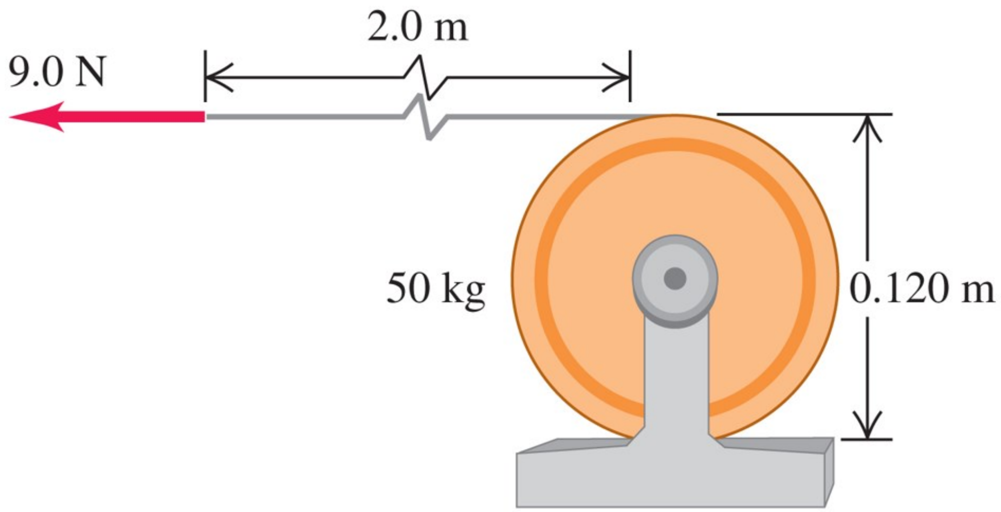
Example:

In the figure, solid cylinder has a mass **50 kg** and diameter **0.120 m**, and it rotates in frictionless bearings about a stationary horizontal axis. We pull the free end of the cable with a constant **9.0 N** force for a **2.0-m** distance; it turns the cylinder as it unwinds without slipping. The cylinder is initially **at rest**. Find its final kinetic energy.



Q1: What info we can get from "a constant **9.0 N** force for a **2.0-m** distance"?

$$W_{\text{other}} = Fs = (9.0\ \text{N})(2.0\ \text{m}) = 18\ \text{J}.$$

Q2: Where does this 18 J energy go?

It is converted to the kinetic energy of the solid cylinder.

$$K_1 + U_1 + W_{other} = K_2 + U_2 \qquad U_1 = 0 = U_2 \qquad K_1 = 0$$

$$W_{other} = K_2 = 18\ J$$

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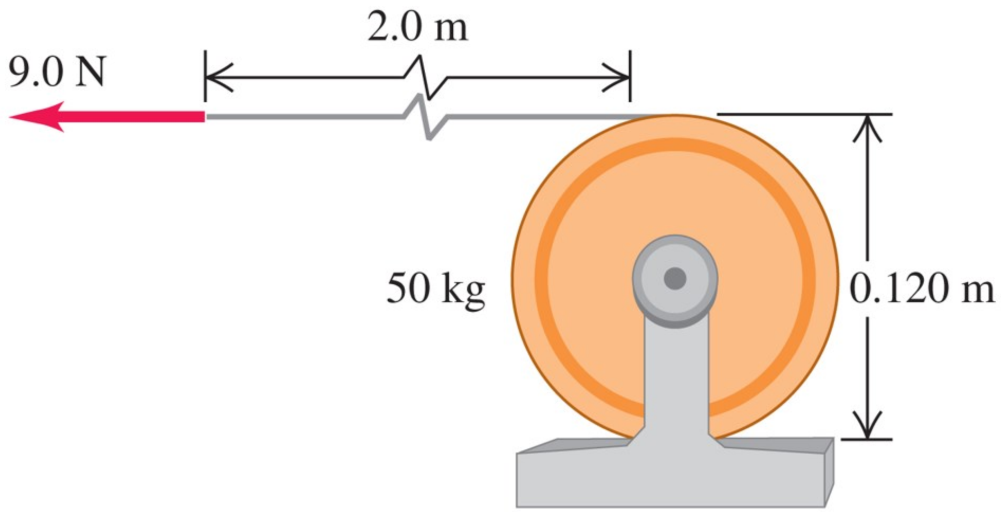
$$W_{other} = K_2 = 18\,J$$



Q1: What is the expression for K_2?

$$K_2 = \frac{1}{2} I\omega^2 = 18\,J$$

$$I = \frac{1}{2}mR^2 = \frac{1}{2}(50\text{ kg})(0.060\text{ m})^2 = 0.090\text{ kg}\cdot\text{m}^2$$

$$\omega = \sqrt{\frac{2(18\text{ J})}{0.090\text{ kg}\cdot\text{m}^2}} = 20\text{ rad/s}$$

Example:

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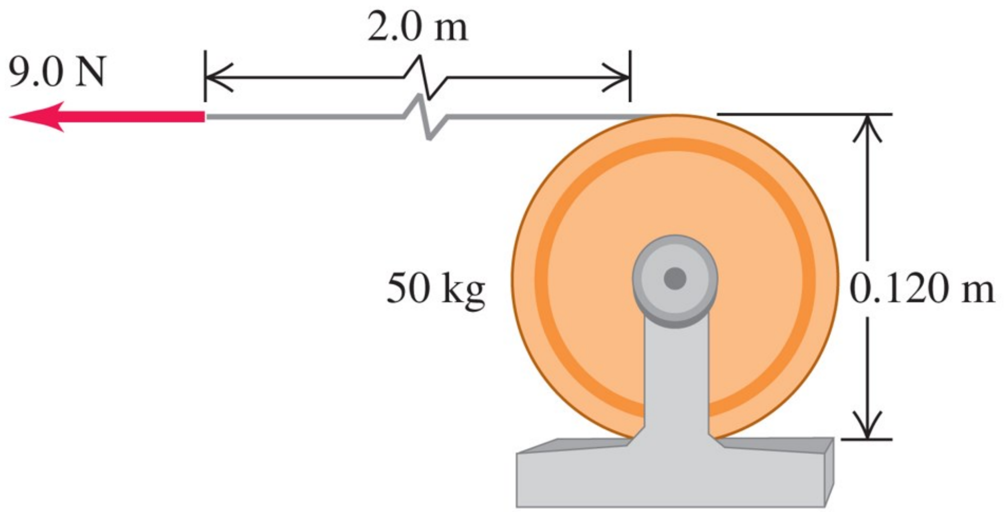
$$\omega = \sqrt{\frac{2(18\ \text{J})}{0.090\ \text{kg}\cdot\text{m}^2}} = 20\ \text{rad/s}$$

$$v = R\omega = (0.060\ \text{m})(20\ \text{rad/s}) = 1.2\ \text{m/s}$$



Example:

A thin, very light wire is wrapped around a drum that is free to rotate. The free end of the wire is attached to a ball of **mass *m***. The drum has the same **mass *m***. Its **radius is *R*** and its moment of inertia is $I = (1/2)mR^2$. As the ball falls, the drum spins.

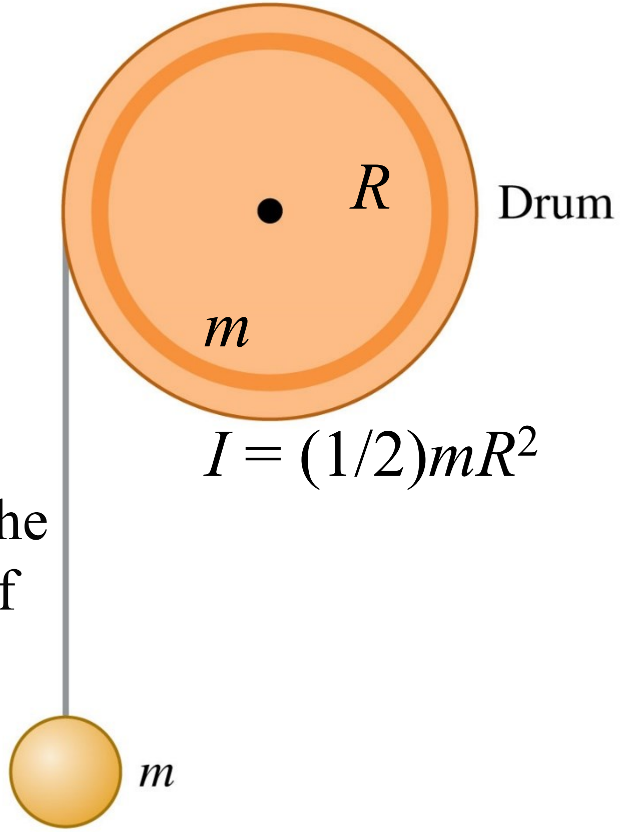
At an instant that the ball has translational kinetic energy K, the drum has rotational kinetic energy

A. K.

B. $2K$.

C. $K/2$. ✔

D. none of these



Suppose the velocity of ball is v

$$K = \tfrac{1}{2} mv^2 \qquad \omega = \frac{v}{R}$$

$$K_R = \frac{1}{2} I\omega^2 = \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2 = \frac{1}{4}mv^2$$

Example

A solid cylinder with **mass *M*** and **radius *R***. The cylinder rotates with negligible friction about a stationary horizontal axis. We tie the free end of the cable to a block of **mass *m*** and release the block **from rest** at a **distance *h*** above the floor. As the block falls, the cable unwinds without stretching or slipping. Find the **speed of the falling block** and the **angular speed of the cylinder** as the block strikes the floor.

Q1: Initially, what kind of energy does the system have?

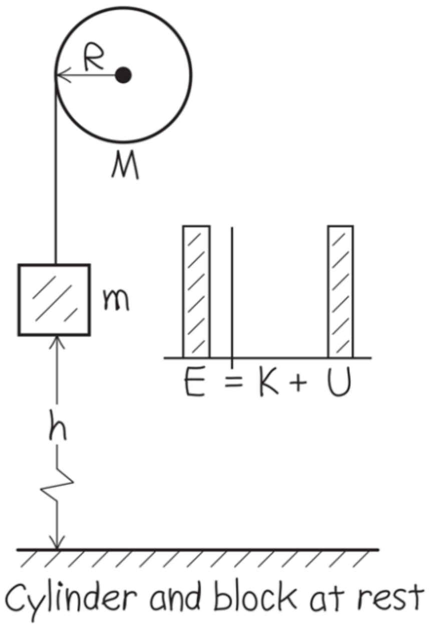
Gravitational potential energy of the block: mgh.

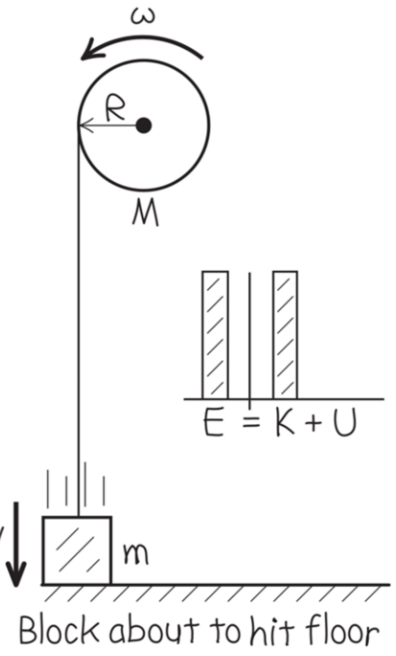
Q2: Final state, what kind of energy does the system have?

Linear kinetic energy of the block $(1/2)mv^2$.

Rotational kinetic energy of the cylinder $(1/2)I\omega^2$.

$$K_1 + U_1 = K_2 + U_2 \qquad K_2 = K_m + K_M$$





Example

A solid cylinder with **mass *M*** and **radius *R***. The cylinder rotates with negligible friction about a stationary horizontal axis. We tie the free end of the cable to a block of **mass *m*** and release the block **from rest** at a **distance *h*** above the floor. As the block falls, the cable unwinds without stretching or slipping. Find the **speed of the falling block** and the **angular speed of the cylinder** as the block strikes the floor.

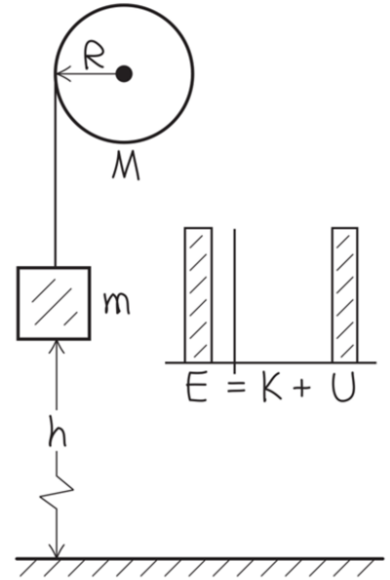
$$K_1 + U_1 = K_2 + U_2 \qquad K_2 = K_m + K_M$$

$$0 + mgh = K_m + K_M + 0 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

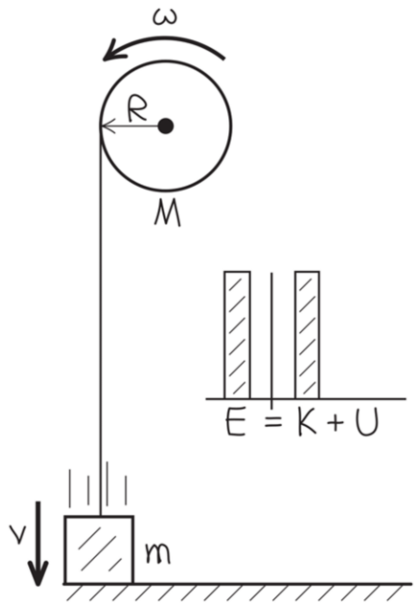
$$\omega = v/R \qquad I = \tfrac{1}{2}MR^2$$

$$0 + mgh = \frac{1}{2}mv^2 + \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\left(\frac{v}{R}\right)^2 = \tfrac{1}{2}\left(m + \tfrac{1}{2}M\right)v^2$$

$$v = \sqrt{\frac{2gh}{1 + M/2m}} \qquad \omega = v/R$$



Cylinder and block at rest



Block about to hit floor

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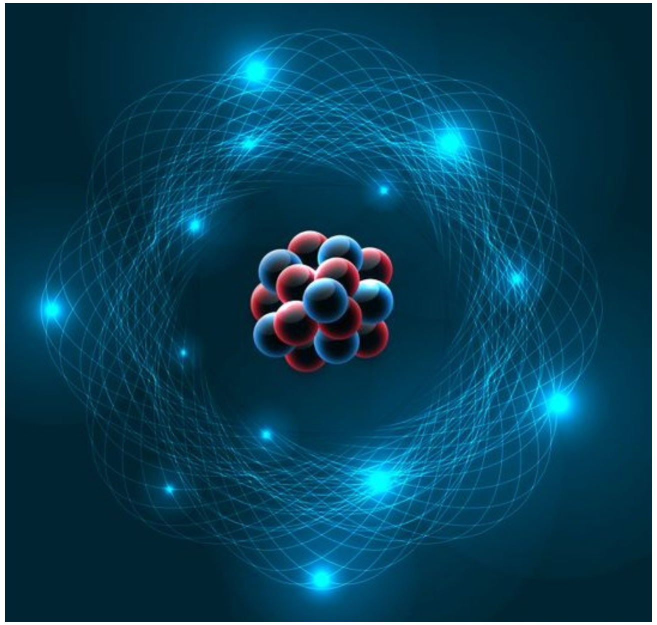

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Angular vs linear motion

$$a_x = \text{ constant} \qquad \alpha_z = \text{ constant}$$

$$v_x = v_{0x} + a_x t \qquad \omega_z = \omega_{0z} + \alpha_z t$$

$$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2 \qquad \theta = \theta_0 + \omega_{0z} t + \frac{1}{2} \alpha_z t^2$$

$$v_x^2 = v_{0x}^2 + 2a_x (x - x_0) \qquad \omega_z^2 = \omega_{0z}^2 + 2\alpha_z (\theta - \theta_0)$$

$$x - x_0 = \frac{1}{2}(v_x + v_{0x}) t \qquad \theta - \theta_0 = \frac{1}{2}(\omega_z + \omega_{0z}) t$$

Linear kinetic energy:
$K = ½\, mv^2$.

Rotational kinetic energy:
$K_R = ½\, I\, \omega^2$.

$$\vec{J} = \Delta\vec{p} = m\vec{v}_f - m\vec{v}_i = \vec{F}\Delta t$$ Impulse-Momentum Theorem

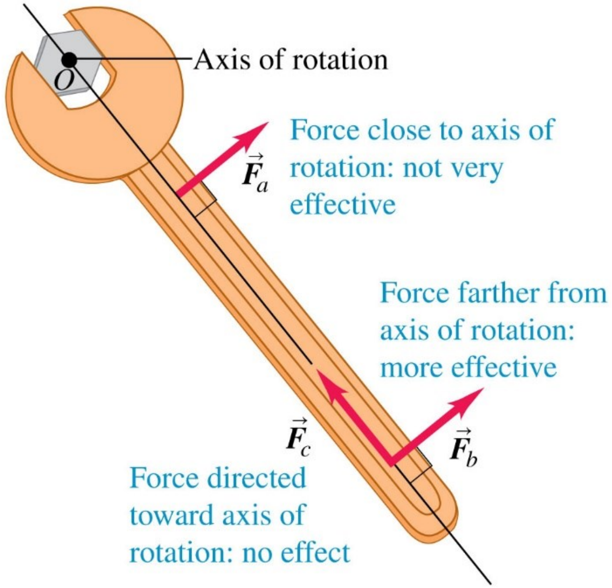
$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$ Conservation of Momentum

Dynamics of Rotational Motion

- What is meant by the **torque produced by a force**.
- How the **net torque** on an object affects the object's **rotational motion**.
- How to analyze the motion of an object that both **rotates and moves as a whole through space**.
- How to solve problems that involve **work and power for rotating objects**.
- How the **angular momentum** of an object can **remain constant** even if the object changes shape.

Force vs Torque

- Newton’s second law: Force cause acceleration
- What causes angular acceleration? What does it take to start a stationary object rotating or bring a spinning object to a halt?
- New concepts, such as **torque** and **angular momentum**, to deepen our understanding of rotational motion. Example?

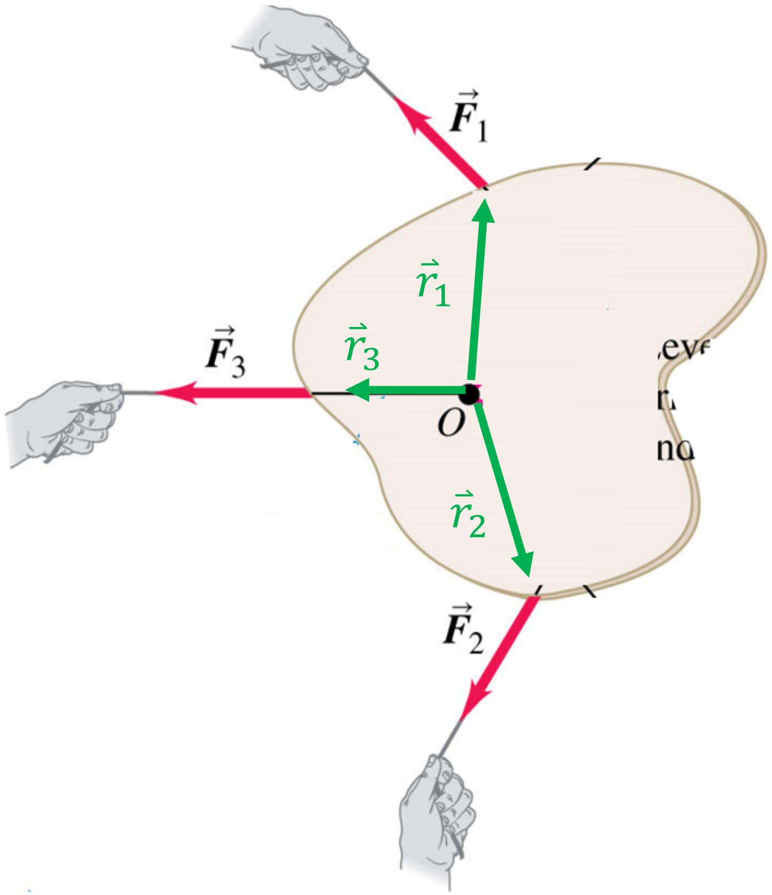


- There are 3 factors that determine the effectiveness of the force in loosening the tight bolt:
  - **The magnitude of the force,**
  - **The position of the application of the force,**
  - **The angle at which the force is applied**

Definition of Torque

- Torque, $\boldsymbol{\tau}$, is the tendency of the force to rotate an object about some axis.
- Let $\boldsymbol{F}$ be a force acting on an object, and let $\boldsymbol{r}$ be a position vector from a rotational center to the point of application of the force. Generally:

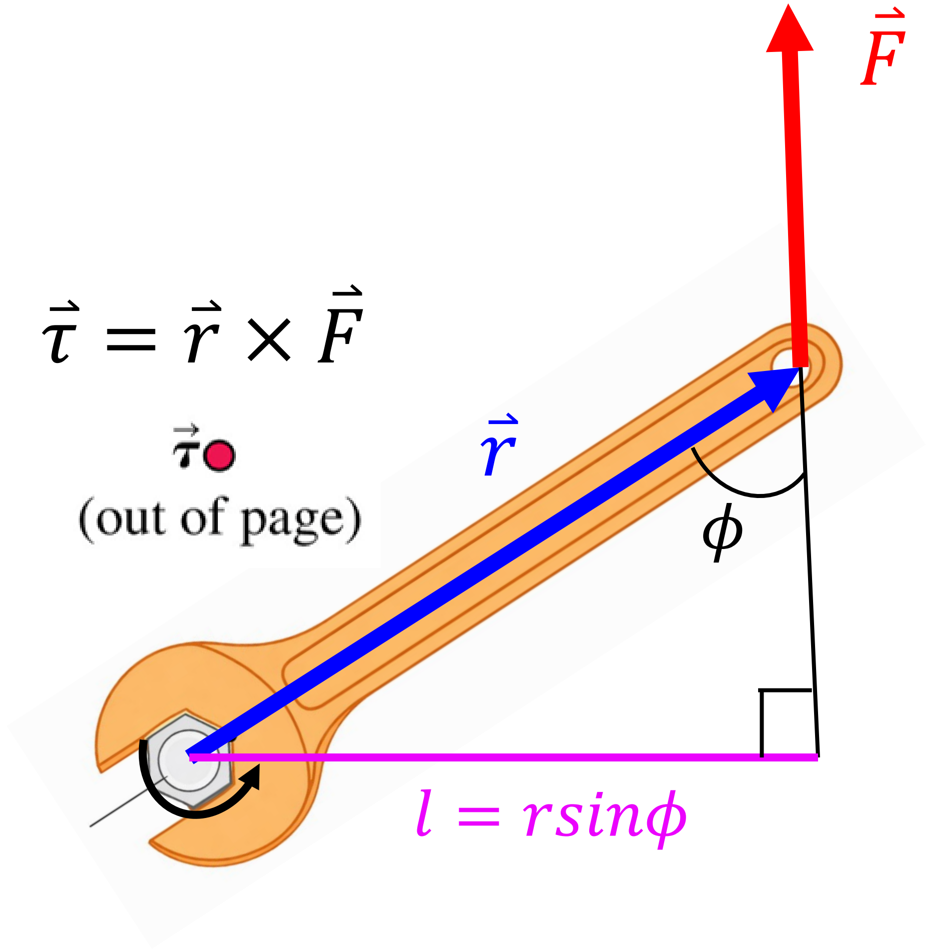
Torque vector due to force $\vec{F}$ relative to point O → $$\vec{\tau} = \vec{r} \times \vec{F}$$ ← Vector from O to where $\vec{F}$ acts; ← Force $\vec{F}$



- Is the torque a vector or scalar?
- Force $\boldsymbol{F}$ is a vector, and the position $\boldsymbol{r}$ is also a vector. Cross produce of $\boldsymbol{F}$ and $\boldsymbol{r}$ can give us the vector result.

Torque Units and moment arm

- The SI units of torque are N·m.
- Torque is a vector quantity, it has direction
  - If the turning tendency of the force is counterclockwise, the torque will be positive
  - If the turning tendency is clockwise, the torque will be negative.
- Torque magnitude is given by $\tau = rF\sin\phi = Fl$
- The **moment arm, *l***, is the perpendicular distance from the axis of rotation to a line drawn along the direction of the force $\boldsymbol{l = r\sin\phi}$



Example:

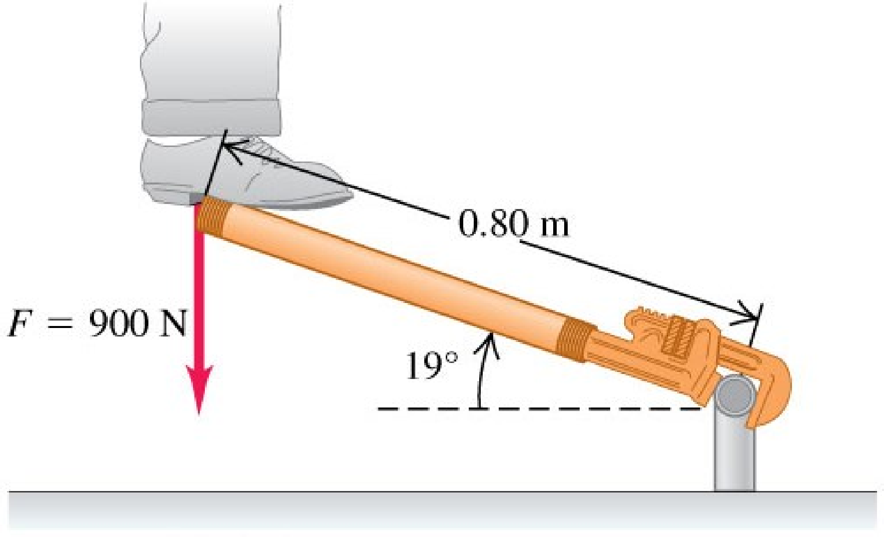
A plumber pushes straight down on the end of a long wrench as shown. What is the magnitude of the torque he applies about the pipe at lower right?

A. (0.80 m)(900 N)sin 19°

B. (0.80 m)(900 N)cos 19°

C. (0.80 m)(900 N)tan 19°

D. none of the above



$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = rFsin\phi = Fl$$

Example:

A force $\vec{F} = \left(4\hat{i} + 3\hat{j}\right)\text{N}$ acts on an object at a point located at the position $\vec{r} = \left(6\hat{k}\right)\text{m}$.

What is the torque that this force applies about the origin?

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\hat{i}\times\hat{j}=\hat{k};\ \hat{i}\times\hat{k}=-\hat{j};\ \hat{j}\times\hat{k}=\hat{i}$$

$$\hat{i}\times\hat{i}=0;\ \hat{j}\times\hat{j}=0;\ \hat{k}\times\hat{k}=0$$

A. zero

B. $\left(24\hat{i} + 18\hat{j}\right)\text{N}\bullet\text{m}$

C. $\left(-24\hat{i} - 18\hat{j}\right)\text{N}\bullet\text{m}$

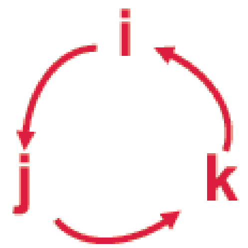
D. ✔ $\left(-18\hat{i} + 24\hat{j}\right)\text{N}\bullet\text{m}$

E. $\left(-18\hat{i} - 24\hat{j}\right)\text{N}\bullet\text{m}$

$$\vec{\tau} = (6\hat{\boldsymbol{k}})\times(4\hat{\boldsymbol{\imath}}+3\hat{\boldsymbol{\jmath}})$$

$$=24(\hat{\boldsymbol{k}}\times\hat{\boldsymbol{\imath}}) + 18(\hat{\boldsymbol{k}}\times\hat{\boldsymbol{\jmath}})$$

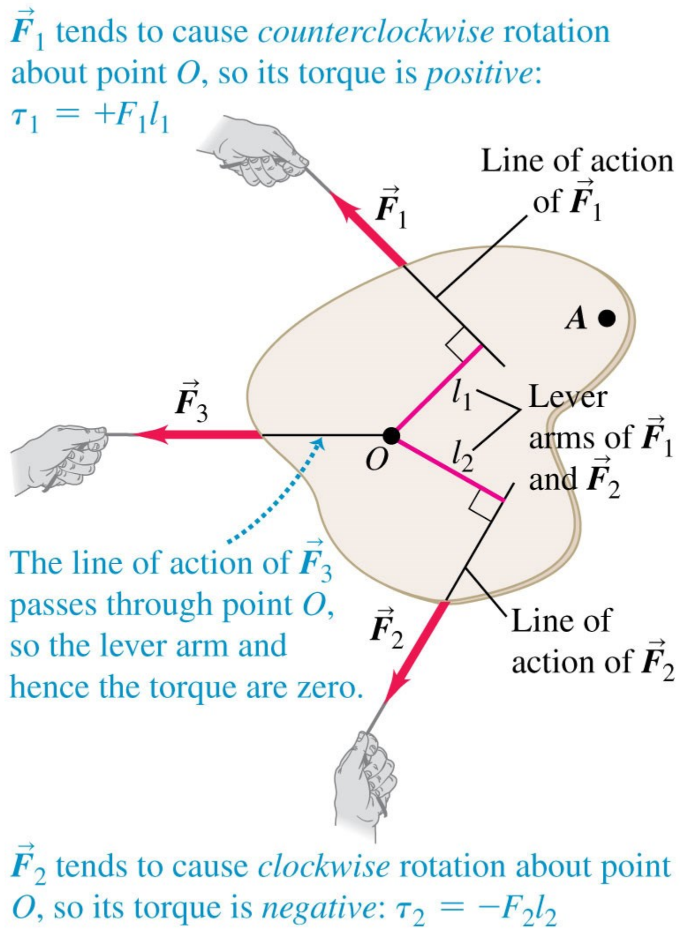
$$=24\hat{\boldsymbol{\jmath}} - 18\hat{\boldsymbol{\imath}}$$



$$\vec{A}\times\vec{B} = (A_yB_z - A_zB_y)\,\hat{\imath} + (A_zB_x - A_xB_z)\,\hat{\jmath} + (A_xB_y - A_yB_x)\,\hat{k}$$

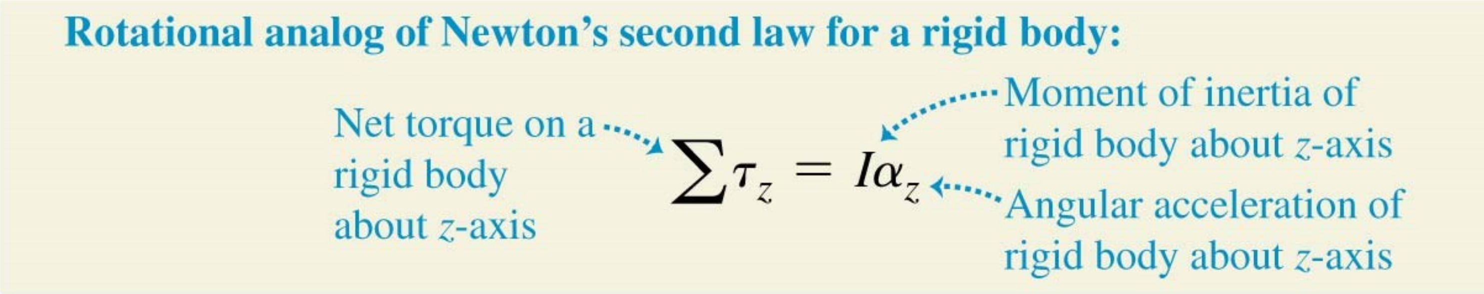
Example: Multiple forces and net torque

- The force $\overrightarrow{F_1}$ will tend to cause a counterclockwise rotation about O.
- The force $\overrightarrow{F_2}$ will tend to cause a clockwise rotation about O.
- Net torque $\sum \tau = \tau_1 + \tau_2 + \tau_3 = F_1 l_1 - F_2 l_2$
- If $\sum \tau = 0$, rotation rate does not change
- If $\sum \tau \neq 0$, rotation starts



Newton's Second Law for a rotating object

- When a rigid object is subject to a **nonzero net torque**, it undergoes an angular acceleration $\sum \boldsymbol{\tau} = \boldsymbol{I\alpha_z}$.



- The angular acceleration is directly proportional to the net torque
- The angular acceleration is inversely proportional to the moment of inertia of the object
- The relationship is analogous to $\sum F = ma$