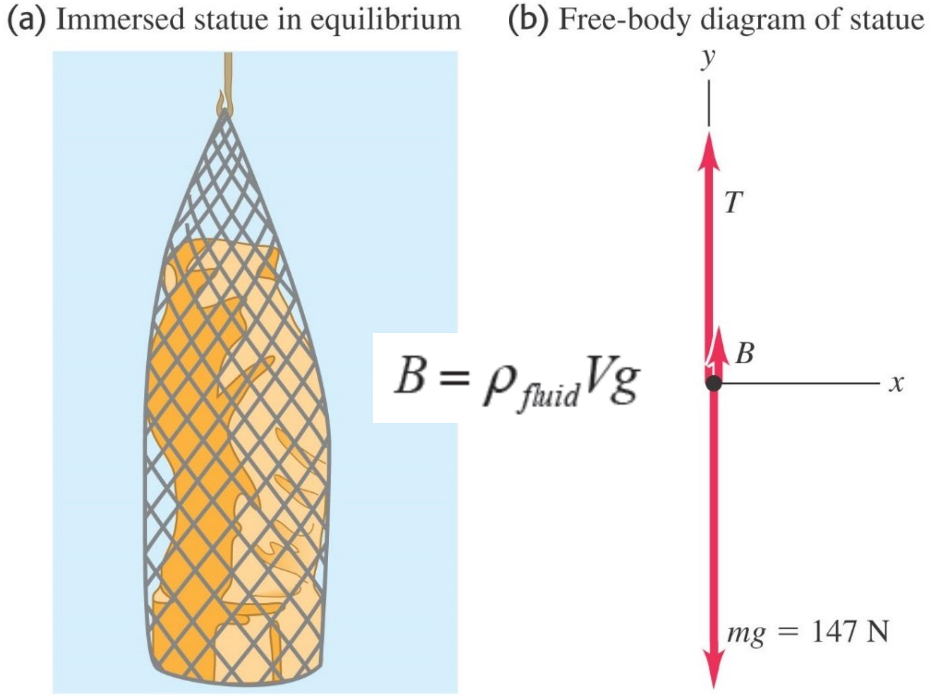
Example: Buoyancy

A 15.0-kg solid gold statue is raised from the sea bottom as shown in the figure. What is the tension in the hoisting cable (assumed massless) when the statue is (a) at rest and completely underwater and (b) at rest and completely out of the water?



$$V = \frac{m_{statue}}{\rho_{statue}} = \frac{15.0\ \text{kg}}{19.3\times10^3\ \text{kg/m}^3} = 7.77\times10^{-4}\ \text{m}^3$$

$$B_{sw} = w_{sw} = m_{sw} g = \rho_{sw} V g = 7.84\ \text{N}$$

$$\sum F_y = B_{sw} + T_{sw} + (-m_{statue} g) = 0$$

$$T_{sw} = m_{statue} g - B_{sw} = 139\ \text{N}$$

$$B_{air} = m_{air} g = \rho_{air} V g = 9.1\times10^{-3}\ \text{N}$$

$$\sum F_y = B_{air} + T_{air} + (-m_{statue} g) = 0$$

$$T_{air} = m_{statue} g - B_{air} = m_{statue} g = 147\ \text{N}$$

Seawater	1.03×10^3
Gold	19.3×10^3

Density (kg/m^3)

Archimedes supposedly was asked to determine whether a crown made for the king consisted of pure gold. According to legend, he solved this problem by weighing the crown first in air and then in water. Suppose the scale read 7.84 N when the crown was in air and 6.84 N when it was in water. What is the density of the crown?

$(\rho_{gold} = 19{,}300\ kg/m^3, \rho_{water} = 1{,}000\ kg/m^3)$

(A). 1,930 kg/m^3

(B). 7,840 kg/m^3 ✓

(C). 2,450 kg/m^3

(D). 5,300 kg/m^3

(E). 12,000 kg/m^3

In air: $\sum \vec{F}_y = T - mg = 0$

$mg = 7.84\ N$

$m = 0.8\ kg$

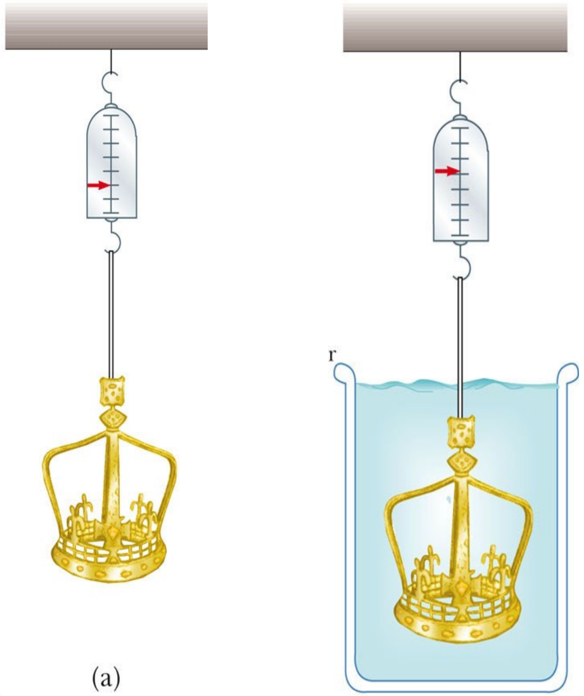
In water: $\sum \vec{F}_y = T + B - mg = 0$

$6.84\ N + B - 7.84\ N = 0$

$B = 1\ N = \rho_{water} g V$

$V = 1.02\ \times 10^{-4}\ m^3$

$\rho = \dfrac{m}{V} = 7{,}843\ kg/m^3$



(a) © Cengage Learning

(b) © Cengage Learning

Example: Buoyancy

The gravitational force exerted on a solid object is 9 N. When the object is suspended from a spring scale and submerged in water, the scale reads 6 N. Find the density of the object.

(A). 350 kg/m^3

(B). 1,500 kg/m^3

(C). 3,000 kg/m^3 ✓

(D). 6,750 kg/m^3

In air: $mg = 9\ N$ $\qquad m = 0.92\ kg$

In water: $\sum \vec{F}_y = T + B - mg = 0$ $\qquad 6.0\ N + B - 9\ N = 0$

$B = 3\ N = \rho_{water} g V$ $\qquad V = 3.06\ \times 10^{-4}\ m^3$

$$\rho = \frac{m}{V} = 3{,}000\ kg/m^3$$

Continuity Equation

conservation of mass and a steady flow

$$\rho_1 dV_1 = \rho_2 dV_2$$

- The figure at the right shows a flow tube with changing cross-sectional area from A_1 to A_2.
- The mass flowing into the tube across A_1 in time dt equal to the mass flowing out across A_2 in time dt

$$\rho_1 A_1 v_1 dt = \rho_2 A_2 v_2 dt$$

- Continuity equation for a compressible fluid:

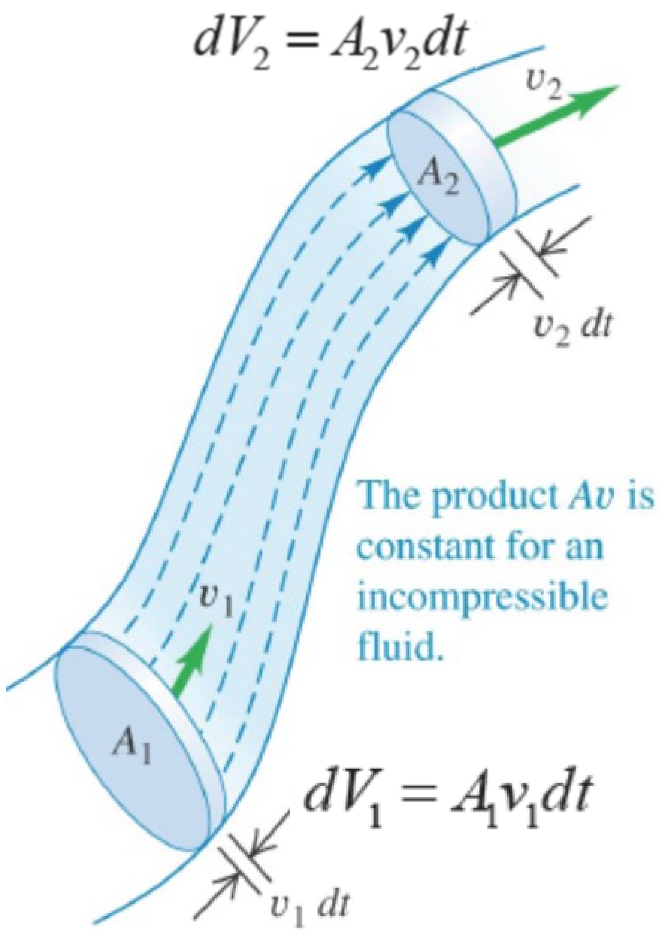
$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

- Continuity equation for an incompressible fluid:

$$\rho_1 = \rho_2 \qquad \boldsymbol{A_1 v_1 = A_2 v_2}$$

- Incompressible fluid: volume flow rate (dV/dt) is constant.

$$\frac{dV_1}{dt} = A_1 v_1 = A_2 v_2 = \frac{dV_2}{dt}$$



Example: Continuity Equation

An incompressible fluid flows through a pipe of varying radius (shown in cross-section). Compared to the fluid at point *P*, the fluid at point *Q* has

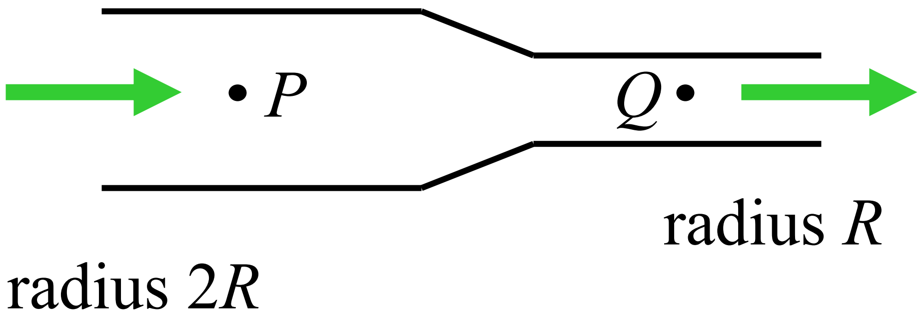
(A). 4 times the fluid speed. ✓

(B). 2 times the fluid speed.

(C). the same fluid speed.

(D). 1/2 the fluid speed.

(E). 1/4 the fluid speed.



$$A_1 v_1 = A_2 v_2$$

$$4\pi R^2 v_P = \pi R^2 v_Q$$

Example: Continuity Equation

An oil company pumps crude oil through a pipe of radius 0.01 m. The volume flow rate is 19.5 L/s (where 1 L=10^{-3} m^3). What is the speed of the flow in m/s?

(A). 0.1 m/s

(B). 0.5 m/s

(C). 12 m/s

✔(D). 62 m/s

(E). 89 m/s

$$\frac{dV}{dt} = Av \qquad Av = 19.5 \text{ L/s}$$

$$\pi(0.01\,m)^2 v = 19.5 \times 10^{-3}\,\text{m}^3/\text{s}$$

Bernoulli's Equation

- Bernoulli's equation relates the pressure, flow speed, and height for flow of an ideal, incompressible fluid

- The net work dW done on this fluid element by the surrounding fluid during this displacement is

$$dW = p_1 A_1 ds_1 - p_2 A_2 ds_2 = (p_1 - p_2) dV$$

- The net change in Kinetic energy dK during time dt is

$$dK = \frac{1}{2} dm(v_2^2 - v_1^2) = \frac{1}{2} \rho dV (v_2^2 - v_1^2)$$

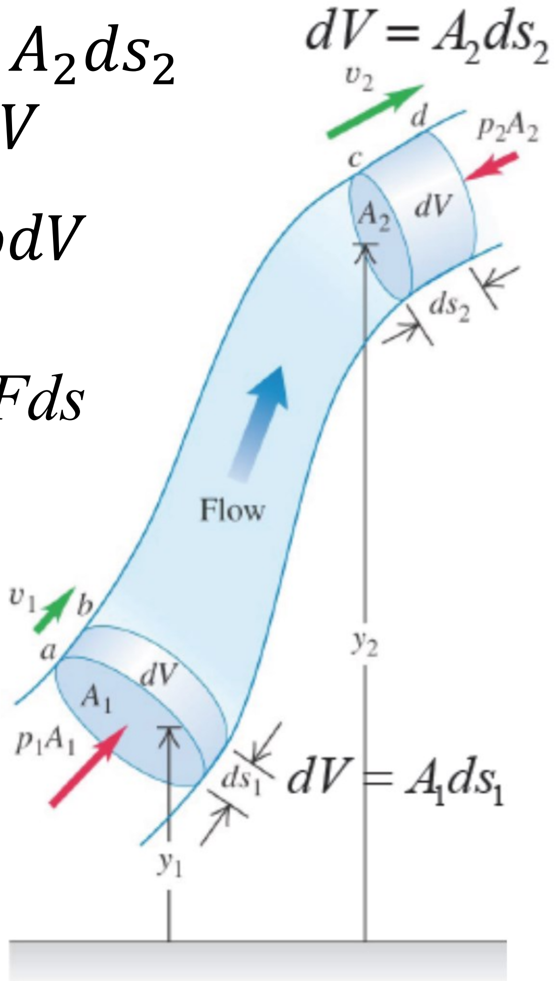
- The net change in potential energy dU during time dt is

$$dU = dm \cdot g(y_2 - y_1) = \rho dV \cdot g(y_2 - y_1)$$

$$A_1 ds_1 = A_2 ds_2 = dV$$

$$dm = \rho dV$$

$$dW = F ds$$



Bernoulli's Equation

- The work done on a unit volume of fluid by the surrounding fluid is equal to the sum of the changes in kinetic and potential energies per unit volume that occurs during the flow.

$$dW = (p_1 - p_2)dV$$

$$dK = \frac{1}{2}\rho dV(v_2^2 - v_1^2)$$

$$dU = \rho dV g(y_2 - y_1)$$

- The work-energy theorem gives $dW = dK + dU$

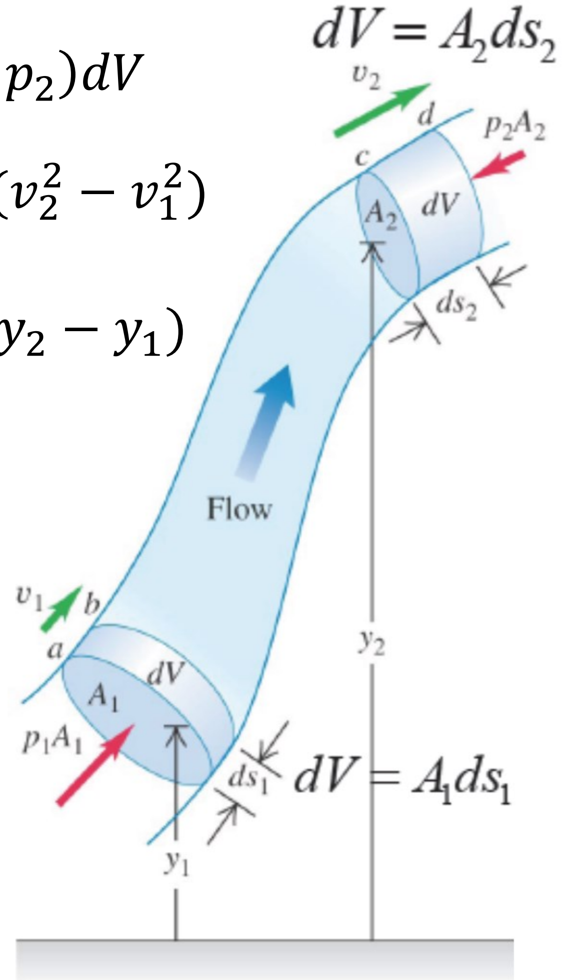
$$(p_1 - p_2)dV = \frac{1}{2}\rho dV(v_2^2 - v_1^2) + \rho g dV(y_2 - y_1)$$

$$p_1 - p_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(y_2 - y_1)$$

- Bernoulli's equation:

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$p + \frac{1}{2}\rho v^2 + \rho g y = constant$$



Example

An incompressible fluid flows through a pipe of varying radius (shown in cross-section). Compared to the fluid at point P, the fluid at point Q has

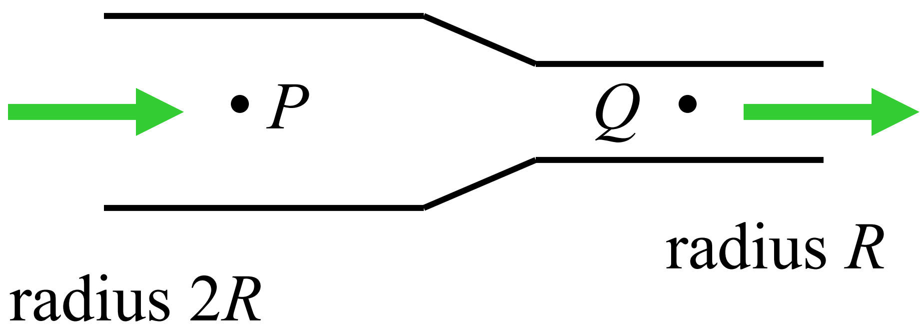
(A). greater pressure and greater volume flow rate.

(B). greater pressure and the same volume flow rate.

(C). the same pressure and greater volume flow rate.

(D). lower pressure and the same volume flow rate. ✓

(E). none of the above



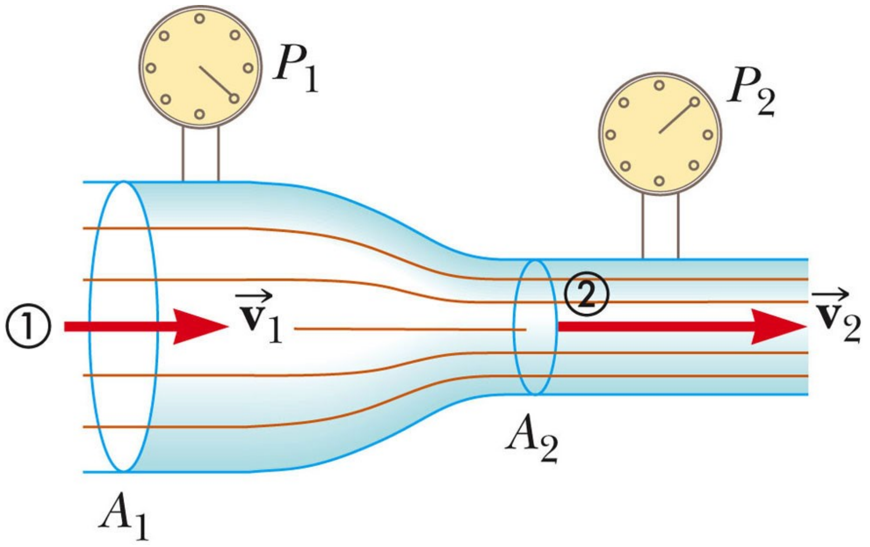
$$A_1 v_1 = A_2 v_2 \qquad \frac{dV}{dt} = Av$$

$$p + \frac{1}{2}\rho v^2 + \rho g y = constant$$

Applications of Bernoulli's Principle: Measuring Speed

The horizontal constricted pipe in the Figure known as *veturi tube*, can be used to measure the flow speed of an incompressible fluid:

p_1, p_2, A_1, A_2 are known, find v_2



$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2 \qquad A_1 v_1 = A_2 v_2$$

$$v_2^2 = \frac{2(p_1 - p_2)}{\rho(1 - \frac{A_2^2}{A_1^2})} \qquad v_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho(1 - \frac{A_2^2}{A_1^2})}}$$

Speed changes as diameter changes

Swiftly moving fluids exert less pressure than do slowly moving fluids

Example: Bernoulli's Equation

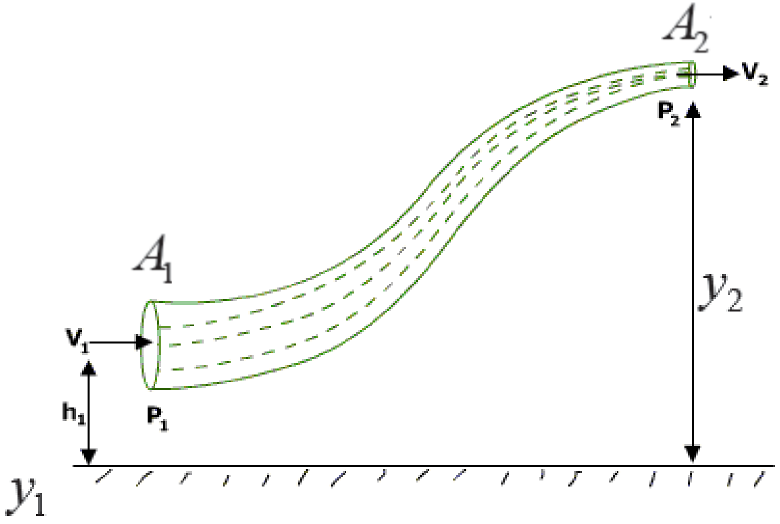
Water enters a house through a pipe with an inside diameter of 2.0 cm at a pressure of 4 × 10^5 Pa. A 1.0-cm-diameter pipe leads to the 2^{nd} floor bathroom 5 m above. When the flow speed at the inlet pipe is 1.5 m/s, Find the pressure in the bathroom.

$$A_1 v_1 = A_2 v_2$$

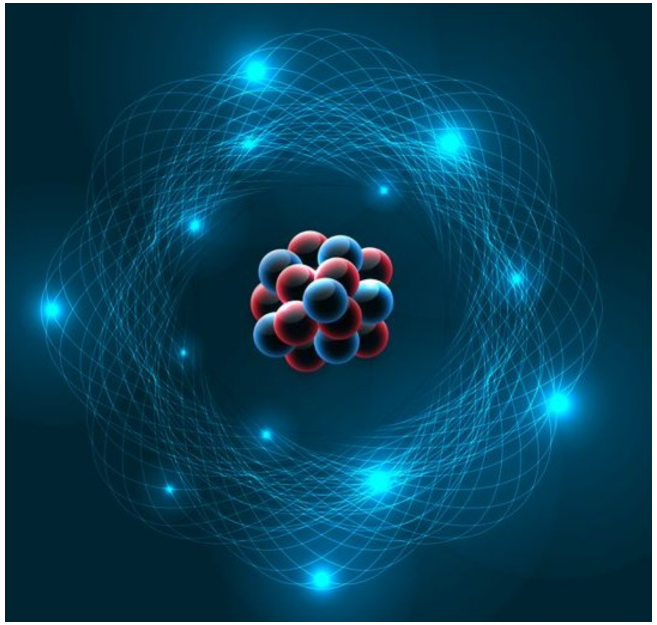
$$p_1 - p_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(y_2 - y_1)$$

$$v_2 = \frac{A_1}{A_2} v_1 = \frac{\pi(1.0 \text{ cm})^2}{\pi(0.50 \text{ cm})^2}(1.5 \text{ m/s}) = 6.0 \text{ m}/s$$

$$\begin{aligned} p_2 &= p_1 - \tfrac{1}{2}\rho(v_2^2 - v_1^2) - \rho g(y_2 - y_1) \\ &= 4.0 \times 10^5 \text{ Pa} - \tfrac{1}{2}\left(1.0 \times 10^3 \text{ kg/m}^3\right)\left(36 \text{ m}^2/\text{s}^2 - 2.25 \text{ m}^2/\text{s}^2\right) \\ &\quad - \left(1.0 \times 10^3 \text{ kg/m}^3\right)\left(9.8 \text{ m/s}^2\right)(5.0 \text{ m}) \\ &= 4.0 \times 10^5 \text{ Pa} - 0.17 \times 10^5 \text{ Pa} - 0.49 \times 10^5 \text{ Pa} \\ &= 3.3 \times 10^5 \text{ Pa} = 3.3 \text{ atm} = 48 \text{ Ib/in.}^2 \end{aligned}$$



Physics 111: Mechanics
Chapter 13

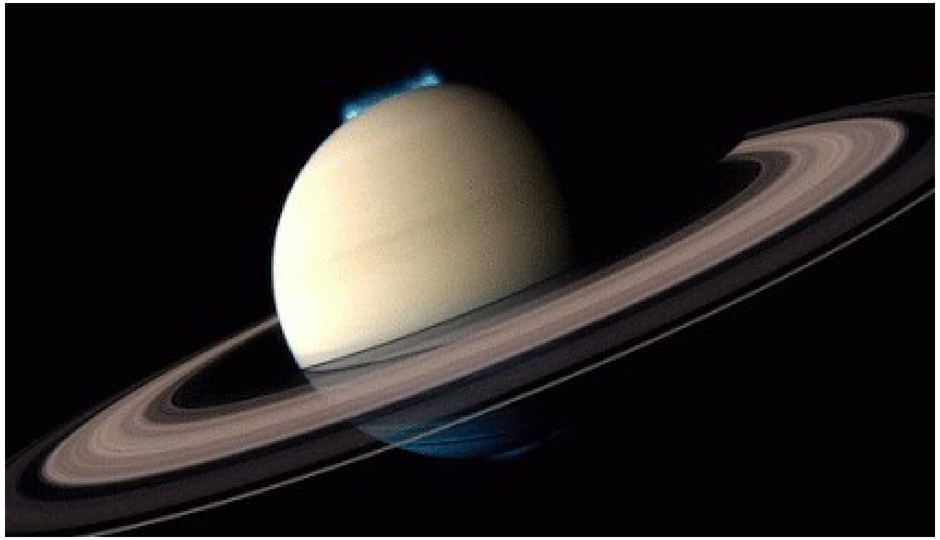


Junjie Yang
Department of Physics, NJIT

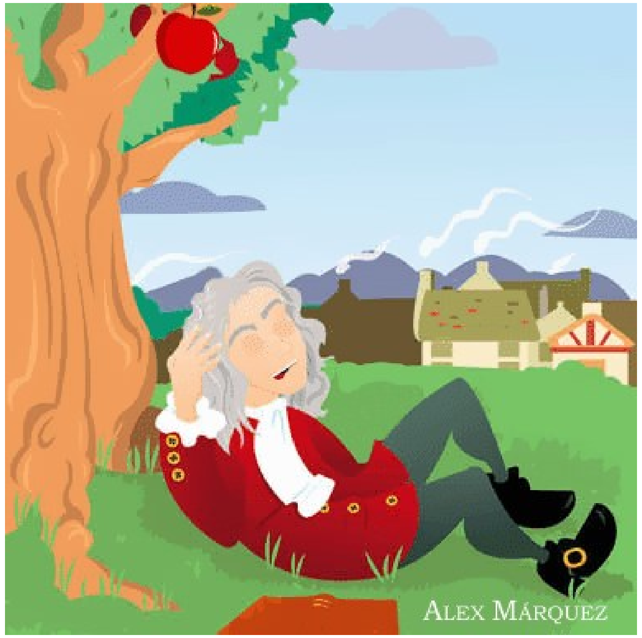
Gravitation

- Newton's Law of Gravitation
- Weight
- Gravitational Potential Energy
- The motion of satellites
- Kepler's law and motion of planets

Newton's Law of Universal Gravitation

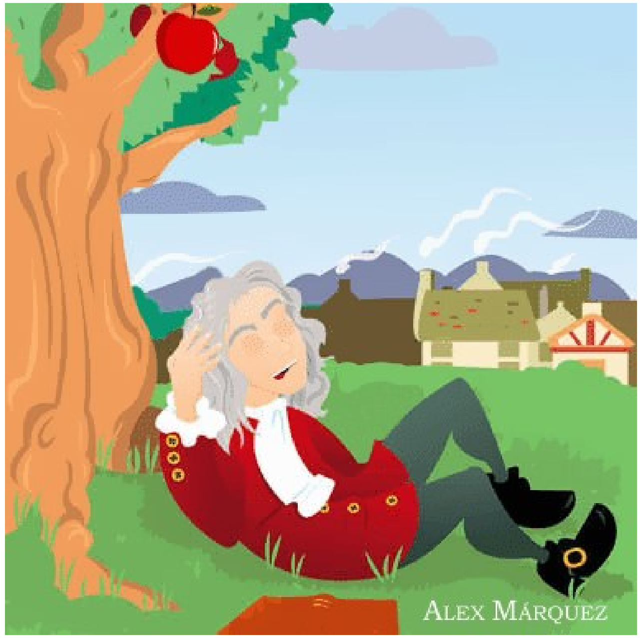


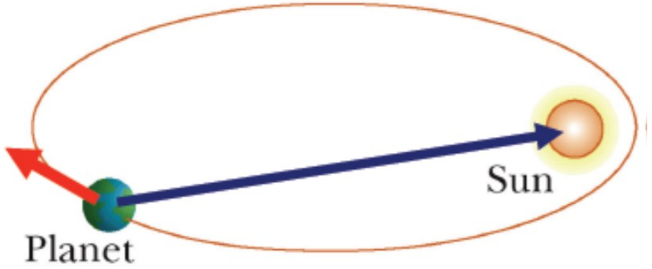
- What can we say about the motion of the particles that make up Saturn's rings?
- Why does the apple fall to earth?
- Why doesn't the moon fall to earth, or the earth into the sun?
- By studying gravitation and celestial mechanics, we will be able to answer these and other questions.



Newton's Law of Universal Gravitation

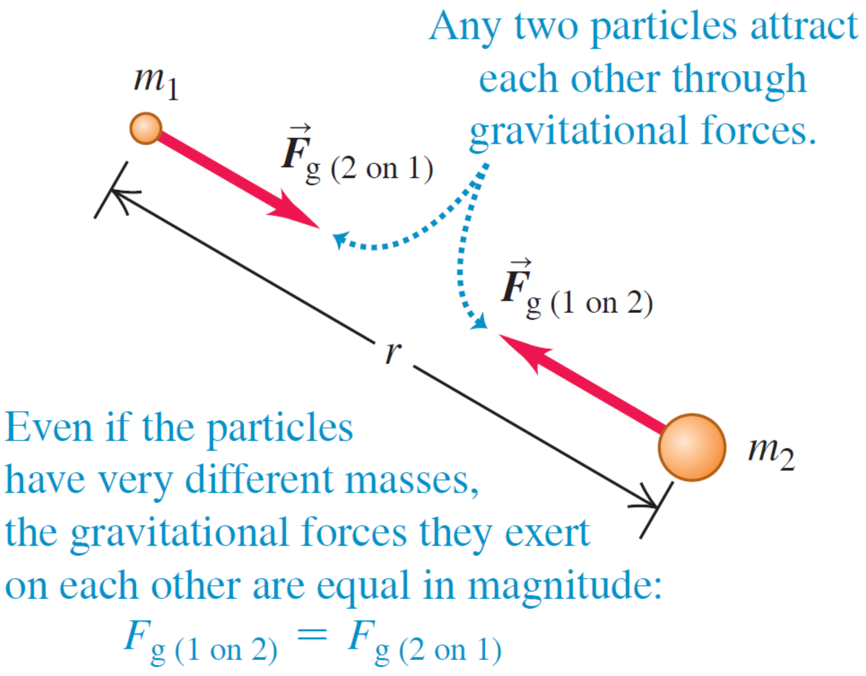
- The apple was attracted to the Earth
- All objects in the Universe were attracted to each other in the same way the apple was attracted to the Earth
- The force law governing the motion of planets was the same as the force law that attracted a falling apple the Earth





Newton's Law of Universal Gravitation

- Every particle in the Universe attracts every other particle with a force that is directly proportional to the product of the masses and inversely proportional to the square of the distance between them.



Newton's **law of gravitation:** Magnitude of attractive gravitational force between any two particles

$$F_g = \frac{Gm_1m_2}{r^2}$$

Gravitational constant (same for any two particles)

Masses of particles

Distance between particles

This is an example of an inverse square law

G is the constant of universal gravitation.

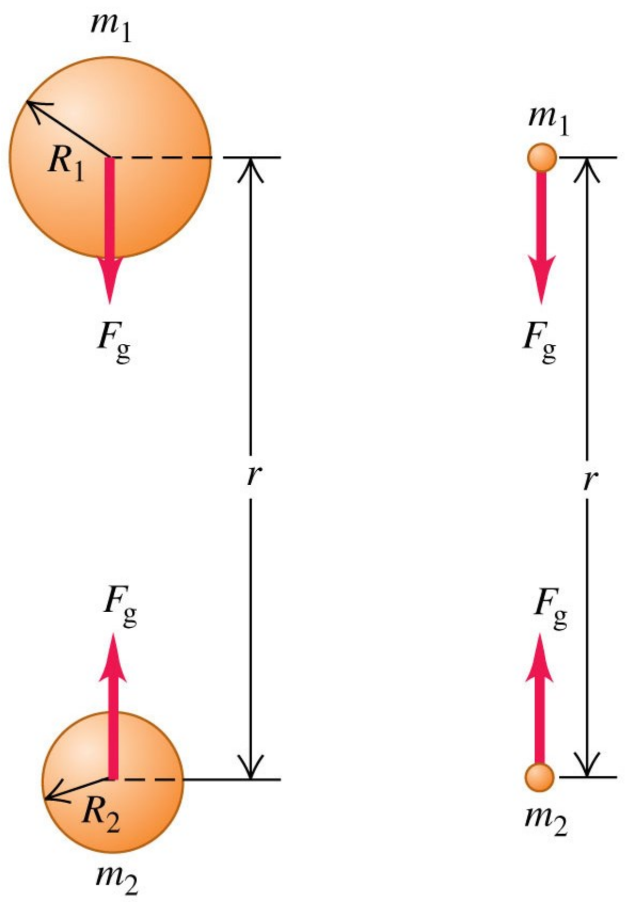
$G = 6.673 \times 10^{-11}\ \mathrm{N\ m^2/kg^2}$

Determined experimentally (Henry Cavendish in 1798)

Universal Gravitation

$$F = G\frac{m_1 m_2}{r^2}$$

- The force that mass 1 exerts on mass 2 is equal and opposite to the force mass 2 exerts on mass 1
- The forces form a Newton's third law action-reaction pair.
- The gravitational force exerted by a uniform sphere on a particle outside the sphere is the same as the force exerted if the entire mass of the sphere were concentrated on its center.



Example

The planet Saturn has 100 times the mass of the Earth and is 10 times more distant from the Sun than the Earth is.

Compared to the Earth's acceleration as it orbits the Sun, the acceleration of Saturn as it orbits the Sun is

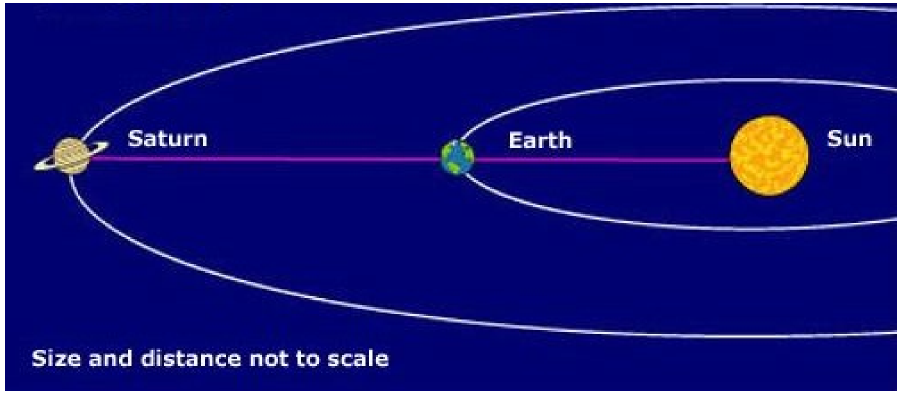
A. 100 times greater.

B. 10 times greater.

C. the same.

D. 1/10 as great.

E. 1/100 as great. ✔

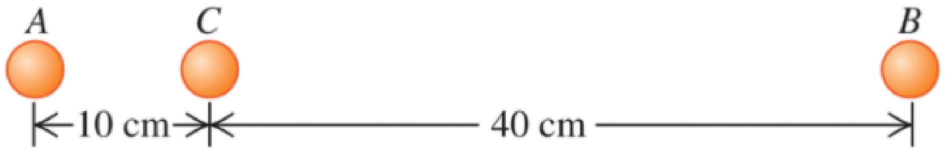


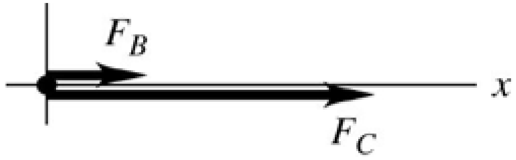
$$F = G\frac{m_1 m_2}{r^2} \qquad F = m a_{rad}$$

$$F = G\frac{\cancel{M_{Saturn}} M_{Sun}}{r^2} = \cancel{M_{Saturn}} a_{rad}$$

Examples: Universal Gravitation

- Find the magnitude and direction of the net gravitational force on the mass A due to masses B and C. Each mass is 2.00 kg.





$$F_B = G\frac{m_A m_B}{r_{AB}^2} = (6.673\times10^{-11}\ \mathrm{N\cdot m^2/kg^2})\frac{(2.00\ \mathrm{kg})^2}{(0.50\ \mathrm{m})^2} = 1.069\times10^{-9}\ \mathrm{N}$$

$$F_C = G\frac{m_A m_C}{r_{AC}^2} = (6.673\times10^{-11}\ \mathrm{N\cdot m^2/kg^2})\frac{(2.00\ \mathrm{kg})^2}{(0.10\ \mathrm{m})^2} = 2.669\times10^{-8}\ \mathrm{N.}$$

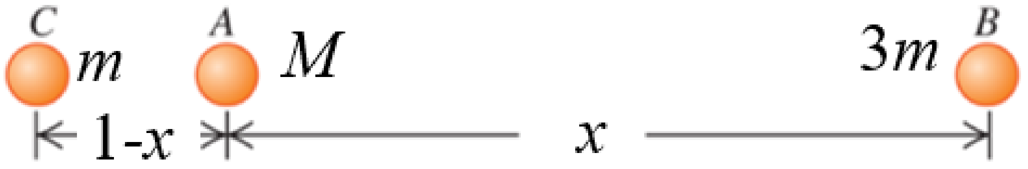
$$F_{\mathrm{net},\,x} = F_{Bx} + F_{Cx} = 1.069\times10^{-9}\ \mathrm{N} + 2.669\times10^{-8}\ \mathrm{N} = 2.8\times10^{-8}\ \mathrm{N}$$

The acceleration of A?

$$a = \frac{F}{m} = 1.4\times10^{-8}\ \mathrm{m/s^2}$$

Examples: Universal Gravitation

- A particle of mass $3m$ is located 1.00 m from a particle of mass m. Where should you put a third mass M so that the net gravitational force on M due to the two masses is exactly zero?



$$F_x = F_{1x} + F_{2x} = -G\frac{3mM}{x^2} + G\frac{mM}{(1.00\text{ m} - x)^2} = 0.$$

Cancelling and simplifying gives $3(1.00\text{ m} - x)^2 = x^2$.

$$x = \frac{1.00\text{ m}}{1 + 1/\sqrt{3}} = 0.634\text{ m}.$$

M must be placed at a point that is 0.634 m from the particle of mass $3m$