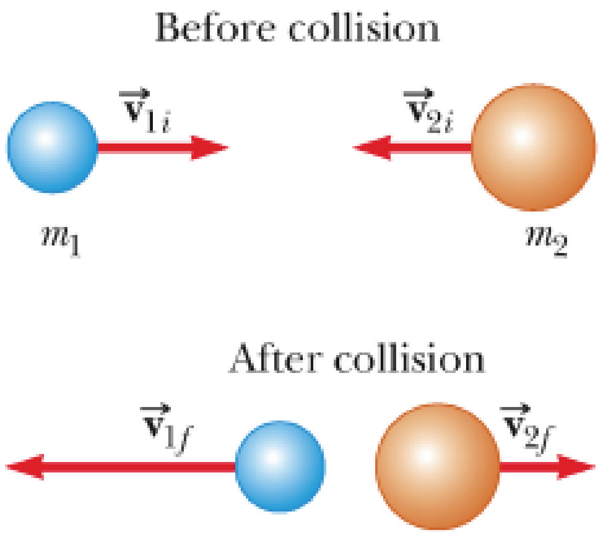
$$\vec{F}_{21}\Delta t = m_1\vec{v}_{1f} - m_1\vec{v}_{1i} \qquad \vec{F}_{12}\Delta t = m_2\vec{v}_{2f} - m_2\vec{v}_{2i}$$

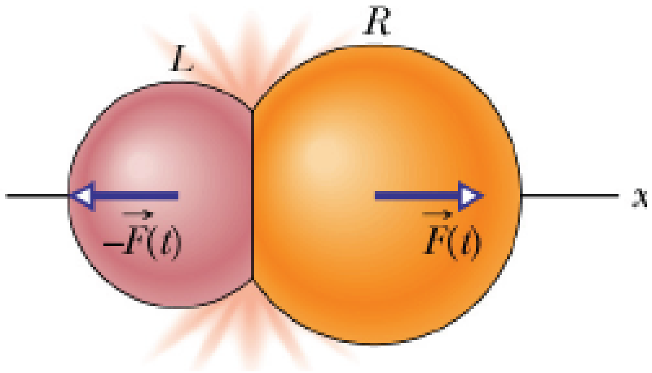
Newton's 3rd law $\quad \vec{F}_{21}\Delta t = -\vec{F}_{12}\Delta t$

$$m_1\vec{v}_{1f} - m_1\vec{v}_{1i} = -(m_2\vec{v}_{2f} - m_2\vec{v}_{2i})$$

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

Looks familiar??





Conservation of Momentum

- The total momentum before the collision equals the total momentum after the collision:

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

- Require: External forces (such as the normal force and gravity) act on the system is zero.

$$\vec{F}_{net}\Delta t = \Delta\vec{p} = \vec{p}_f - \vec{p}_i,\ \vec{F}_{net} = 0, \text{ then } \Delta\vec{p} = 0$$

- Or the system is isolated system: no external force.
- **Conservation of momentum**: If the **vector sum of the external forces** on a system is **zero**, the total momentum of the system is constant.

Types of Collisions

- Elastic collisions: billiard ball.

  Both momentum and kinetic energy are conserved

- Inelastic collisions: Kinetic energy is not conserved
- Completely inelastic: collision occur when the objects stick together

Most collisions fall between elastic and completely inelastic collisions.

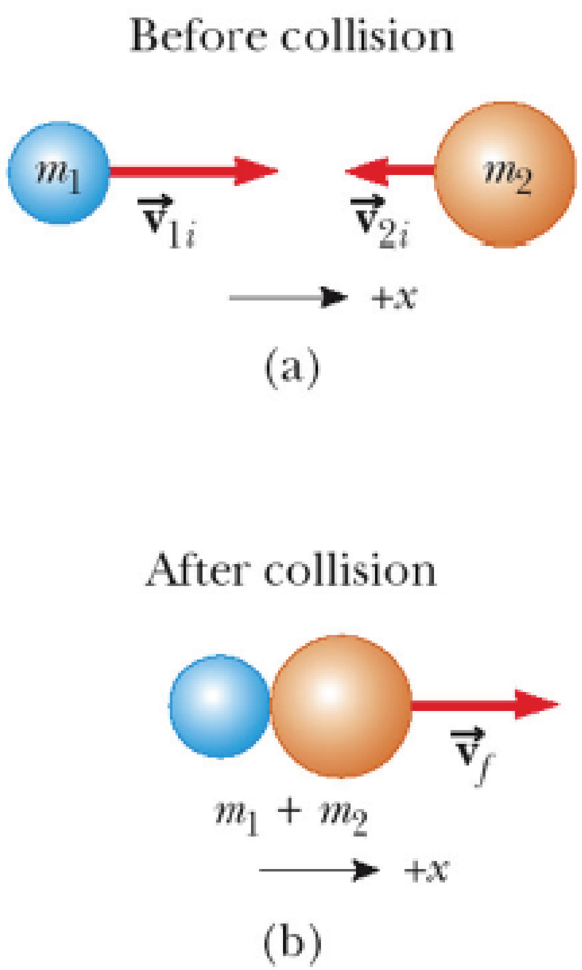
Completely Inelastic Collisions

- When two objects stick together after the collision, they have undergone a completely inelastic collision
- Conservation of momentum (external net force = 0)

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

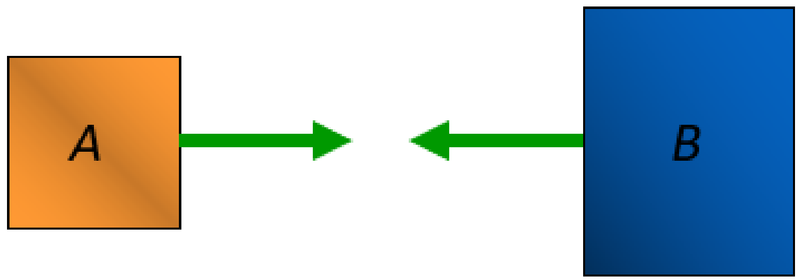
$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$

- Kinetic energy is NOT conserved.



Example:

Two objects with different masses collide and *stick* to each other. Compared to *before* the collision, the system of two objects *after* the collision has



(A). the same total momentum and the same total kinetic energy.

(B). the same total momentum but less total kinetic energy. ✓

(C). less total momentum but the same total kinetic energy.

(D). less total momentum and less total kinetic energy.

(E). not enough information given to decide

Example:

A 10-g bullet with a speed of 402 m/s is fired on a 2-kg duck sitting on an ice lake. Ignore friction.

Find the final speed of the duck and bullet in m/s?

Q: If we consider this as a collision problem, which type of collision? Elastic or inelastic?

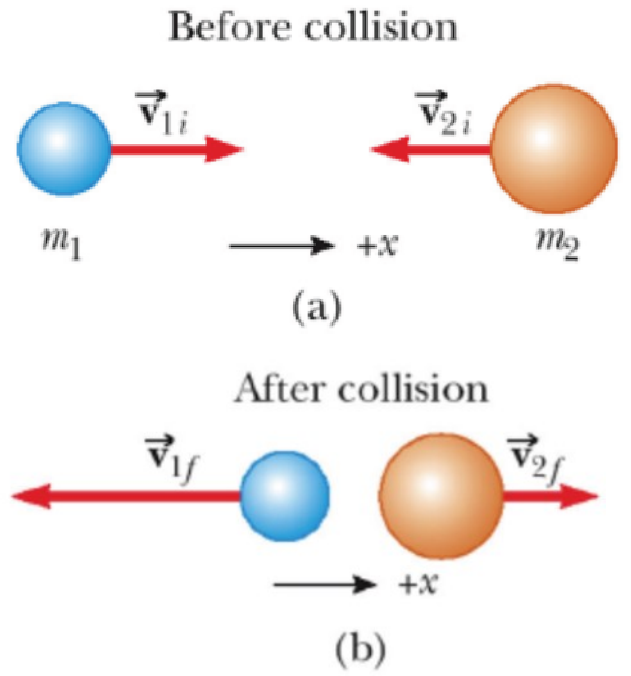
Total inelastic.

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1 + m_2)\vec{v}_f$$

$$(0.01\,kg)(402\,m/s) + (2\,kg)(0) = (0.01\,kg + 2\,kg)\vec{v}_f$$

$$\vec{v}_f = 2\,m/s$$

Elastic Collisions



- Conservation of momentum and kinetic energy.

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \longrightarrow m_1(v_{1i} - v_{1f}) = m_2(v_{2f} - v_{2i})$$

$$\cancel{\frac{1}{2}} m_1 v_{1i}^2 + \cancel{\frac{1}{2}} m_2 v_{2i}^2 = \cancel{\frac{1}{2}} m_1 v_{1f}^2 + \cancel{\frac{1}{2}} m_2 v_{2f}^2$$

- A simpler equation can be use in place of kinetic energy.

$$m_1(v_{1i}^2 - v_{1f}^2) = m_2(v_{2f}^2 - v_{2i}^2) \qquad \cancel{m_1(v_{1i} - v_{1f})}(v_{1i} + v_{1f}) = \cancel{m_2(v_{2f} - v_{2i})}(v_{2f} + v_{2i})$$

$$\boxed{v_{1i} + v_{1f} = v_{2f} + v_{2i}} \qquad \boxed{v_{1f} - v_{2f} = -(v_{1i} - v_{2i})} \qquad \boxed{m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}}$$

Momentum is vector: Be sure to have the **correct sign**.