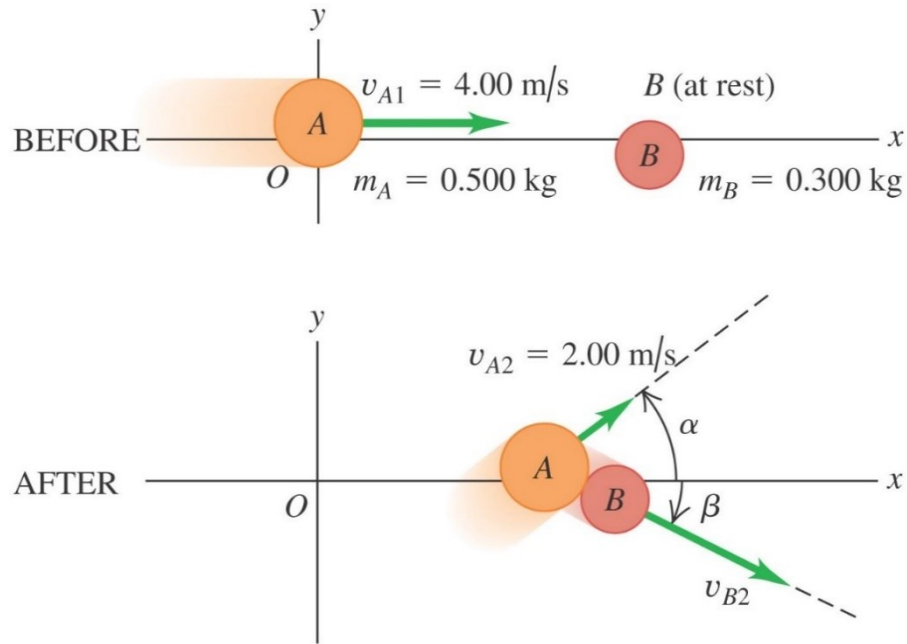


Example: two-dimensional elastic collision

The following figure shows an **elastic** collision of two pucks (masses $m_A = 0.5 \text{ kg}$ and $m_B = 0.3 \text{ kg}$) on a **frictionless** air-hockey table. Puck A has an initial velocity of **4.0 m/s** in the positive x -direction and a final velocity of **2.0 m/s** in the unknown direction α . Puck B is initially **at rest**. Find the final speed v_{B2} of puck B and the angles α and β



Step 1 and 2: Setup coordinate and draw diagram

Step 3: Conservation of momentum?

No external force, true for both x and y .

$$m_A v_{A1x} = m_A v_{A2x} + m_B v_{B2x}$$

$$0 = m_A v_{A2y} + m_B v_{B2y}$$

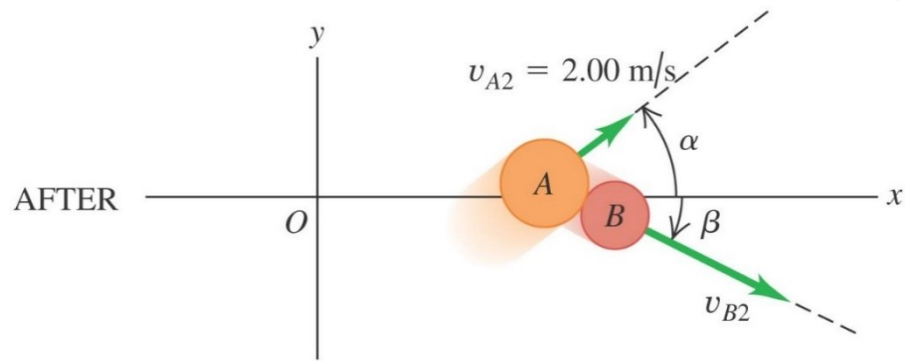
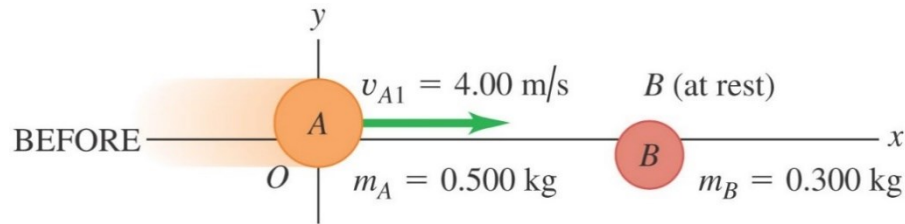
Step 4: Type of collision?

Elastic: conservation of total kinetic energy

$$\frac{1}{2} m_A v_{A1}^2 = \frac{1}{2} m_A v_{A2}^2 + \frac{1}{2} m_B v_{B2}^2$$

Example: two-dimensional elastic collision

The following figure shows an **elastic** collision of two pucks (masses $m_A = 0.5 \text{ kg}$ and $m_B = 0.3 \text{ kg}$) on a **frictionless** air-hockey table. Puck A has an initial velocity of **4.0 m/s** in the positive x -direction and a final velocity of **2.0 m/s** in the unknown direction α . Puck B is initially **at rest**. Find the final speed v_{B2} of puck B and the angles α and β



$$\frac{1}{2}m_A v_{A1}^2 = \frac{1}{2}m_A v_{A2}^2 + \frac{1}{2}m_B v_{B2}^2$$

$$v_{B2}^2 = \frac{m_A v_{A1}^2 - m_A v_{A2}^2}{m_B}$$

$$v_{B2}^2 = \frac{(0.500 \text{ kg})(4.0 \text{ m/s})^2 - (0.500 \text{ kg})(2.00 \text{ m/s})^2}{0.300 \text{ kg}} \quad v_{B2} = 4.47 \text{ m/s}$$

Conservation of the x - and y -components of total momentum gives

$$m_A v_{A1x} = m_A v_{A2x} + m_B v_{B2x}$$

$$(0.500 \text{ kg})(4.00 \text{ m/s}) = (0.500 \text{ kg})(2.00 \text{ m/s})(\cos \alpha) + (0.300 \text{ kg})(4.47 \text{ m/s})(\cos \beta)$$

$$\text{Get, } \cos \alpha = 2 - 1.341 \cos \beta \quad (1)$$

$$0 = m_A v_{A2y} + m_B v_{B2y}$$

$$0 = (0.500 \text{ kg})(2.00 \text{ m/s})(\sin \alpha) - (0.300 \text{ kg})(4.47 \text{ m/s})(\sin \beta)$$

$$\text{So, } \sin \alpha = 1.341 \sin \beta \quad (2)$$

$$\sin^2 \alpha + \cos^2 \alpha = 1 \quad \text{So, } (1.341 \sin \beta)^2 + (2 - 1.341 \cos \beta)^2 = 1$$

$$(1.341)^2 (\sin^2 \beta + \cos^2 \beta) + 4 - 4 \times 1.341 \cos \beta = 1 \quad \text{so, } \cos \beta = 0.8945$$

$$\text{Because (1), } \cos \alpha = 2 - 1.341 \cos \beta = 0.8 \quad \alpha = 36.9^\circ \quad \beta = 26.6^\circ$$

Example: James (mass **90 kg**) and Ramon (mass **60 kg**) are **20.0 m** apart on a frozen pond. Midway between them is a mug of their favorite beverage. They pull on the ends of a light rope stretched between them. When James has moved **6.0 m** toward the mug, how far and in what direction has Ramon moved?

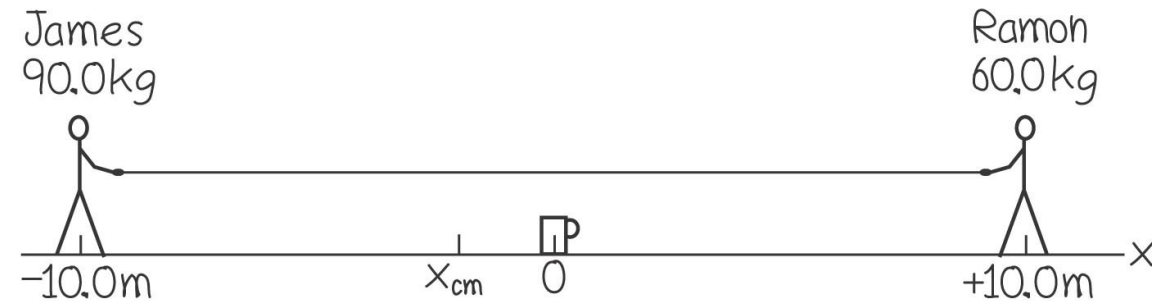
Q1: Which knowledge can we use to solve this problem?

No external force.

Conservation of momentum.

But velocity is not given.

Total momentum is conserved.



Q2: What is the initial total momentum? Zero.

Q3: What is the final total momentum? Zero.

This means that the mass center does not move, and the velocity of the mass center is zero.

Example:

James (mass **90 kg**) and Ramon (mass **60 kg**) are **20.0 m** apart on a frozen pond. Midway between them is a mug of their favorite beverage. They pull on the ends of a light rope stretched between them. When James has moved **6.0 m** toward the mug, how far and in what direction has Ramon moved?

No net external force, the mass center does not change.

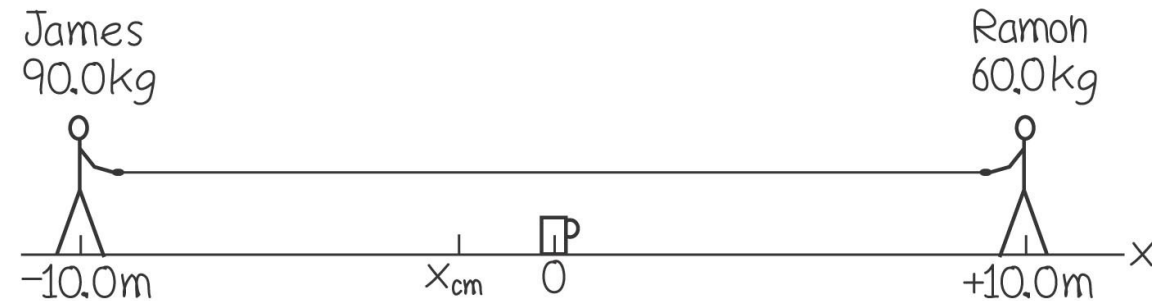
$$x_{\text{cm}} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \dots}{m_1 + m_2 + m_3 + \dots}$$

Before James moves, the mass center is:

$$x_{\text{cm}} = \frac{(90.0 \text{ kg})(-10.0 \text{ m}) + (60.0 \text{ kg})(10.0 \text{ m})}{90.0 \text{ kg} + 60.0 \text{ kg}} = -2.0 \text{ m}$$

The mass center does not change, after James moves:

$$x_{\text{cm}} = \frac{(90.0 \text{ kg})(-4.0 \text{ m}) + (60.0 \text{ kg})x_2}{90.0 \text{ kg} + 60.0 \text{ kg}} = -2.0 \text{ m} \quad x_2 = 1.0 \text{ m}$$



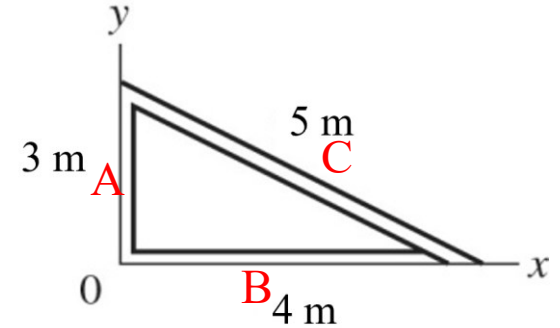
Example: As shown in the figure, the uniform triangle has a total mass of 12 kg. What are the x and y coordinates of the center of mass (or center of gravity) of this triangle?

Q1: How can we find the mass center for this “continuous” object?

Decompose this triangle to three objects, as “A”, “B”, and “C”.

Find the mass center for each “A”, “B”, and “C”, separately.

Find the mass center for the combined object of “A+B+C”.



$$\text{Density} = 12/(3+4+5) = 1 \text{ kg/m}$$

Bar A: (0 m, 1.5 m), 3 kg

Bar B: (2 m, 0 m), 4 kg

Ball C: (2 m, 1.5 m), 5 kg

$$x = \frac{x_A m_A + x_B m_B + x_C m_C}{m_A + m_B + m_C} = 1.5 \text{ m}$$

$$y = \frac{y_A m_A + y_B m_B + y_C m_C}{m_A + m_B + m_C} = 1.0 \text{ m}$$

Example: A 2.0-m rope is lying on a table. You pick up one end and start raising it vertically. How high above the table is the center of mass (or center of gravity) of the rope when half of the rope has lifted off the table?

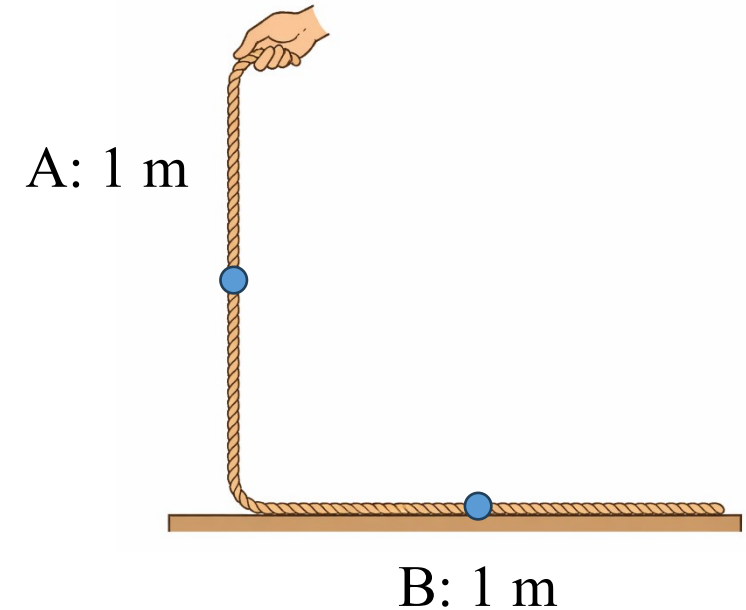
Step 1: Draw diagram

Bar A: (0 m, 0.5 m), 1 kg

Bar B: (0.5 m, 0 m), 1 kg

$$x = \frac{x_A m_A + x_B m_B}{m_A + m_B} = 0.25 \text{ m}$$

$$y = \frac{y_A m_A + y_B m_B}{m_A + m_B} = 0.25 \text{ m}$$

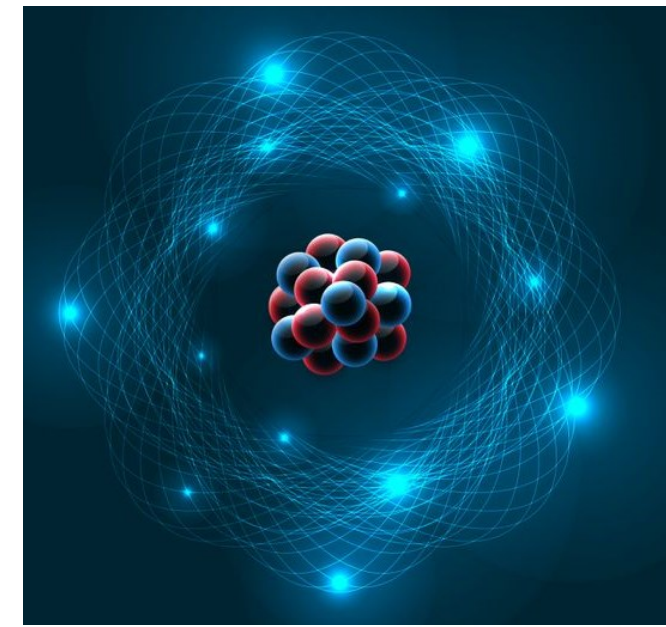


Physics 111: Mechanics

Chapter09

Junjie Yang

Department of Physics, NJIT



Rotational Mechanics

- **Translational Mechanics: chapter 1 to chapter 8**

Motion of point bodies: do not consider their size and shape

Conservation of energy and momentum

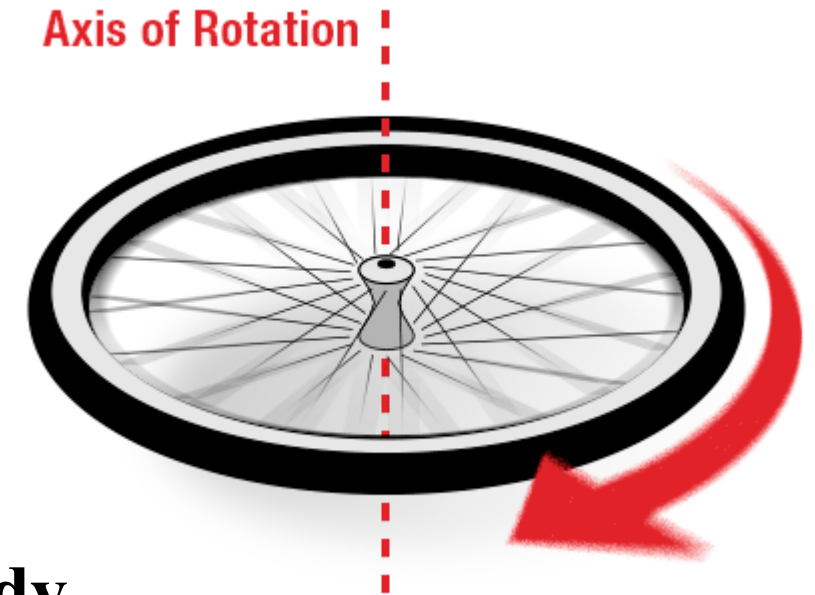
- **Rotational Mechanics: chapter 9 to ...**

Motion of Rigid bodies: rotation and translation

Need to consider their shape and size et al.

Conservation of energy and **angular momentum**.

We do not consider the change of the shape: **Rigid body**



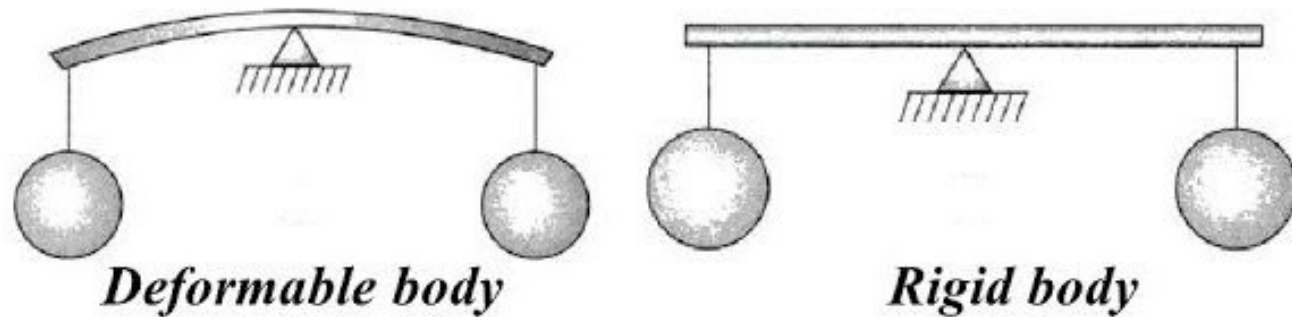
Rigid Bodies

- A rigid body is one that is **nondeformable**

The relative locations of all particles making up the object remain constant.

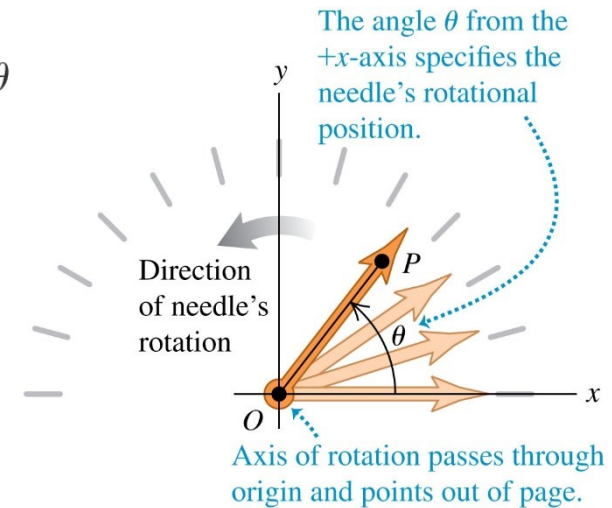
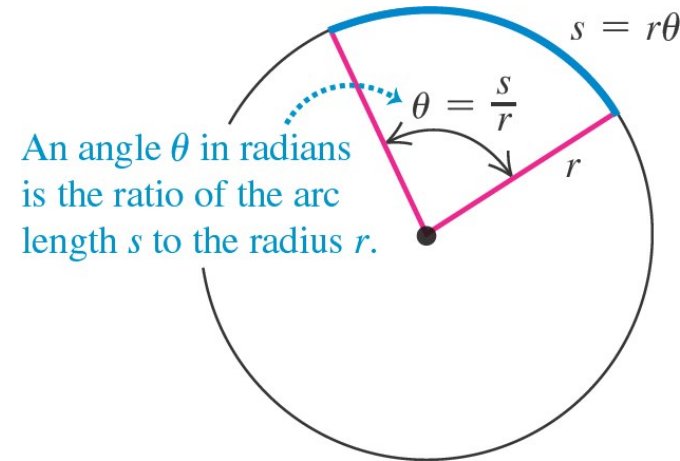
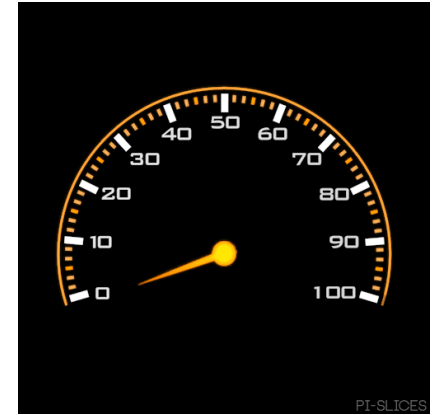
All real objects are deformable to some extent, but the rigid object model is very useful where the deformation is negligible.

- Example

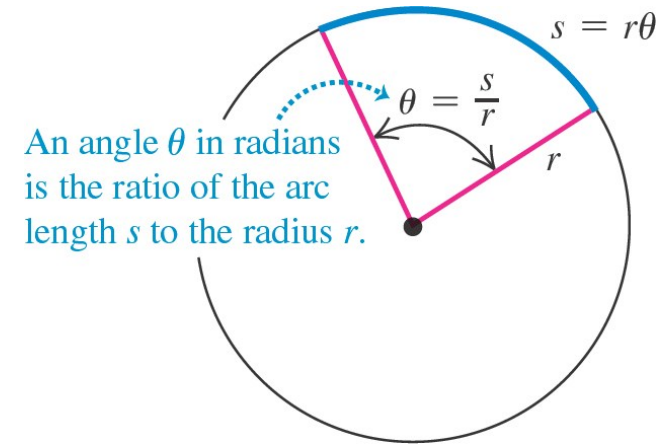


Angle and Radian

- Rotation: A car's speedometer needle rotates about a fixed axis.
- Important quantities: **angle**, **circle/circumference**, **radius**.
- Circumference: $s = (2\pi)r$, so $2\pi = s/r$
- **Angle θ** : defined as the arc length s along a circle divided by radius r : $\theta = s/r$
- θ is a **pure number**, but commonly is given the **artificial unit**, radian “rad”.



Unit of angles



- Comparing degrees and radians: $\pi(\text{rad}) = 180^\circ$

- Converting from degrees to radians:

$$\theta(\text{rad}) = \frac{\pi}{180^\circ} \theta(\text{degrees}) \qquad 57.3^\circ \rightarrow \frac{\pi}{180^\circ} (57.3^\circ) = 1.0 \text{ rad}$$

- Converting from radians to degrees

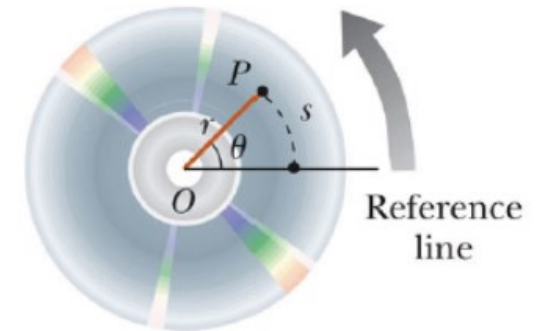
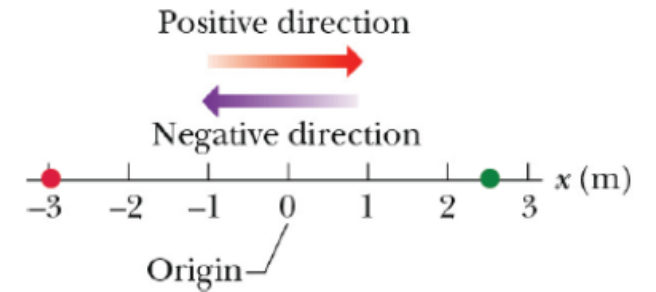
$$\theta(\text{degrees}) = \frac{180^\circ}{\pi} \theta(\text{rad}), \qquad 1.0 \text{ rad} \rightarrow \frac{180^\circ}{\pi} (1.0) = 57.3^\circ$$

- Converting from revolutions to radians

$$1 \text{ revolution} = 2\pi(\text{rad}) = 360^\circ$$

One-dimensional position x vs angular position

- One-dimensional motion: x relative to the **origin**.
- Axis of rotation: the center of the disc.
- “**Origin**” for angular motion: a **fixed reference line**
- Point P is at a fixed distance r from the origin (center of disc)
- As the particle moves, the only coordinate that **change** is θ
- As the particle **moves through** θ , it moves through an **arc length** s .
- The angle θ , measured in radians, is called the **angular position**.



Displacement vs angular displacement

- Displacement: change of position in time $\Delta x = x_f(t_f) - x_i(t_i)$

Vector quantity, magnitude and direction, +/- sign

- **Angular displacement:** the angle the object rotates through during some time interval

$$\Delta\theta = \theta_f(t_f) - \theta_i(t_i)$$

- SI unit: radian (rad)
- A **counter-clockwise** rotation is **positive**
- A **clockwise** rotation is **negative**

Counterclockwise rotation:

θ increases, so angular velocity is positive.

$\Delta\theta > 0$, so

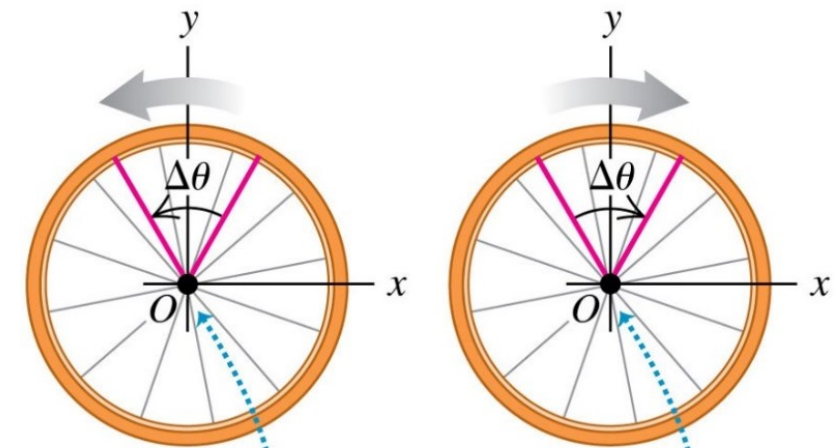
$$\omega_{\text{av-z}} = \Delta\theta/\Delta t > 0$$

Clockwise rotation:

θ decreases, so angular velocity is negative.

$\Delta\theta < 0$, so

$$\omega_{\text{av-z}} = \Delta\theta/\Delta t < 0$$



Axis of rotation (z-axis) passes through origin and points out of page.

Velocity vs angular velocity

- One-dimensional: rate of change of position in time

$$\text{average } v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}, \text{ instantaneous } v = \frac{dx}{dt} = \lim_{\Delta t \rightarrow 0} \frac{x_f - x_i}{\Delta t}$$

- **Average angular velocity** ω_{avg} : the ratio of the angular

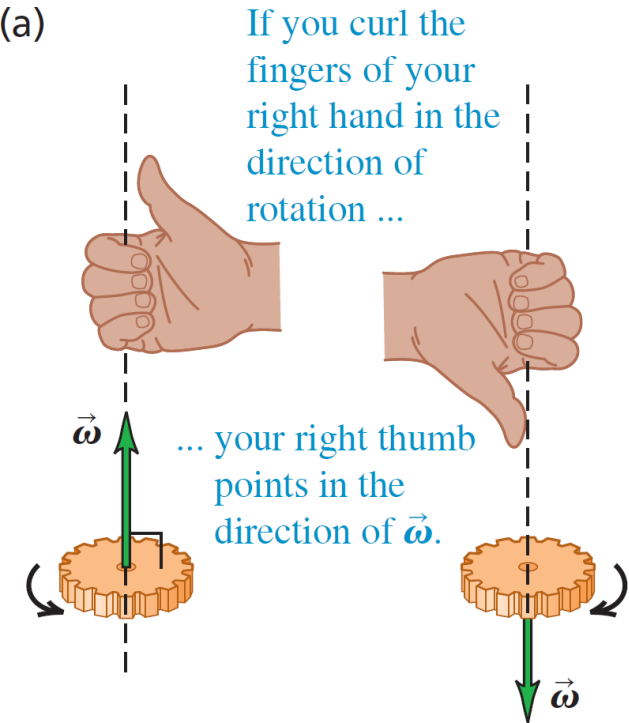
$$\text{displacement to the time interval } \omega_{avg} = \frac{\Delta \theta}{\Delta t} = \frac{\theta_f - \theta_i}{\Delta t}$$

- **Instantaneous angular velocity**: the limit of the average

$$\text{velocity as the time interval approaches zero } \omega = \lim_{\Delta t \rightarrow 0} \frac{\Delta \theta}{\Delta t} = \frac{d\theta}{dt}$$

- **Direction**: positive for counter-clockwise, negative for clockwise.

(a)



SI unit: rad/s

Example: The angular position θ of a 0.36-m-diameter flywheel is given by $\theta = (2.0 \text{ rad/s}^2)t^2$. Find the instantaneous angular velocity at $t = 0.5 \text{ s}$. (use unit rad/s)

$$\omega = \frac{d\theta}{dt} = 4t \quad t = 0.5 \text{ s} \quad \omega = 2 \text{ rad/s}^2$$

Angular acceleration

- **Average angular acceleration α_{avg} :** the ratio of the change in the angular speed to the time it takes for the object to undergo the change

$$\alpha_{avg} = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{\Delta t}$$

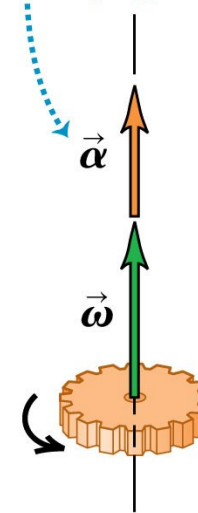
SI unit: rad/s²

- **Instantaneous angular acceleration:** the limit of the average angular acceleration as the time interval approaches zero

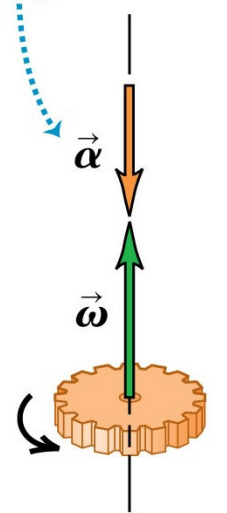
$$\alpha = \lim_{\Delta t \rightarrow 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

- **Positive α :** counter-clockwise speeding up,
(If ω is clockwise, slowing down)
- **Negative α :** counter-clockwise slowing down,
(If ω is clockwise, speeding up.)

$\vec{\alpha}$ and $\vec{\omega}$ in the **same** direction: Rotation speeding up.

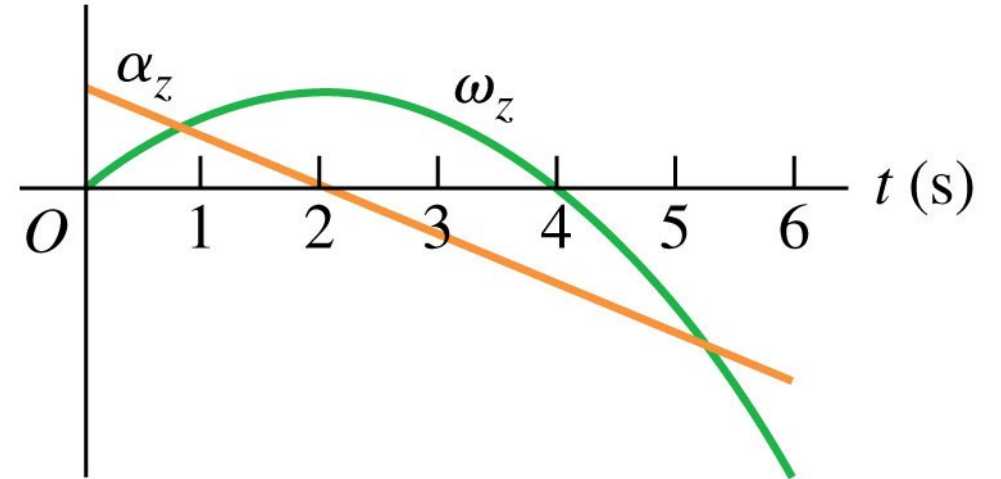


$\vec{\alpha}$ and $\vec{\omega}$ in the **opposite** directions: Rotation slowing down.



Example: The graph shows the angular velocity ω_z and angular acceleration α_z versus time t for a rotating body. At which of the following times is the rotation speeding up at the greatest rate?

- (A) $t = 1$ s
- (B) $t = 2$ s
- (C) $t = 3$ s
- (D) $t = 4$ s
- (E) $t = 5$ s



Comparison between rotational and linear equations

Straight-Line Motion with Constant Linear Acceleration	Fixed-Axis Rotation with Constant Angular Acceleration
$a_x = \text{constant}$	$\alpha_z = \text{constant}$
$v_x = v_{0x} + a_x t$	$\omega_z = \omega_{0z} + \alpha_z t$
$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$	$\theta = \theta_0 + \omega_{0z} t + \frac{1}{2} \alpha_z t^2$
$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$	$\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$
$x - x_0 = \frac{1}{2}(v_{0x} + v_x)t$	$\theta - \theta_0 = \frac{1}{2}(\omega_{0z} + \omega_z)t$