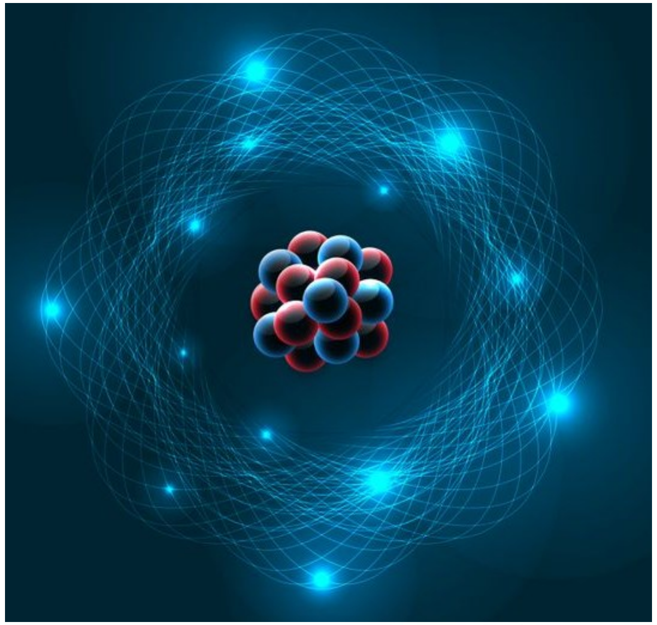
Physics 111: Mechanics
Chapter 11



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Summary: Conditions for Equilibrium

- The net external force on the object must equal zero.

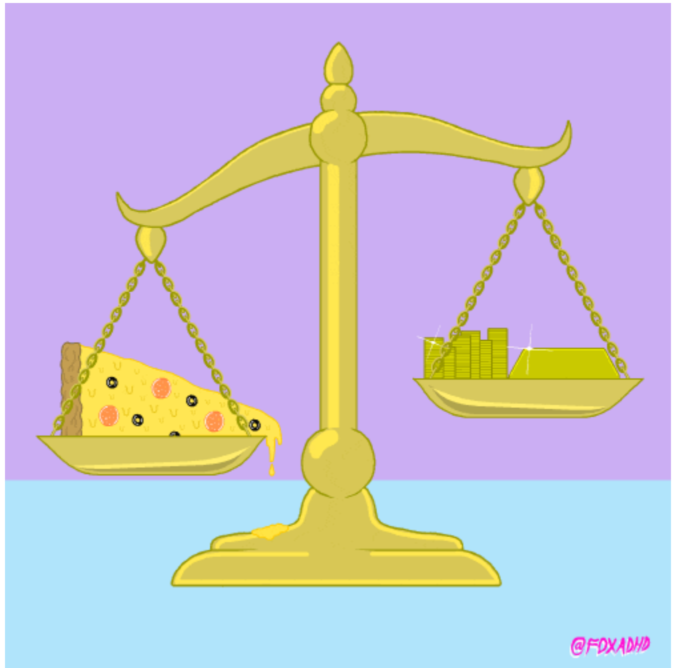
$$\vec{F}_{net} = \sum \vec{F}_{ext} = m\vec{a} = 0$$

- The net external torque on the object must equal zero.

$$\vec{\tau}_{net} = \sum \vec{\tau}_{ext} = I\vec{\alpha} = 0$$

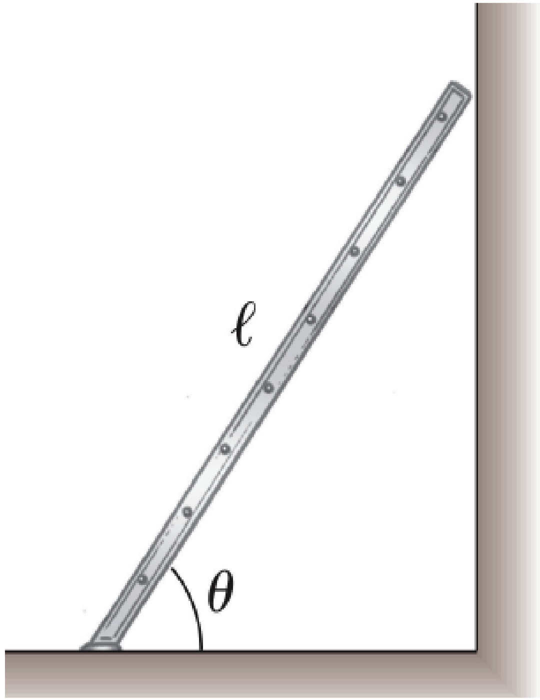
$$\boxed{\begin{aligned} F_{net,\,x} &= \sum F_{ext,x} = 0 \\ F_{net,\,y} &= \sum F_{ext,y} = 0 \\ \tau_{net,\,z} &= \sum \tau_{ext,z} = 0 \end{aligned}}$$

F is confined in *xy* plane.



Ladder problem

- A uniform ladder of length l rest against a smooth, vertical wall. The mass of the ladder is m, and the coefficient of static friction between the ladder and the ground is $\mu_s = 0.40$. The wall is frictionless. Find the minimum angle θ at which the ladder does not slip.

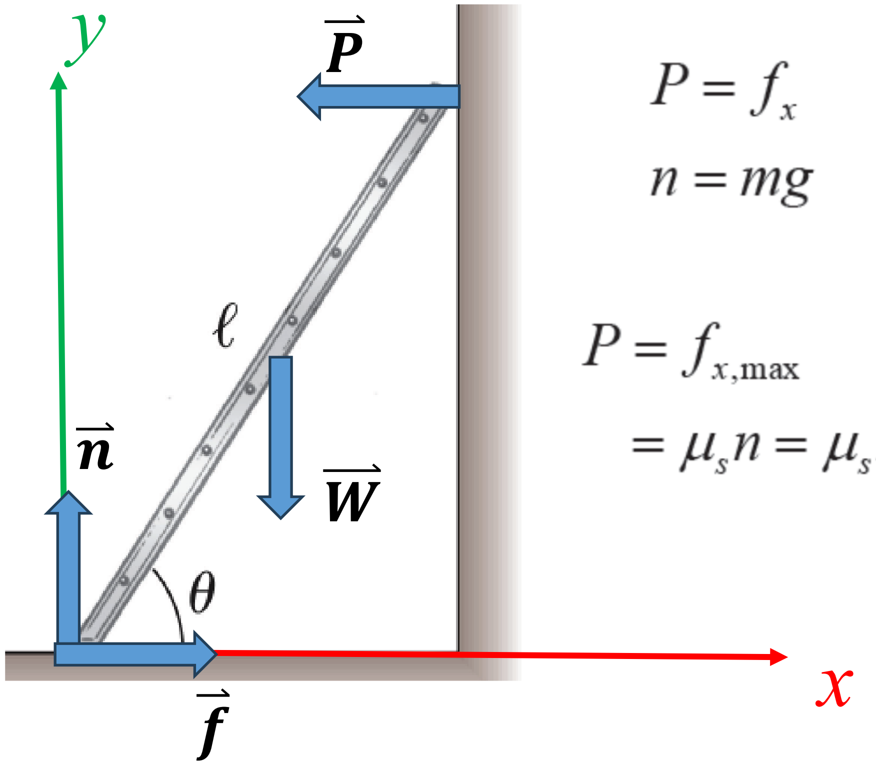


Problem-Solving Strategy 1

- Draw sketch, decide what is in or out the system.
- Draw a free body diagram
- Show and label all external forces acting on the object
- Indicate the locations of all the forces
- Establish a convenient coordinate system
- Find the components of the forces along the two axes
- Apply the first condition for equilibrium
- Be careful of signs

$$\sum F_x = f_x - P = 0$$

$$\sum F_y = n - mg = 0$$



$$P = f_x$$

$$n = mg$$

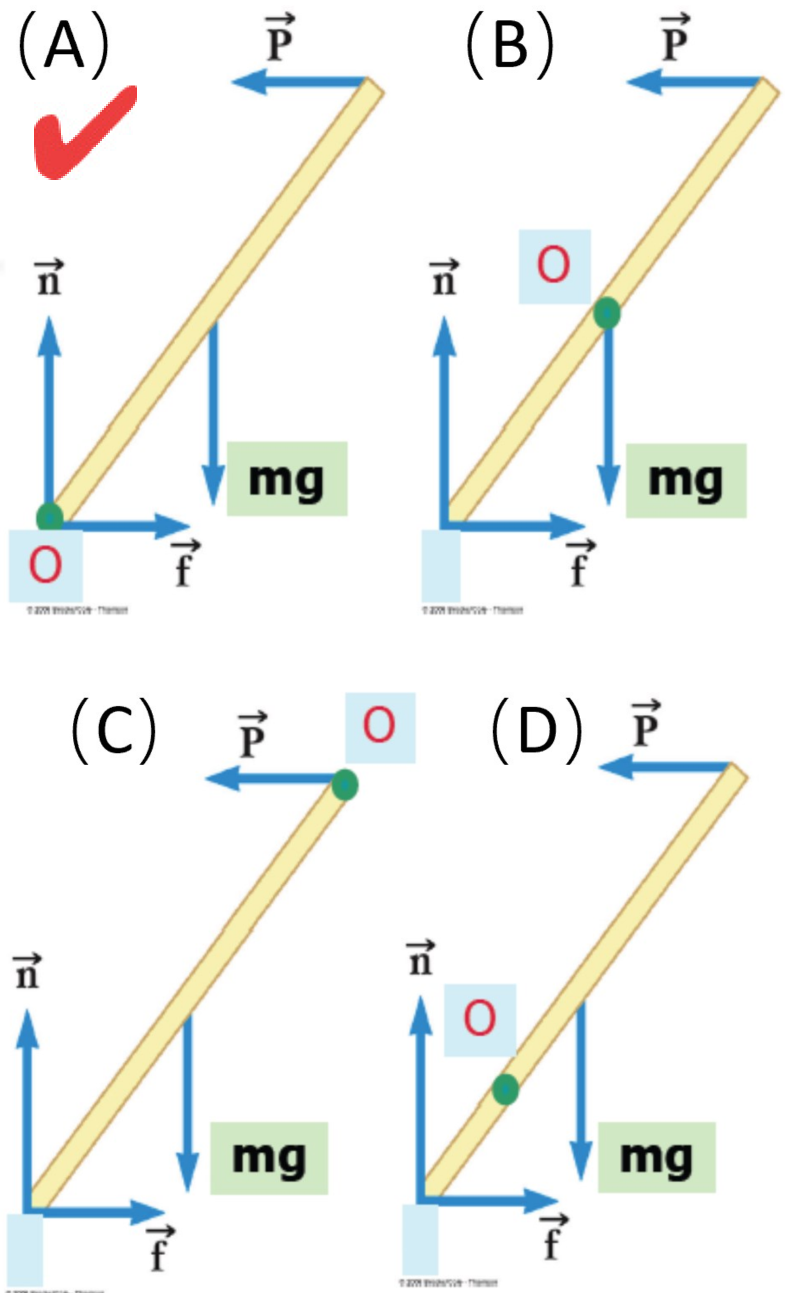
$$P = f_{x,\max}$$

$$= \mu_s n = \mu_s mg$$

Problem-Solving Strategy 2

- Choose a convenient axis for calculating the net torque on the object: remember the choice of the axis is arbitrary
- Choose an origin that simplifies the calculations as much as possible: A force that acts along a line passing through the origin produces a zero torque
- Be careful of sign with respect to rotational axis
    - positive if force tends to rotate object in CCW
    - negative if force tends to rotate object in CW
    - zero if force is on the rotational axis
- Apply the second condition for equilibrium

$$\tau_{net,z} = \sum \tau_{ext,z} = 0$$



Problem-Solving Strategy 3

- The two conditions of equilibrium will give a system of equations
- Solve the equations simultaneously
- Make sure your results are consistent with your free body diagram
- If the solution gives a negative for a force, it is in the opposite direction to what you drew in the free body diagram
- Check your results to confirm

$$\sum F_x = f_x - P = 0$$

$$\sum F_y = n - mg = 0$$

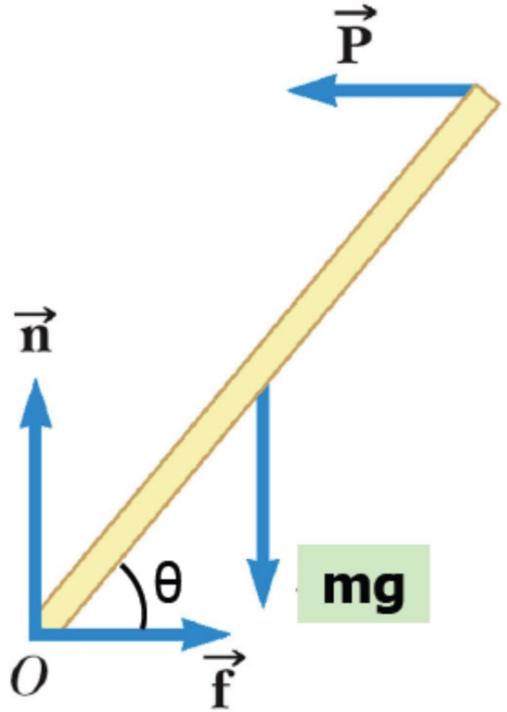
$$P = f_{x,\max}$$

$$= \mu_s n = \mu_s mg$$

$$\sum \tau_O = \tau_n + \tau_f + \tau_g + \tau_P \; = 0 + 0 + Pl\sin\theta_{\min} - mg\frac{l}{2}\cos\theta_{\min} = 0$$

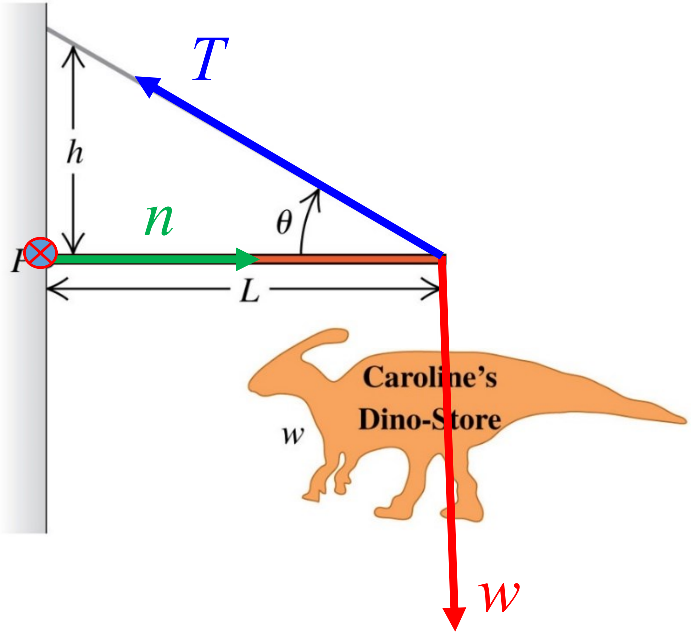
$$\frac{\sin\theta_{\min}}{\cos\theta_{\min}} = \tan\theta_{\min} = \frac{mg}{2P} = \frac{mg}{2\mu_s mg} = \frac{1}{2\mu_s}$$

$$\theta_{\min} = \tan^{-1}\left(\frac{1}{2\mu_s}\right) = \tan^{-1}\left[\frac{1}{2(0.4)}\right] = 51^\circ$$



Example Problem

A metal advertising sign (weight *w*) is suspended from the end of a massless rod of length *L*. The rod is supported at one end by a hinge at point *P* and at the other end by a cable at an angle θ from the horizontal. What is the tension in the cable?



$$\sum F_{ext,x} = 0 \qquad \sum F_{ext,y} = 0$$

$$\sum \tau_{ext,z} = 0$$

(A) $T = w \sin \theta$

(B) $T = w \cos \theta$

(C) $T = w/(\sin \theta)$ ✓

(D) $T = w/(\cos \theta)$

(E) none of the above

$$\sum \vec{\tau}_{net} = TL\sin\theta - wL = 0$$

So $T = w/(\sin\theta)$

Raising a ladder

A ladder (weigh $w = 3400$ N) carried by a fire truck is 20.0 m long. The ladder is pivoted at one end (A) about a frictionless pin. The ladder is raised into position by a force applied at C. Point C is 8.0 m from A, and the force $\vec{F}$ makes an angle of 40° with the ladder. What magnitude must $\vec{F}$ have to just lift the ladder off the support bracket at B? Start with a free-body diagram of the ladder.

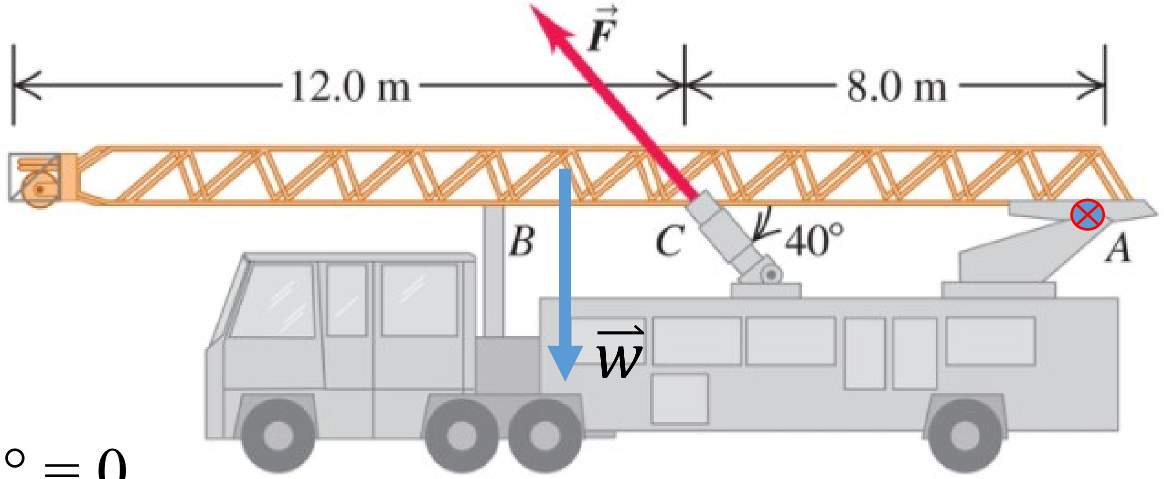
$$\sum F_{ext,x} = 0 \qquad \sum F_{ext,y} = 0$$

$$\sum \tau_{ext,z} = 0$$

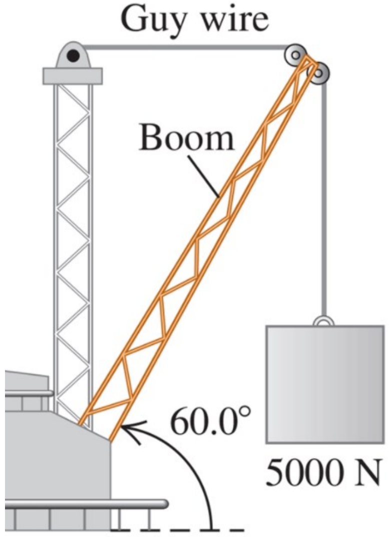
$$\sum \tau_{ext,z} = 0 = \tau_w + \tau_F$$

$$\tau_w + \tau_F = (3400\ \mathrm{N})(10.0\ \mathrm{m}) - F(8.0\ \mathrm{m})sin40° = 0$$

$$F = 6.6\ \mathrm{kN}$$



The boom weighs 2600 N and is attached to a frictionless pivot at its lower end. The distance of its center of gravity from the pivot is 35% of its length. Find **(a)** the tension in the guy wire and **(b)** the horizontal and vertical components of the force exerted on the boom at its lower end.



$\sum \tau = 0, \ \sum F_x = 0, \ \sum F_y = 0.$ L: the length of the boom.

$\sum \tau = 0$ gives

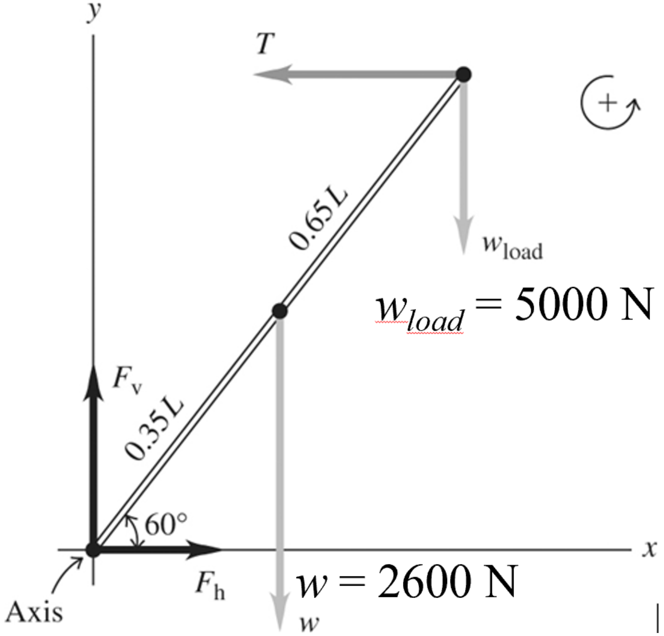
$$T(L\sin 60.0°) - w_{\text{load}}(L\cos 60.0°) - w(0.35L\cos 60.0°) = 0$$

$$T = \frac{w_{\text{load}}\cos 60.0° + w(0.35\cos 60.0°)}{\sin 60.0°} = 3.41\times 10^3 \text{ N.}$$

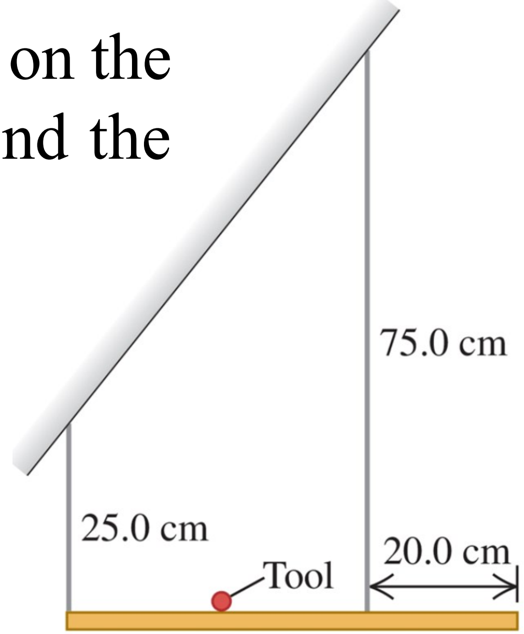
$\sum F_x = 0$ gives $F_h - T = 0$ and $F_h = 3410$ N.

$\sum F_y = 0$ gives $F_v - w - w_{\text{load}} = 0$

$$F_v = 5000 \text{ N} + 2600 \text{ N} = 7600 \text{ N}$$



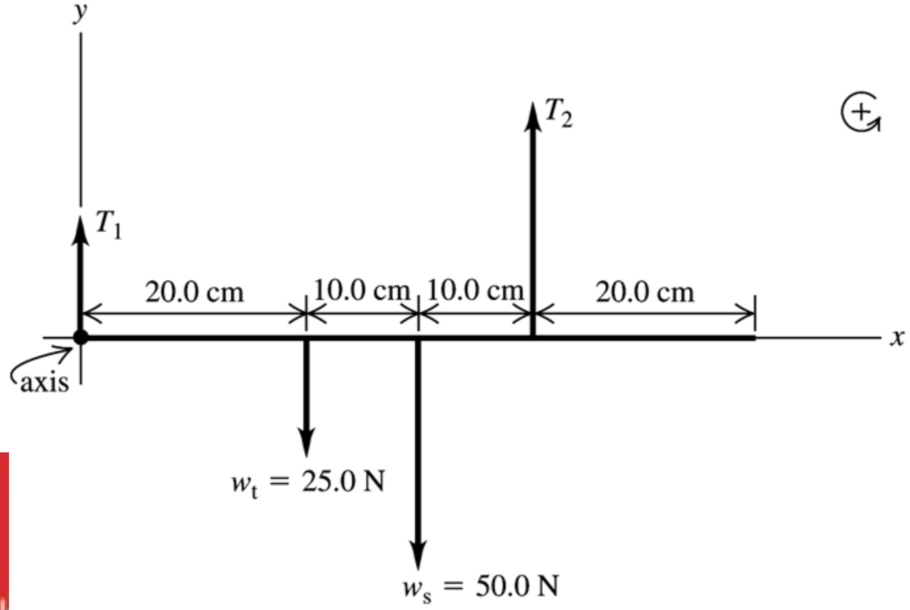
A 60.0 cm, uniform, 50.0 N shelf is supported horizontally by two vertical wires attached to the sloping ceiling. A very small 25.0 N tool is placed on the shelf midway between the points where the wires are attached to it. Find the tension in each wire. Begin by making a free-body diagram of the shelf.



$$\sum \tau_z = 0 \text{ gives } -w_t(0.200\text{ m}) - w_s(0.300\text{ m}) + T_2(0.400\text{ m}) = 0.$$

$$T_2 = \frac{(25.0\text{ N})(0.200\text{ m}) + (50.0\text{ N})(0.300\text{ m})}{0.400\text{ m}} = 50.0\text{ N}$$

$$\sum F_y = 0 \text{ gives } T_1 + T_2 - w_t - w_s = 0 \text{ and } T_1 = 25.0\text{ N}.$$



One end of a uniform 4.0-m-long rod of weight F_g is supported by a cable at an angle of $\theta = 37°$ with the rod. The other end rests against the wall, where it is held by friction as shown in the figure below. The coefficient of static friction between the wall and the rod is $\mu_s = 0.50$. Determine the minimum distance x from point A at which an additional object, also with the same weight F_g, can be hung without causing the rod to slip at point A.

[✓ (A). 2.81 m]

(B). 1.55 m

(C). 3.05 m

(D). 2.98 m

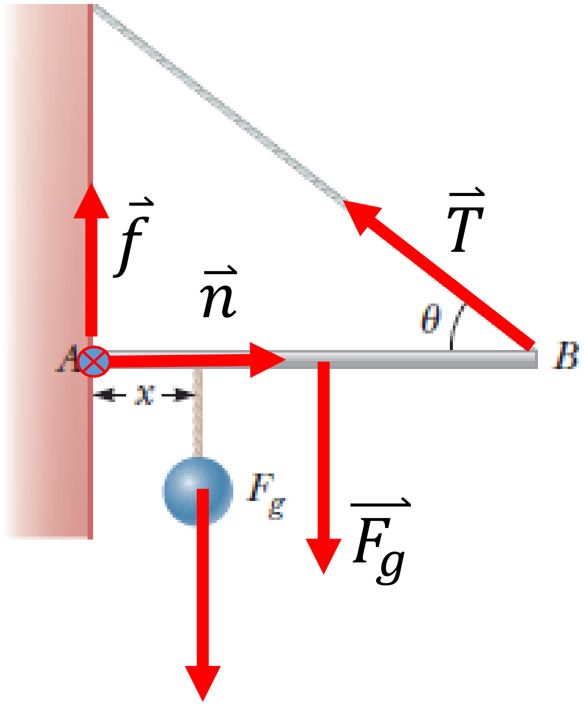
$$\sum \vec{F}_x = -T\cos\theta + n = 0$$

$$\sum \vec{F}_y = T\sin\theta + f - 2F_g = T\sin\theta + 0.5n - 2F_g = 0$$

$$0.5T(\sin\theta + 0.5\cos\theta) = F_g$$

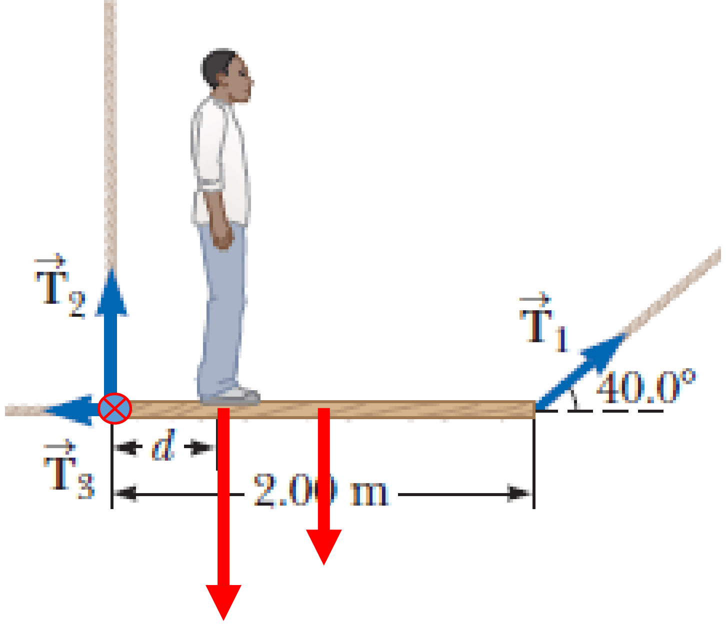
$$\sum \vec{\tau}_{net} = TL\sin\theta - F_g\left(\frac{L}{2}\right) - F_g x = 0$$

$$(\sin\theta + 0.5\cos\theta)(0.5L + x) = 2L\sin\theta$$



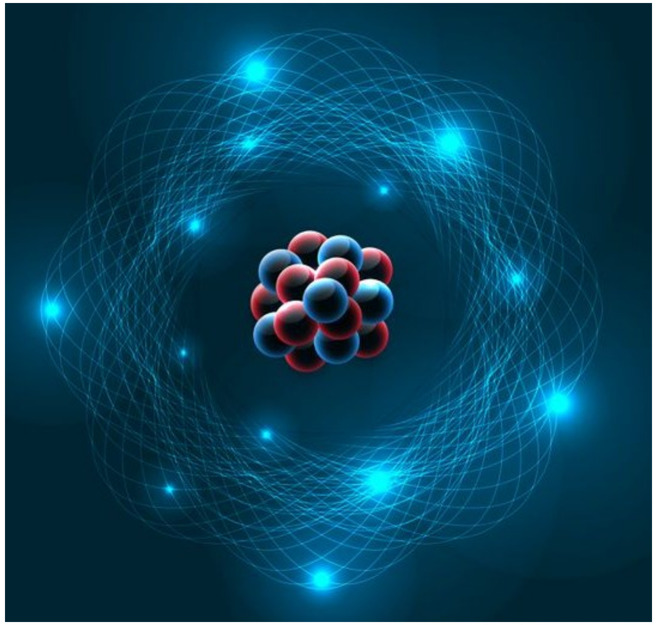
A uniform plank of length 2.00 m and mass 30.0 kg is supported by three ropes as indicated by the blue vectors in the figure below. Find the tension in each rope when a 700 N person is d = 0.5 m from the left end.

- [✓ (A)] $T_1 = 500$ N, $T_2 = 672$ N, $T_3 = 383$ N
- (B) $T_1 = 600$ N, $T_2 = 869$ N, $T_3 = 482$ N
- (C) $T_1 = 300$ N, $T_2 = 875$ N, $T_3 = 755$ N
- (D) $T_1 = 400$ N, $T_2 = 333$ N, $T_3 = 555$ N



$$\sum \vec{\tau}_{net} = -(700\,N)(0.5\,m) - (30.0\,kg)g(1.00\,m) + T_1(2.00\,m)sin40° = 0$$

Physics 111: Mechanics
Chapter 12



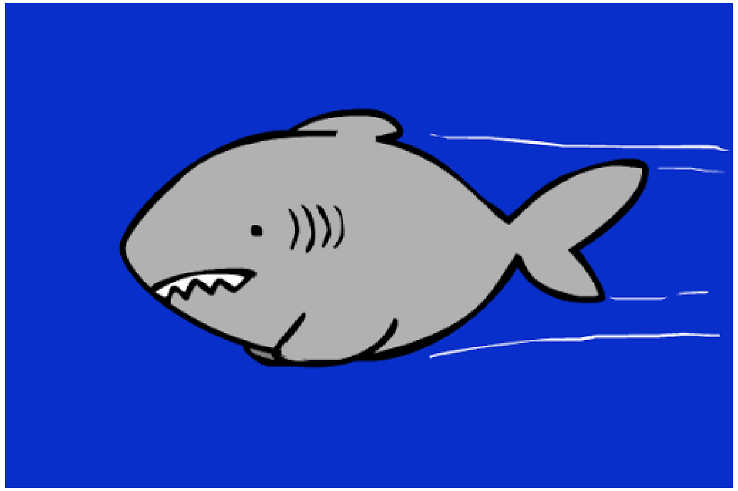
Physics at NJIT

New Jersey's Science & Technology University

THE EDGE IN KNOWLEDGE

Fluid mechanics

- Meaning of density, and how it is measured
- Meaning of pressure in a fluid, and how it is measured
- Buoyancy force, and how to calculate it
- Fluid flow
- Bernoulli's equation, how to use it to relate pressure and flow speed at different points in certain types of flow.



Density

- The density of a material is its mass per unit volume.
- For a homogeneous material:

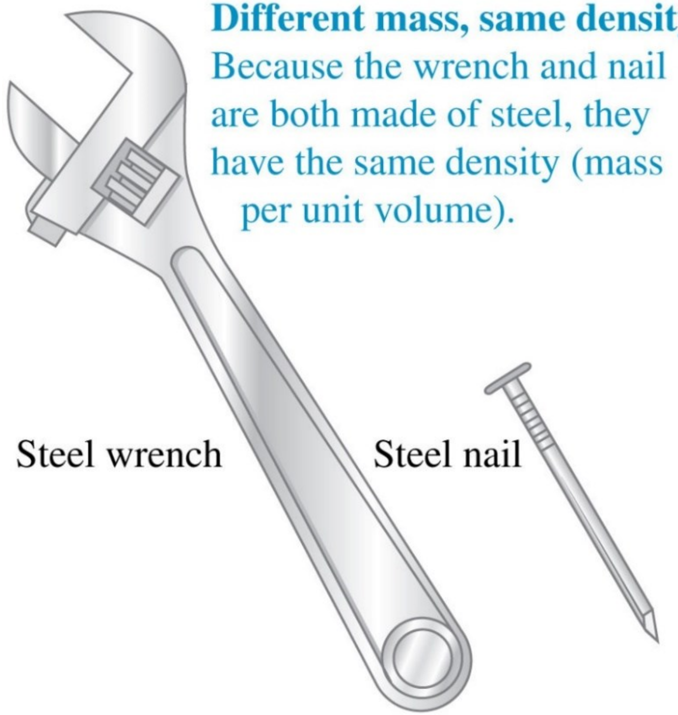
Density of a homogeneous material

$$\rho = \frac{m}{V}$$

Mass of material

Volume occupied by material

- SI units of density is kg/m^3.
- Object made of the same material have the same density even though they may have different masses and different volumes
- Specific gravity: the density compared to that of water at 4 °C



Different mass, same density: Because the wrench and nail are both made of steel, they have the same density (mass per unit volume).

Table 12.1 Densities of Some Common Substances

Material	Density (kg/m^3)*	Material	Density (kg/m^3)*
Air (1 atm, 20°C)	1.20	Iron, steel	7.8×10^3
Ethanol	0.81×10^3	Brass	8.6×10^3
Benzene	0.90×10^3	Copper	8.9×10^3
Ice	0.92×10^3	Silver	10.5×10^3
Water	1.00×10^3	Lead	11.3×10^3
Seawater	1.03×10^3	Mercury	13.6×10^3
Blood	1.06×10^3	Gold	19.3×10^3
Glycerine	1.26×10^3	Platinum	21.4×10^3
Concrete	2×10^3	White dwarf star	10^{10}
Aluminum	2.7×10^3	Neutron star	10^{18}

specific gravity

*To obtain the densities in grams per cubic centimeter, simply divide by 10^3.

Example: Density

How much does the air in a room with a 4.0 m × 5.0 m floor and a ceiling 3.0 m high weight?

(A). 72 kg.

[✓ (B). 705 N.]

(C). 120 N.

(D). 25 kg.

Material	Density (kg/m^3)*
Air (1 atm, 20°C)	1.20

$$W = mg = \rho V g$$

$$= \left(1.2\frac{kg}{m^3}\right)\left(4.0 \times 5.0 \times 3.0\ m^3\right) \times 9.8 = 705\ N$$