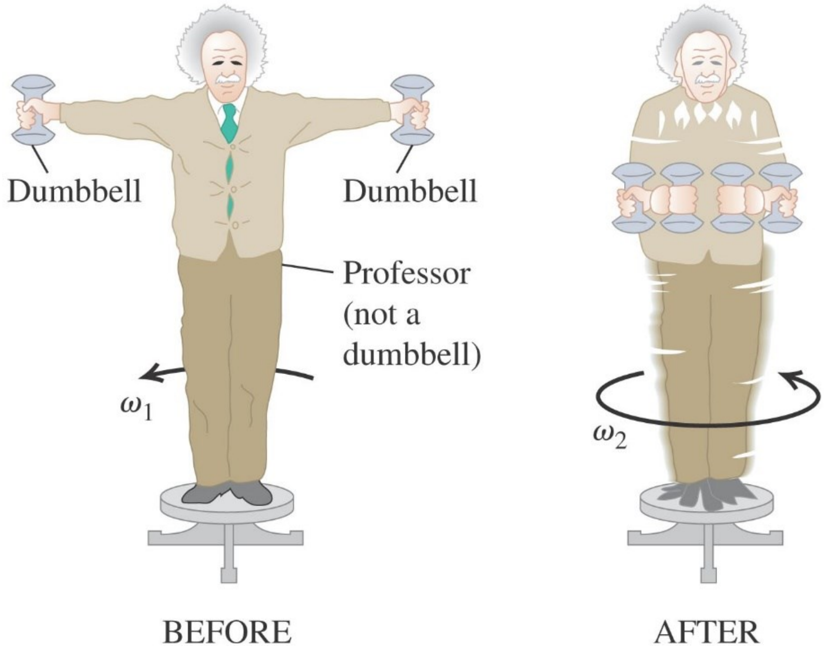
A physics professor stands on a frictionless turntable with arms outstretched and a **5.0 kg** dumbbell in each hand. He is rotating (**one revolution in 2.0 s**).
Find his final angular velocity if he pulls the dumbbells inward to his stomach.

Moment of inertia (without dumbbells) is **3.0 kg·m²** with arms outstretched.

Moment of inertia (without dumbbells) is **2.2 kg·m²** with his hands at his stomach.



The dumbbells are **1.0 m** from the axis initially and **0.20 m** at the end.

Q1: "Before" and "after", which quantities are changed? Which quantities are unchanged?

"Frictionless", no net torque, angular moment unchanged

Moment of inertia changes

$$L = I_1\omega_{1z} = I_2\omega_{2z}$$

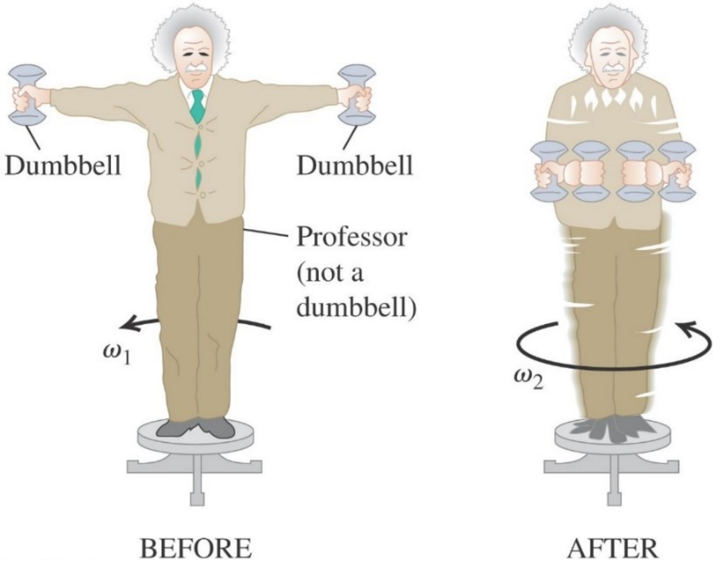
Need to find I_1 and I_2.

A physics professor stands on a frictionless turntable with arms outstretched and a **5.0 kg** dumbbell in each hand. He is rotating (**one revolution in 2.0 s**). Find his final angular velocity if he pulls the dumbbells inward to his stomach.

I_A **(before):** Moment of inertia (without dumbbells) is **3.0 kg·m²** with arms outstretched.

I_B **(after):** Moment of inertia (without dumbbells) is **2.2 kg·m²** with his hands at his stomach.

I_D: The dumbbells are **1.0 m (before)** from the axis initially and **0.20 m (after)** at the end.



$$L = I_1\omega_{1z} = I_2\omega_{2z} \qquad \omega_{1z} = 1\text{ rev}/2.0\text{ s} = 0.50\text{ rev/s}$$

$$I_1 = I_A + I_D = I_A + 2mr^2 = 3.0\text{kg}\cdot\text{m}^2 + 2(5.0\text{kg})(1.0\text{m})^2 = 13\text{kg}\cdot\text{m}^2$$

$$I_2 = I_B + I_D = I_B + 2mr^2 = 2.2\text{kg}\cdot\text{m}^2 + 2(5.0\text{kg})(0.2\text{m})^2 = 2.6\text{kg}\cdot\text{m}^2$$

$$\omega_{2z} = I_1\omega_{1z}/I_2 = (13\text{kg}\cdot\text{m}^2/2.6\text{kg}\cdot\text{m}^2)(0.50\text{rev/s}) = 2.5\text{rev/s}$$

$$K_1 = (1/2)I_1\omega_{1z}^2 = (1/2)(13\text{kg}\cdot\text{m}^2)(3.14\text{rad/s})^2 = 64\text{J}$$

$$K_2 = (1/2)I_2\omega_{2z}^2 = (1/2)(2.6\text{kg}\cdot\text{m}^2)(15.7\text{rad/s})^2 = 320\text{J}$$

Summary and comparison

Translation		Rotation	
Force	$\vec{F}$	Torque	$\vec{\tau} = \vec{r} \times \vec{F}$
Linear Momentum	$\vec{p} = m\vec{v}$	Angular Momentum	$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$
Kinetic Energy	$K = \frac{1}{2}mv^2$	Kinetic Energy	$K = \frac{1}{2}I\omega^2$

Systems and Rigid Bodies

Translation		Rotation	
Linear Momentum	$\vec{P} = \sum \vec{p}_i = M\vec{v}_{cm}$	Angular Momentum	$\vec{L} = \sum \vec{L}_i = \sum I_i \vec{\omega}_i$
Second Law	$\vec{F}_{net} = \frac{d\vec{P}}{dt}$	Second Law	$\vec{\tau}_{net} = \frac{d\vec{L}_{sys}}{dt}$

Momentum conservation - for closed, isolated systems

$\vec{P}_{sys} = \text{constant}$

$\vec{L}_{sys} = \text{constant}$

Example

A light nylon cord is wound around a uniform cylindrical spool of **radius 0.5-m** and **mass 1.2 kg**. The spool is mounted on a frictionless rotational axis and is **initially at rest**. The cord is pulled from the spool with a constant acceleration of magnitude **1.3 m/s²**. How much work has been done on the spool, when it reaches an angular speed of **10 rad/s**?

Q1: Which energies are involved in this problem?

Kinetic energy of the spool, and work done by "pulling".

$$K_1 + U_1 + W_{other} = K_2 + U_2$$

$$0 + 0 + W_{other} = K_2 + 0$$

$$K_2 = 1/2\, I_{cm}\omega^2$$

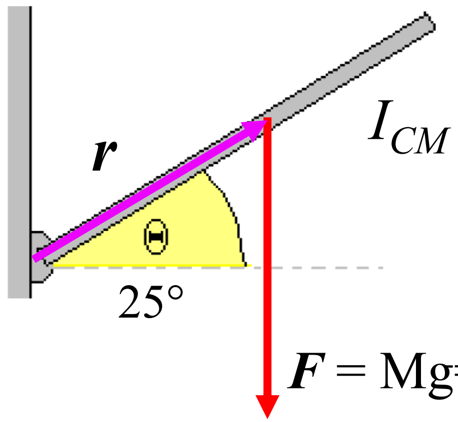
$$K_2 = 1/2\, I_{cm}\omega^2 = (1/2)(1/2\times 1.2\text{ kg} \times 0.5^2)(10\text{ rad/s})^2 = 7.5\text{ J}$$

$$I_{cm} = \tfrac{1}{2} m r^2$$

$$W_{other} = K_2 = 7.5\text{ J}$$

Example

A thin uniform rod (length **1.2 m** and **mass 2.0 kg**) is pivoted about a horizontal, frictionless pin through one end of the rod. The rod is release when it makes an angle of **25 degrees** with the horizontal. What is the angular acceleration of the rod at the instant it is released?



Q1: Which forces are applied on the rod?

Gradational force and normal force

Q2: Which force provide non-zero torque? Gradational force

$$\vec{\tau} = \vec{r} \times \vec{F} = I\alpha_z$$

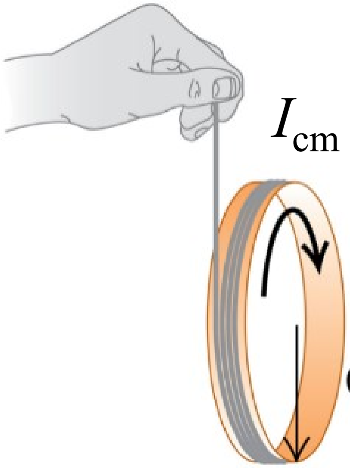
$$\vec{r} \times \vec{F} = -(0.6\,m)(19.6\,N)\sin[180° - (90° - 25°)] = -10.66 \qquad I\alpha_z = -10.66$$

$$I = I_{CM} + MD^2 = 1/12ML^2 + M(L/2)^2 = 1/3ML^2 \qquad I = 1/3ML^2 = 0.96 \qquad \alpha_z = -11.1\,rad/s^2$$

Example

A lightweight string is wrapped around the rim of a small hoop. If the free end of the string is held in place and the hoop is released from rest, the string unwinds and the hoop descends **2.0 m**. What is the speed of the center of mass of the hoop at that position?

Q1: Total mechanical energy conserved or not?



Except for the gradational fore, no other force does work. Conserved.

$K_1 + U_1 = K_2 + U_2$ $0 + mgh = K_2 + 0$

$K_2 = 1/2\, mv_{cm}^2 + 1/2\, I_{cm}\omega^2$ $I_{cm} = mR^2$ $v_{cm} = R\omega$

$mgh = 1/2\, mv_{cm}^2 + 1/2\, mv_{cm}^2$ $2g = v_{cm}^2$ $v_{cm} = 4.4$ m/s

Example:

A **392-N** wheel comes off a moving truck and **rolls with-out slipping** along a highway. At the bottom of a hill it is rotating at **25.0 rad/s**. The radius of the wheel is **0.6** m, and its moment of inertia about its rotation axis is **0.8 *MR²***. The wheel rolls up the hill to a stop, a height ***h*** above the bottom of the hill. Calculate *h*.

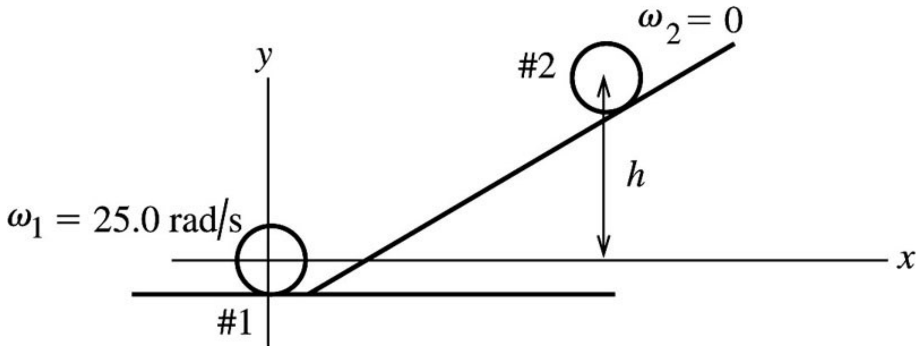
$$K_1 + U_1 = K_2 + U_2$$

$$K_2 = 0,\ \ U_1 = 0,\ \ U_2 = Mgh$$

$$K_1 = \tfrac{1}{2} I\omega_1^2 + \tfrac{1}{2} M v_1^2;\ \ v = R\omega \ \text{ and } \ I = 0.800 MR^2$$

$$K_1 = \tfrac{1}{2}(0.800) MR^2\omega_1^2 + \tfrac{1}{2} MR^2\omega_1^2 = 0.900 MR^2\omega_1^2$$

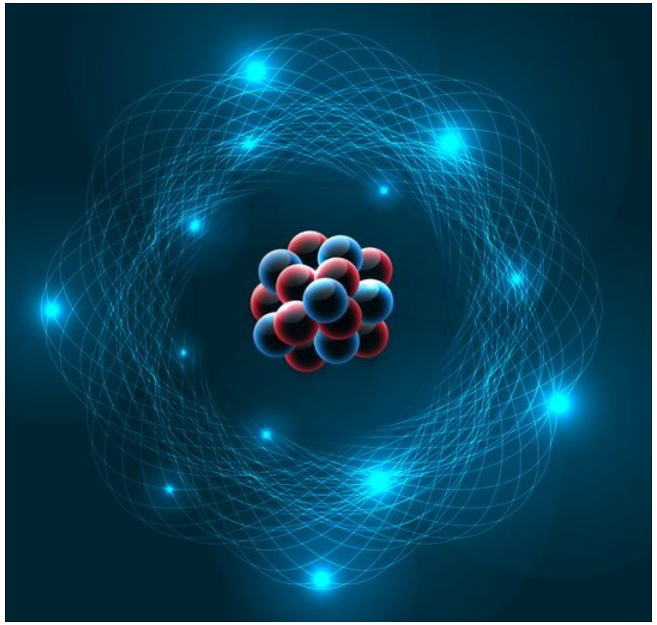
$$\text{Thus } 0.900 MR^2\omega_1^2 = Mgh$$



Take $y = 0$ at the center of the wheel when it is at the bottom of the hill.

$$h = \frac{0.9 R^2\omega_1^2}{g} = \frac{0.9(0.6\,m)^2(25\,rad/s)^2}{9.8\,m/s^2} = 20.7\,m$$

Physics 111: Mechanics
Chapter 11



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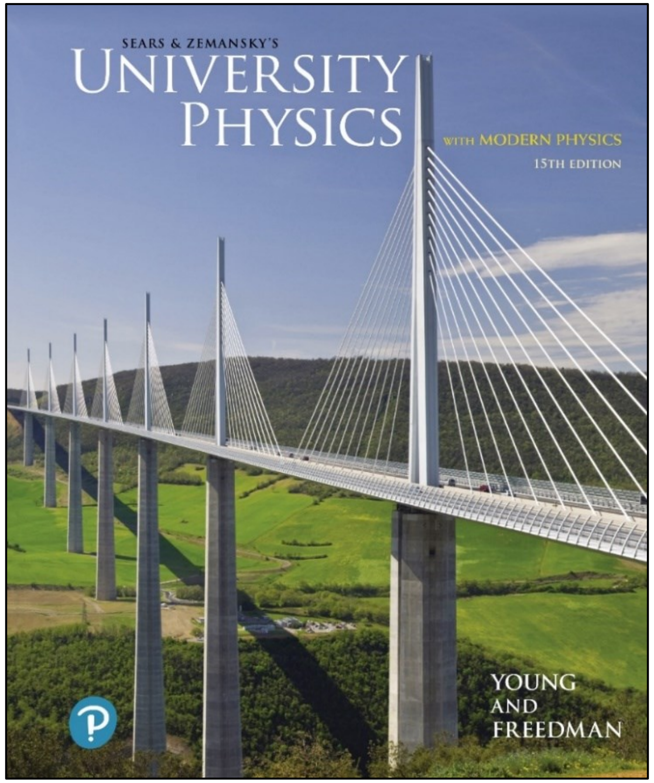
THE EDGE IN KNOWLEDGE

Equilibrium

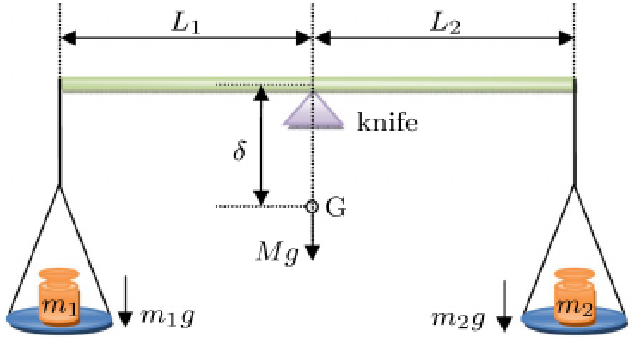
- In construction we're interested in making sure that objects don't accelerate.

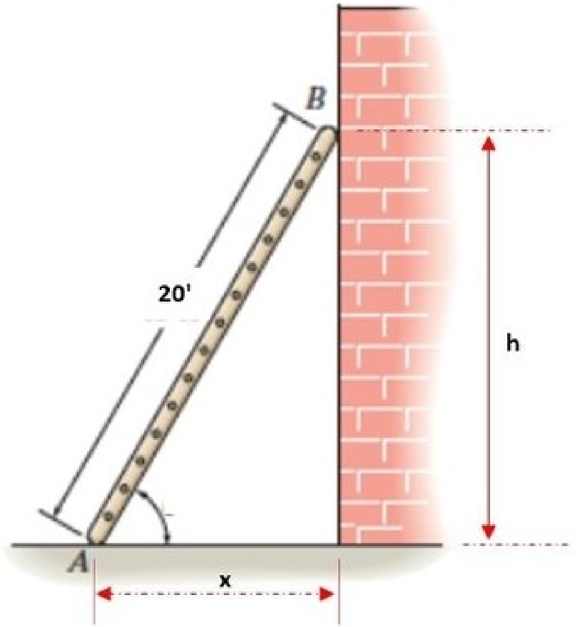
- **Equilibrium**
  - the conditions that must be satisfied for an object to be in equilibrium.
  - the center of gravity of an object and how it relates to the object's stability.
  - how to solve problems that involve rigid bodies in equilibrium.



Static and dynamic equilibrium

- **Equilibrium (definition):**
  - the object is at rest (static): **linear** and **angular** velocities are both zero.
  - Its center of mass moves with a constant velocity (dynamic)
- Example:
  - Book on table
  - **Balance scale**
  - **Ladder leaning against wall**
  - Puck sliding on ice in a constant velocity
  - ……





Conditions for equilibrium

- The first condition is that the vector sum of all external forces acting on the object must be zero:

First condition for equilibrium:
For the center of mass of an object at rest to remain at rest ...

$$\sum \vec{F} = 0$$

... the *net external force* on the object must be *zero*.

It states that the translational acceleration of the object's center of mass must be zero.

- The second condition is that the sum of external torques must be zero about any point:

Second condition for equilibrium:
For a nonrotating object to remain nonrotating ...

$$\sum \vec{\tau} = 0$$

...the *net external torque around any point* on the object must be *zero*.

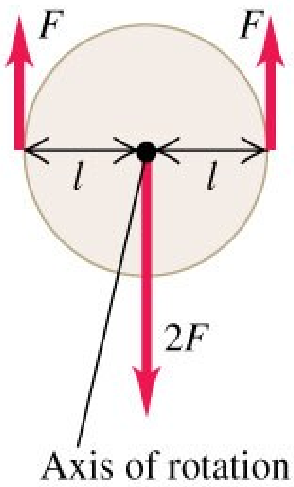
It states that the angular acceleration of the object must be zero.

For any axis of rotation

Conditions for equilibrium: examples

$$\sum F = 0 \qquad \sum \tau = 0$$

(a) This body is in static equilibrium.

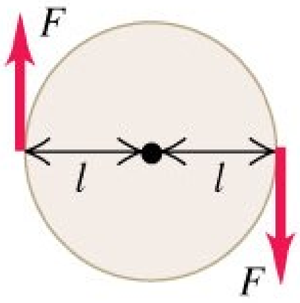


Equilibrium conditions:

First condition satisfied: Net force = 0, so body at rest has no tendency to start moving as a whole.

Second condition satisfied: Net torque about the axis = 0, so body at rest has no tendency to start rotating.

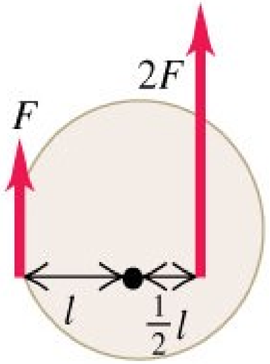
(b) This body has no tendency to accelerate as a whole, but it has a tendency to start rotating.



First condition satisfied: Net force = 0, so body at rest has no tendency to start moving as a whole.

Second condition NOT satisfied: There is a net clockwise torque about the axis, so body at rest will start rotating clockwise.

(c) This body has a tendency to accelerate as a whole but no tendency to start rotating.



First condition NOT satisfied: There is a net upward force, so body at rest will start moving upward.

Second condition satisfied: Net torque about the axis = 0, so body at rest has no tendency to start rotating.

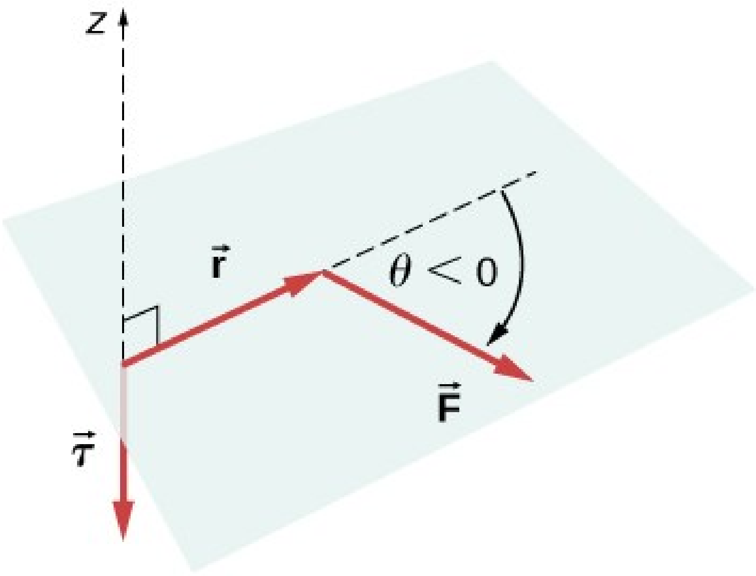
Equilibrium Equations

- Equation for 1st condition: $\sum \vec{F}_{net} = 0$, $F_{net,\,x} = 0$, $F_{net,\,y} = 0$, ~~$F_{net,\,z} = 0$~~
- Equation for 2nd condition: $\sum \vec{\tau}_{net} = 0$, ~~$\tau_{net,\,x} = 0$~~, ~~$\tau_{net,\,y} = 0$~~, $\tau_{net,\,z} = 0$
- We will restrict the applications to situations in which all the forces lie in the *xy* plane.
- There are three resulting equations

$$F_{net,\,x} = \sum F_{ext,x} = 0$$

$$F_{net,\,y} = \sum F_{ext,y} = 0$$

$$\tau_{net,\,z} = \sum \tau_{ext,z} = 0$$

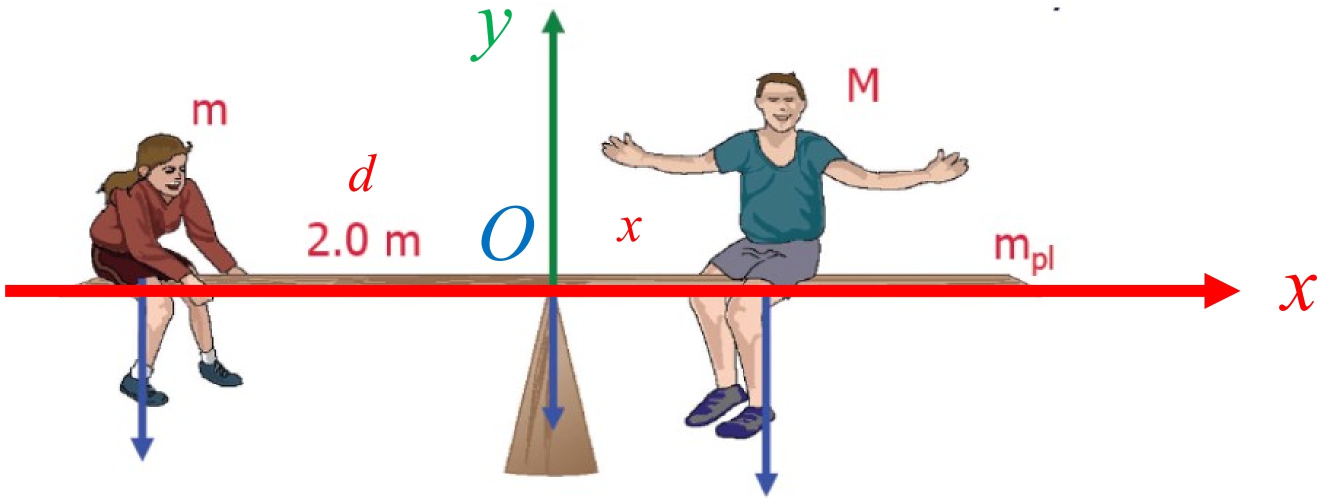


Example: Equilibrium Equations

- A seesaw consisting of a uniform board of mass m_{pl} and length L supports at rest a father and daughter with masses M and m, respectively. The support is under the center of mass of the board, the father is a distance x from the center, and the daughter is a distance d = 2.00 m from the center.

  (a) Find the magnitude of the upward force ***n*** exerted by the support on the board

  (b) Find the where the father should sit to balance the system at rest.



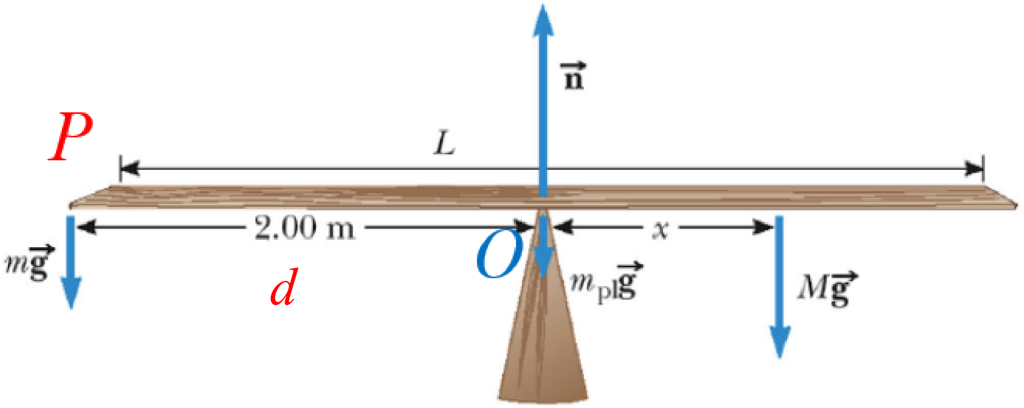
$$F_{net,x} = \sum F_{ext,x} = 0$$

$$F_{net,y} = \sum F_{ext,y} = 0$$

$$\tau_{net,z} = \sum \tau_{ext,z} = 0$$

Example: Equilibrium Equations

(a) The magnitude of the upward force ***n*** exerted by the support on the board
(b) Find the where the father should sit to balance the system at rest.



$$F_{net,\,x} = \sum F_{ext,x} = 0$$

$$F_{net,\,y} = \sum F_{ext,y} = 0$$

$$\tau_{net,\,z} = \sum \tau_{ext,z} = 0$$

$$F_{net,\,y} = n - mg - Mg - m_{pl}g = 0$$

$$n = mg + Mg + m_{pl}g$$

$$\tau_{net,\,z} = \tau_d + \tau_f + \tau_{pl} + \tau_n$$

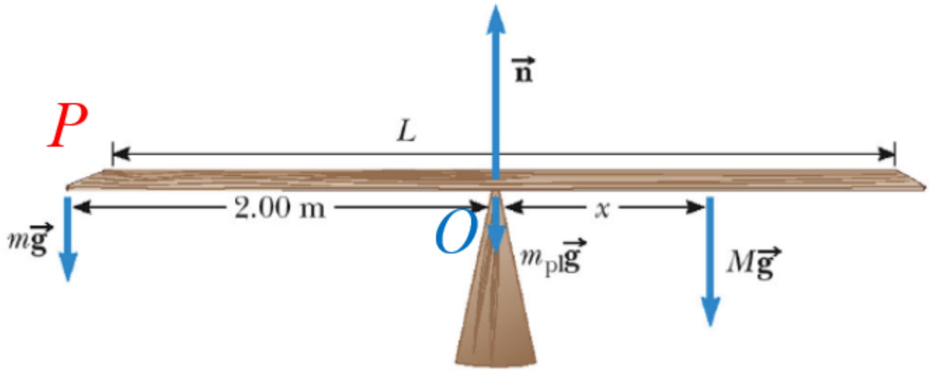
$$\tau_{net,\,z} = mgd - Mgx + 0 + 0 = 0$$

$$mgd = Mgx$$

$$x = \frac{md}{M} = \frac{2m}{M} < 2.0\text{ m}$$

Axis of rotation

- The net torque is about an axis through any point in the *xy* plane.
- For static equilibrium, does it matter which axis you choose for calculating torques?
- No. The choice of an axis is *arbitrary*.
- **If an object is in equilibrium and the net torque is zero about one axis, then the net torque must be zero about any other axis.**
- We should be smart to choose a rotation axis to simplify problems.
- Look at the previous example again, a different axis?



Example: Equilibrium Equations

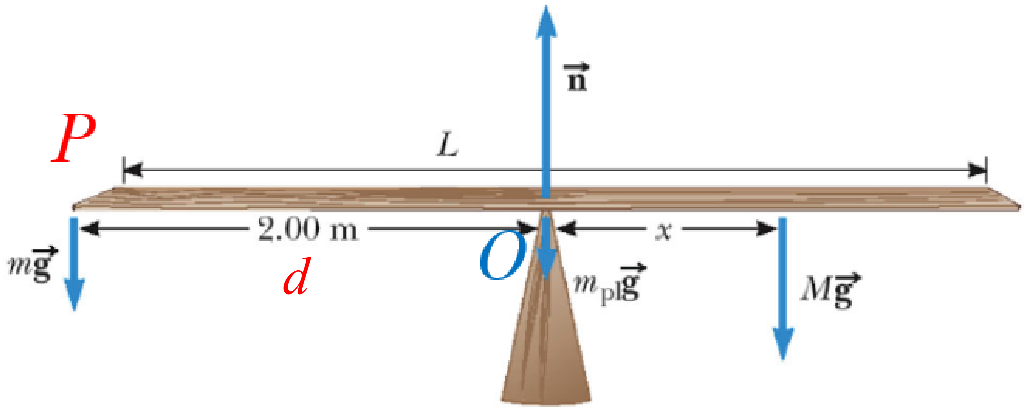
Rotation axis at *O*.

$$\tau_{net,z} = \tau_d + \tau_f + \tau_{pl} + \tau_n$$

$$\tau_{net,z} = mgd - Mgx + 0 + 0 = 0$$

$$mgd = Mgx$$

$$x = \frac{md}{M} = \frac{2m}{M} < 2.0\text{ m}$$



Rotation axis at *P*.

$$\tau_{net,z} = \tau_d + \tau_f + \tau_{pl} + \tau_n$$

$$\tau_{net,z} = 0 - Mg(d + x) - m_{pl}gd + nd = 0$$

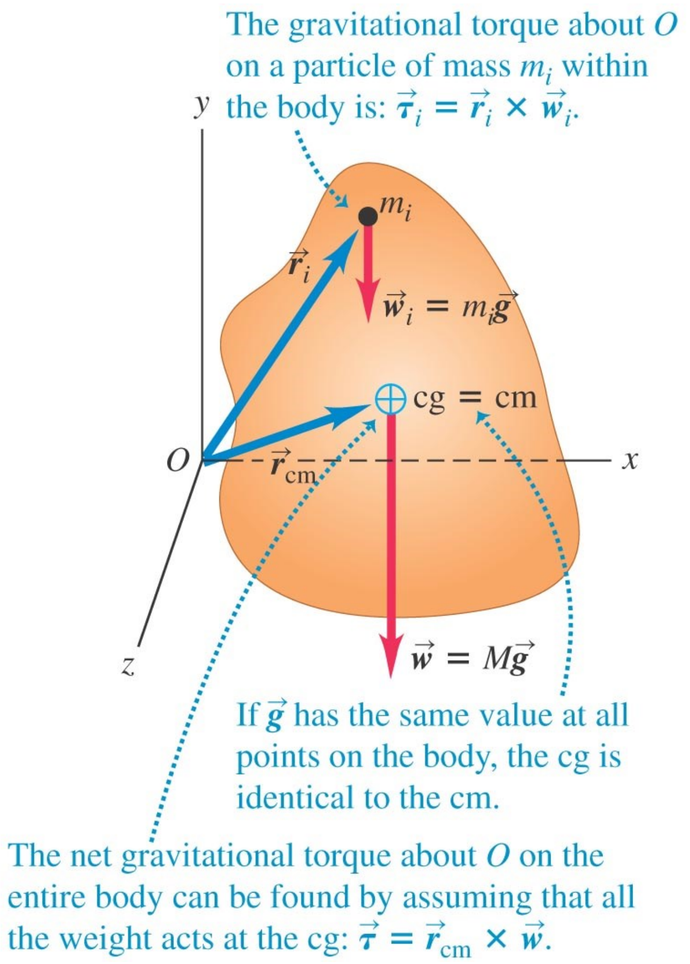
$$-Mg(d + x) - m_{pl}gd + nd = 0$$

$$mgd = Mgx$$

$$x = \frac{md}{M} = \frac{2m}{M} < 2.0\text{ m}$$

Center of gravity

- We can treat a body's ***weight*** as though it all acts at a single point—the ***center of gravity***.
- If we can ignore the variation of gravity with altitude, the center of gravity is the same as the center of mass.
- If the object is homogeneous and symmetrical, the center of gravity is at its geometric center
- The torque due to the gravitational force on an object of mass M is the force Mg acting at the center of gravity of the object.



Center of gravity

$$\vec{\boldsymbol{\tau}}_i = \vec{\boldsymbol{r}}_i \times \vec{\boldsymbol{w}}_i = \vec{\boldsymbol{r}}_i \times m_i\,\vec{\boldsymbol{g}} \qquad \vec{\boldsymbol{\tau}} = \sum_i \vec{\boldsymbol{\tau}}_i = \vec{\boldsymbol{r}}_1 \times m_1\,\vec{\boldsymbol{g}} + \vec{\boldsymbol{r}}_2 \times m_2\,\vec{\boldsymbol{g}} + \cdots = \left(m_1\,\vec{\boldsymbol{r}}_1 + m_2\,\vec{\boldsymbol{r}}_2 + \cdots\right) \times \vec{\boldsymbol{g}}$$

$$M = m_1 + m_2 + \cdots = \sum_i m_i \qquad \vec{\boldsymbol{\tau}} = \frac{m_1\,\vec{\boldsymbol{r}}_1 + m_2\,\vec{\boldsymbol{r}}_2 + \cdots}{m_1 + m_2 + \cdots} \times M\,\vec{g} = \frac{\sum_i m_i\,\vec{\boldsymbol{r}}_i}{\sum_i m_i} \times M\,\vec{g}$$

$$\vec{\boldsymbol{r}}_{\rm cm} = \frac{m_1\vec{\boldsymbol{r}}_1 + m_2\vec{\boldsymbol{r}}_2 + m_3\vec{\boldsymbol{r}}_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{\sum_i m_i\vec{\boldsymbol{r}}_i}{\sum_i m_i}$$

$$\boxed{\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{r}}_{\rm cm} \times M\vec{\boldsymbol{g}} = \vec{\boldsymbol{r}}_{\rm cm} \times \vec{\boldsymbol{w}}}$$

The total gravitational torque is the same as though the total weight were acting at the position $\boldsymbol{r}_{cm}$ of the center of mass, also called center of gravity.

g has the same value at all points on an object, center of mass = center of gravity