Physics 111: Mechanics
Chapter 13

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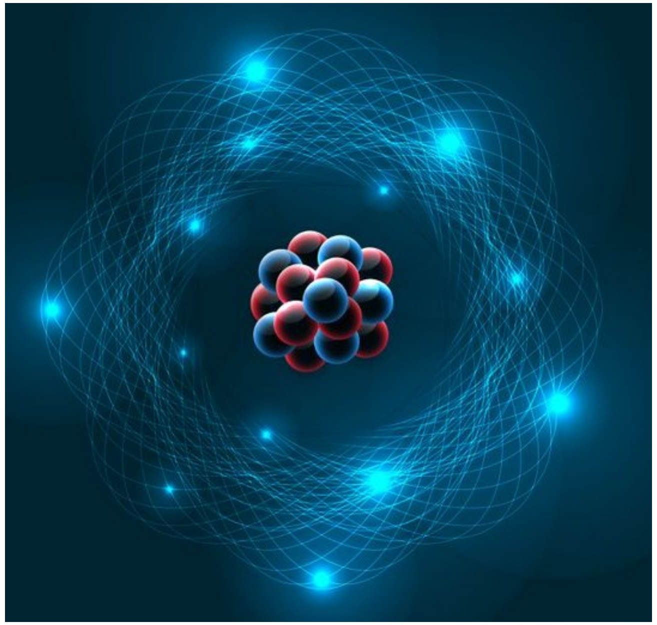
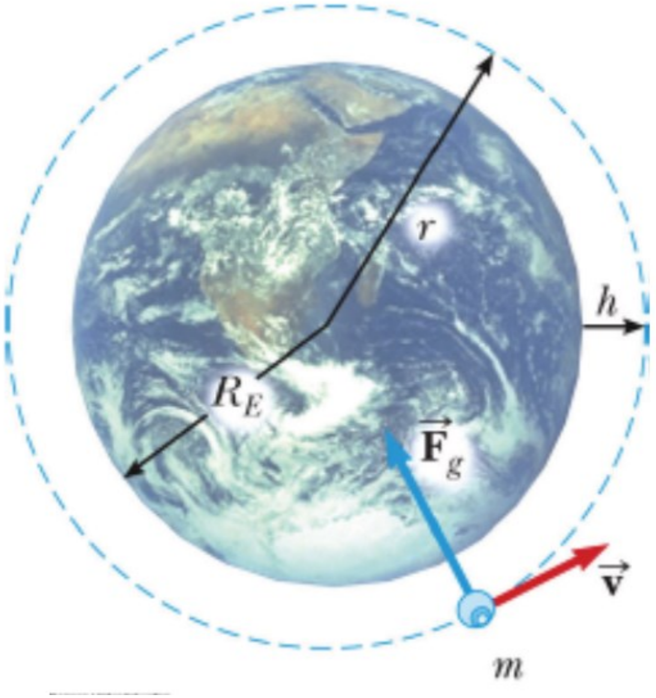


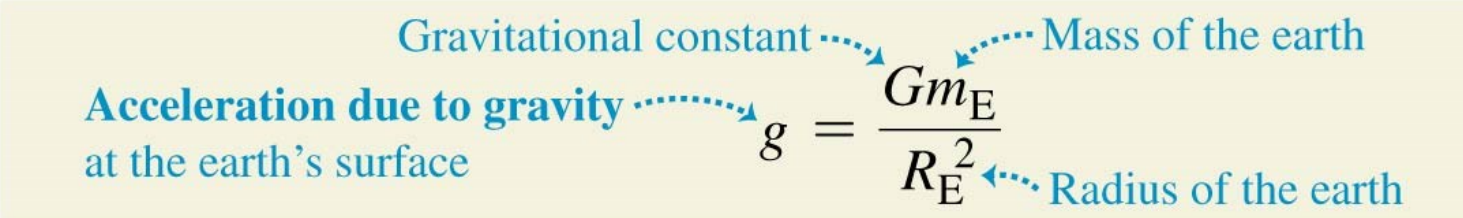
Free-fall acceleration: near Earth's surface

- Consider an object mass m near the Earth's surface

$$F = G\frac{m_1 m_2}{r^2} = G\frac{mM_E}{R_E^2}$$



- Acceleration g due to gravity $F = G\frac{mM_E}{R_E^2} = mg$



- Since $M_E = 5.9742 \times 10^{23}\ kg,\ R_E = 6378.1\ km$
- Near the Earth's surface $g = 9.8$

$G = 6.673 \times 10^{-11}\ N\ m^2/kg^2$

Free-fall acceleration: above the Earth's surface

- Consider an object mass m at a height h above the Earth's surface

$$F = G\frac{m_1 m_2}{r^2} = G\frac{mM_E}{(R_E + h)^2}$$

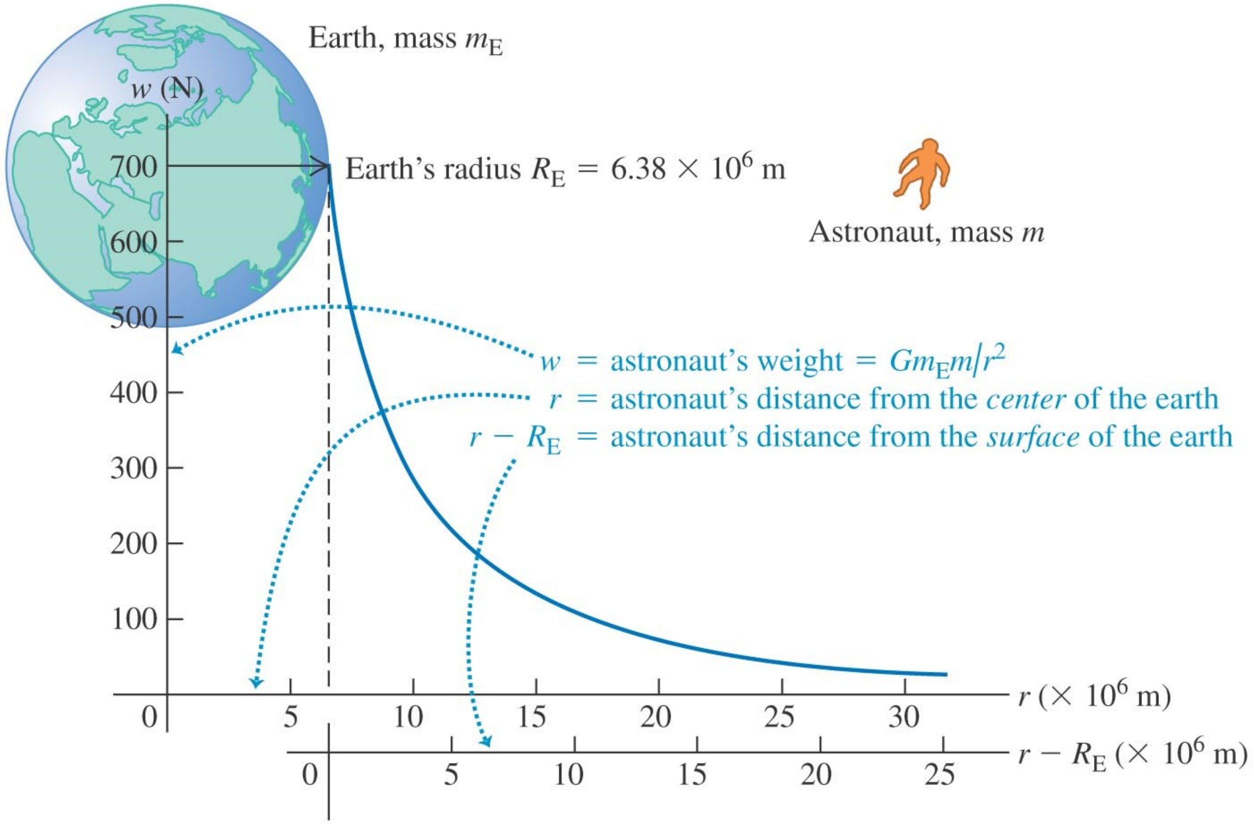
- Acceleration g due to gravity

$$F = G\frac{mM_E}{(R_E + h)^2} = mg$$

- Acceleration g vary with altitude

$$g = G\frac{M_E}{(R_E + h)^2}$$

$$w = mg = mG\frac{M_E}{(R_E + h)^2}$$



Example

I weigh 600 N on the surface of the earth. If I travel on a space shuttle orbiting at a distance of 3 times the Earth's radius above ground. What is my mass in kg at the space shuttle?

A. 37.5

B. 150

C. 15.0

D. 3.83

E. 61.2 ✓

$$g = G\frac{M_E}{(R_E)^2} = 9.8$$

$$g = G\frac{M_E}{(R_E + h)^2}$$

$M_E = 5.9742 \times 10^{23}\ kg$
$R_E = 6378.1\ km$

$G = 6.673 \times 10^{-11}$ N m^2/kg^2

On earth surface: w =600 N = mg, g = 9.8
So m = 61.2 kg

Example: Free-fall acceleration

At what distance above the surface of the earth is the acceleration due to the earth's gravity 0.980 m/s^2, if the acceleration due to gravity at the surface has magnitude 9.8 m/s^2?

$$g = G\frac{M_E}{[r = (R_E + h)]^2} \qquad R_E = 6.37 \times 10^6 \text{ m}$$

The g value changes from 0.980 m/s^2 to 0.980 m/s^2

The r must be increased by a factor of $\sqrt{10}$

so the distance above the surface of the earth is $(\sqrt{10} - 1)R_E = 1.38 \times 10^7$ m.

Astronauts in Orbit

- Acceleration g vary with altitude

$$g = G\frac{M_E}{(R_E + h)^2}$$

- What is an Astronaut's weight in outer space?
- How will the g and our gravitational potential energy change, for example, when we travel to Mars from Earth?

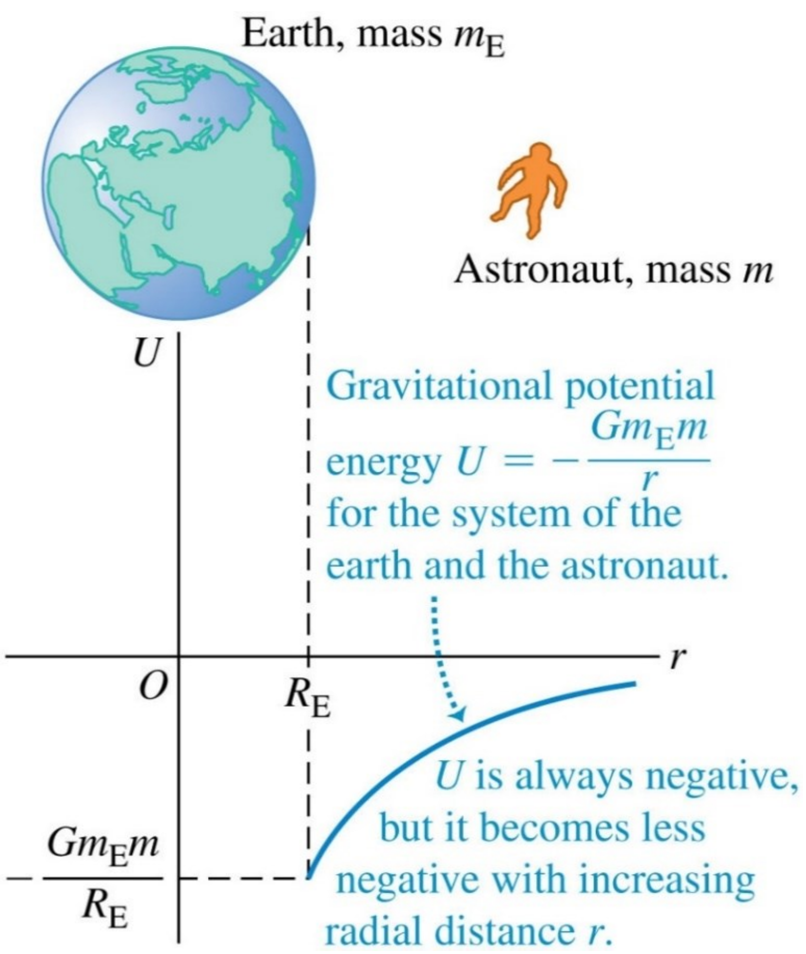
Astronauts in Orbit

$$g = G\frac{M_E}{(R_E + h)^2}$$

- Astronauts are weightless in space, therefore there is no gravity in space?
- It is true that if an astronaut on the International Space Station tried to step on a scale, he/she will weigh nothing.
- It may seem reasonable to think that if weight = mg, since weight = 0, $g = 0$, but this is NOT true.
- If you stand on a scale in an elevator and then the cables are cut, you will also weigh nothing ($ma = N - mg$, but in free-fall $a = g$, so the normal force $N = 0$). This does not mean $g = 0$!
- Astronauts in orbit are in free-fall around the Earth, just as you would be in the elevator. They do not fall to Earth, only because of their very high tangential speed.

Gravitational Potential Energy

- $U = mgy$ is valid only near the Earth's surface
- $U = mgy$: Zero reference level is the Earth's surface.
- For objects high above the Earth's surface?
- An alternate expression is needed $U = -G\frac{M_E m}{r}$
- r changes from R_E to infinitely far. Reference level?
- **Zero reference level** is infinitely far from the Earth, so potential energy is everywhere negative.
- Energy conservation $E = K + U = \frac{1}{2}mv^2 - G\frac{M_E m}{r}$



Example: Gravitational Potential Energy

Compared to the Earth, Planet X has twice the mass and twice the radius. This means that compared to the amount of energy required to move an object from the Earth's surface to infinity, the amount of energy required to move that same object from Planet X's surface to infinity is

A. 4 times as much.

B. twice as much.

✓C. the same.

D. 1/2 as much.

E. 1/4 as much.

$$U = -G\frac{M_E m}{r}$$

$$U = -G\frac{2M_E m}{2r} = -G\frac{M_E m}{r}$$

Example: Gravitational Potential Energy

10 days after it is launched toward Mars, the Starship (mass 5×10^6 kg) is 2.87×10^6 km from the earth and traveling at 1.20×10^4 km/h relative to the earth. At this time instant:

(a) What is the Starship's kinetic energy relative to the earth?

(b) What is the potential energy of the earth-Starship system?

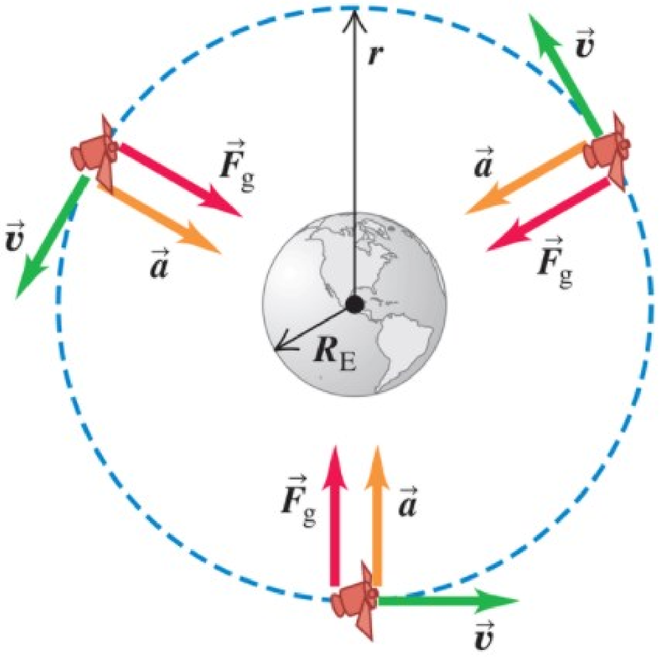
$$K = \tfrac{1}{2} m v^2 \qquad K = \tfrac{1}{2}(5 \times 10^6 \text{ kg})(3.33 \times 10^3 \text{ m/s})^2 = 2.77 \times 10^{13} \text{ J}$$

$$U = -\frac{G M_{\mathrm{E}} m}{r} = -\frac{(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.97 \times 10^{24} \text{ kg})(5 \times 10^6 \text{ kg})}{2.87 \times 10^6 \text{ m}} = -6.94 \times 10^{14} \text{ J.}$$

Satellites: Circular Orbits

- In a circular orbit the speed is just right to keep the distance from the satellite to the center of the earth constant.

$$a_{\mathrm{rad}} = v^2/r \qquad F_g = G m_E\, m/r^2 \qquad \sum \vec{F} = m\vec{a}$$



$$\frac{G m_E\, m}{r^2} = \frac{m v^2}{r}$$

Gravitational constant

Speed of satellite in a circular orbit around the earth

$$v = \sqrt{\frac{G m_E}{r}}$$

Mass of the earth

Radius of orbit

Examples: Circular Orbits

A star Rho is 57 light-years from the earth and has a mass 0.85 times that of our sun. A planet has been detected in a circular orbit around Rho with an orbital radius equal to 0.11 times the radius of the earth's orbit around the sun. What are (a) the orbital speed and (b) the orbital period of the planet?

$v = \sqrt{Gm/r},$ Mass of sun: $m_S = 1.99 \times 10^{30}$ kg. orbit radius of earth 1.50×10^{11} m.

$$v = \sqrt{(6.673 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})(0.85 \times 1.99 \times 10^{30}\ \mathrm{kg})/((1.50 \times 10^{11}\ \mathrm{m})(0.11))} = 8.27 \times 10^4\ \mathrm{m/s}.$$

$T = \dfrac{2\pi r}{v}.$ $2\pi r/v = 1.25 \times 10^6\ \mathrm{s} = 14.5$ days

Example:

A satellite is moving around the Earth in a circular orbit. Over the course of an orbit, the Earth's gravitational force

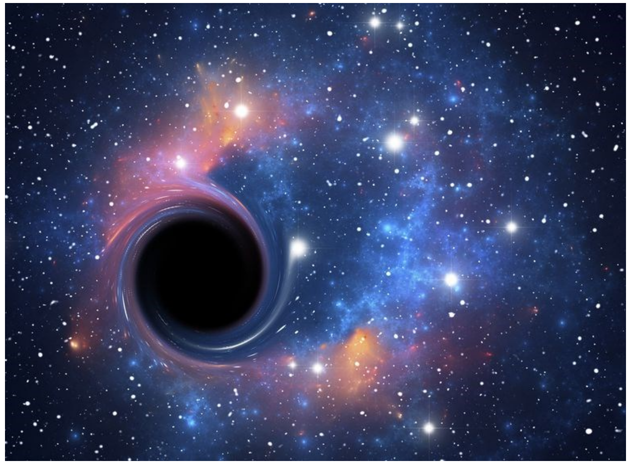
(A). does positive work on the satellite.

(B). does negative work on the satellite.

(C). does positive work on the satellite during part of the orbit and negative work on the satellite during the other part.

✓ (D). does zero work on the satellite at all points in the orbit.

Escape Speed



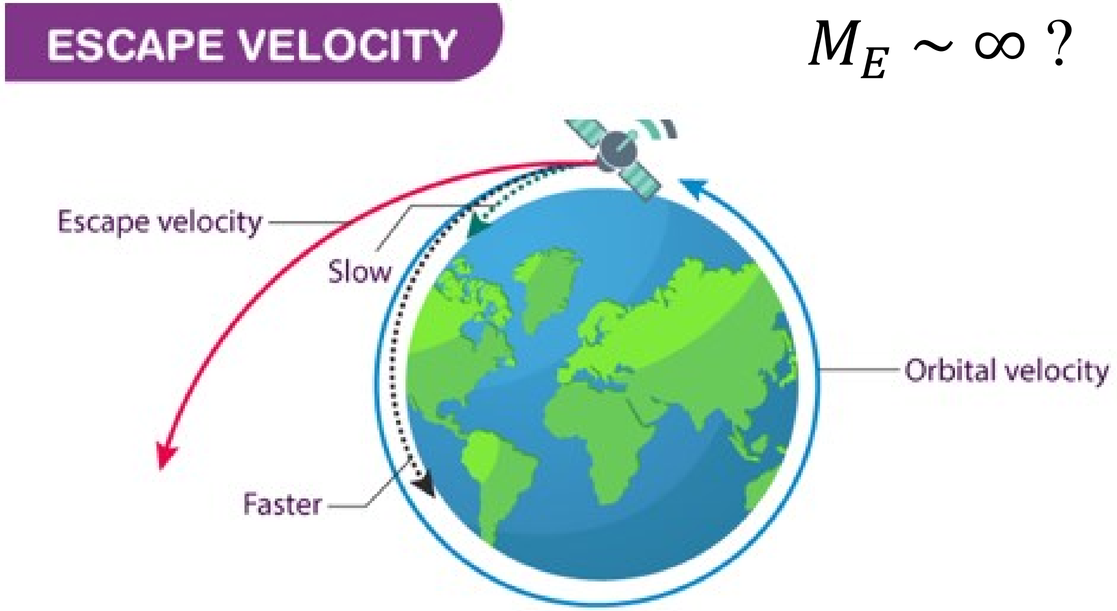
- To escape from the earth, the Starship must have the escape speed. Total mechanical energy is conserved:

$$E = K + U = \frac{1}{2}mv^2 - G\frac{M_E m}{r}$$

$r \sim \infty$ $$E = \frac{1}{2}mv_\infty^2 \geq 0$$

$r \sim R_E$ $$E = \frac{1}{2}mv^2 - G\frac{M_E m}{R_E} \geq 0$$

$$\frac{1}{2}mv^2 \geq \mathrm{G}\frac{M_E m}{r} \qquad v = \sqrt{\frac{2GM_E}{R_E}}$$



$M_E \sim \infty$?

Example: Escape Speed

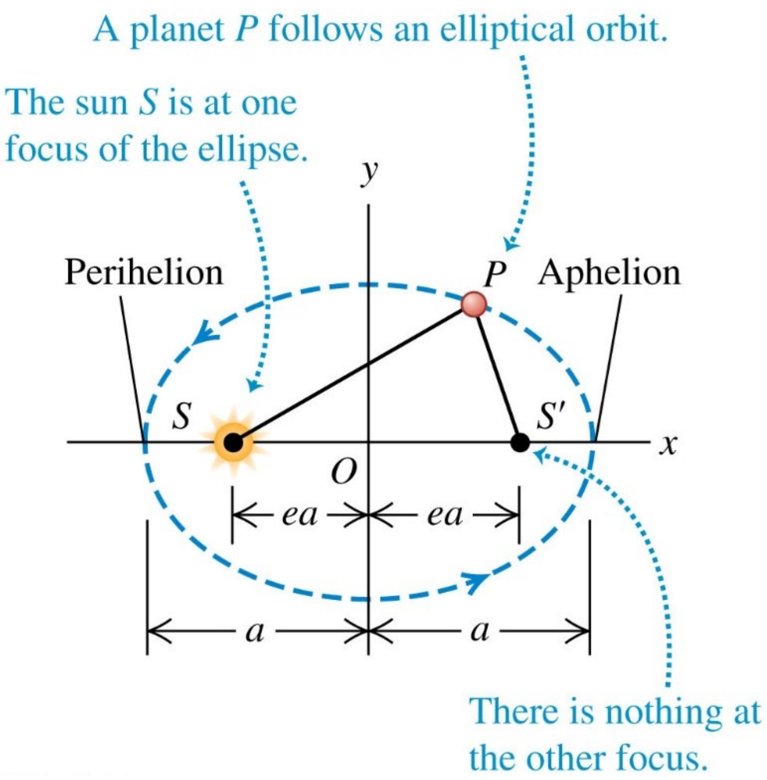
A planet orbiting a distant star has radium 3.24×10^6 m. The escape speed for an object launched from this planet's surface is 7.65×10^3 m/s. What is the acceleration (m/s^2) due to gravity at the surface of the planet?

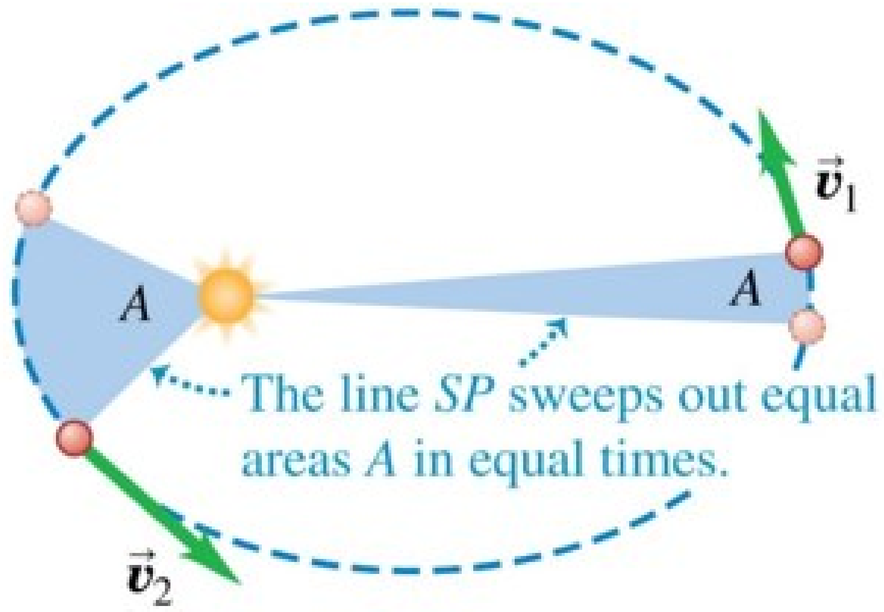
$$v = \sqrt{\frac{2GM_P}{R_P}} \qquad v^2 R_P = 2GM_P \qquad g = G\frac{M_P}{{R_P}^2} = \frac{v^2}{2R_P}$$

$g = (7.65 \times 10^3 \text{ m/s})^2/[2(3.24 \times 10^6 \text{ m})] = 9.03 \text{ m/s}^2$.

Kepler's Laws

- **First Law:** Each planet moves in an elliptical orbit with the sun at one focus of the ellipse.
- **Second Law:** A line from the sun to a given planet sweeps out equal areas in equal times.





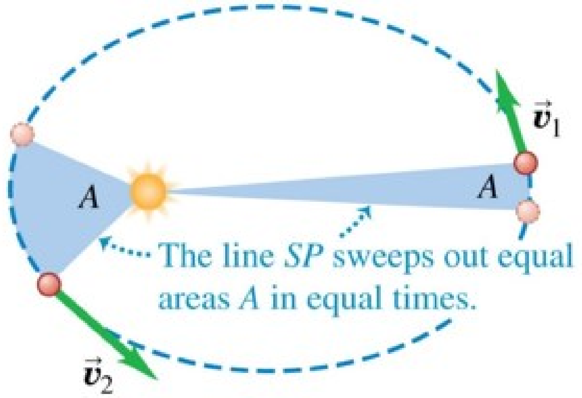
Kepler's Laws

- **Second Law:** A line from the sun to a given planet sweeps out equal areas in equal times.

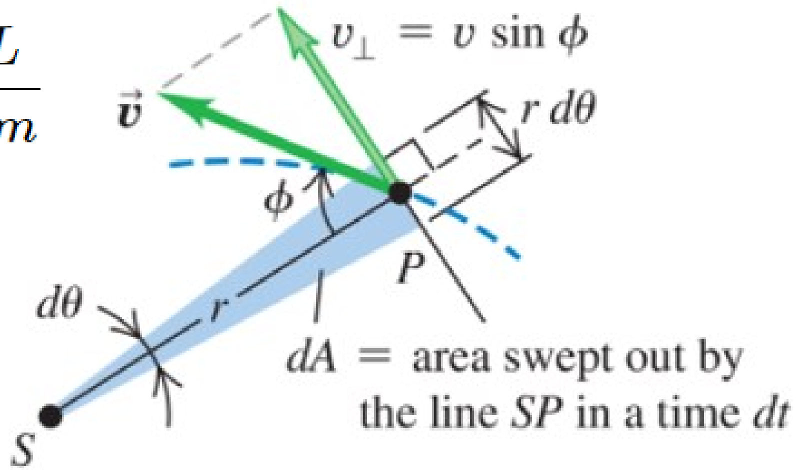
Area of the blue triangle $dA = \frac{1}{2}(r)(rd\theta) = \frac{1}{2}r^2 d\theta$

Tangential velocity $v_\perp = r\omega = r\frac{d\theta}{dt}$

Area sweeping rate: $\frac{dA}{dt} = \frac{1}{2}r^2\frac{d\theta}{dt} = \frac{1}{2}rv_\perp$



$rv_\perp = \vec{r} \times \vec{v}$ $\quad \vec{L} = \vec{r} \times m\vec{v}$ $\quad \frac{dA}{dt} = \frac{1}{2}\vec{r} \times \vec{v} = \frac{1}{2m}|\vec{r} \times m\vec{v}| = \frac{L}{2m}$



Kepler's second law tells us the angular moment of the planet is a constant (conserved)!