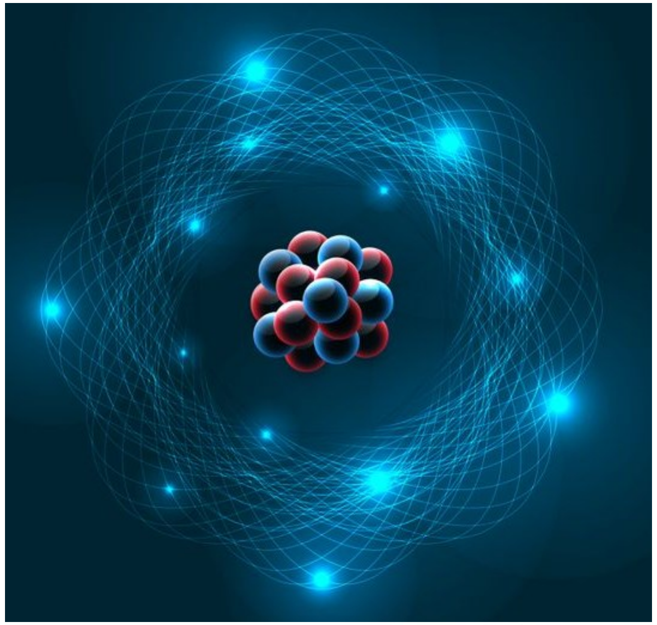
Physics 111: Mechanics
Chapter 13

Junjie Yang
Department of Physics, NJIT



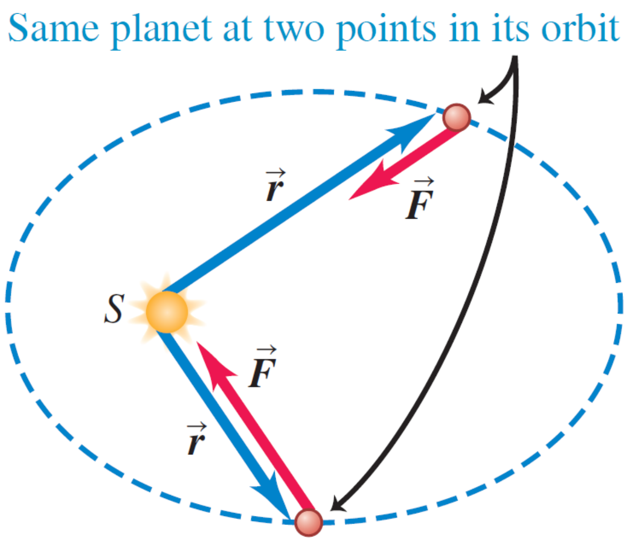
Kepler's Second Laws and angular momentum

- **Second Law:** A line from the sun to a given planet sweeps out equal areas in equal times.

$$\frac{dA}{dt} = \frac{1}{2m}\,|\vec{r} \times m\vec{v}| = \frac{L}{2m}$$

$$\frac{d\vec{L}}{dt} = \vec{\tau} = \vec{r} \times \vec{F} = rF\sin\theta = 0$$

- The gravitational force that the sun exerts on a planet produces **zero torque** around the sun, the planet's angular momentum around the sun remains constant.



- Gravitational force $\vec{F}$ on planet has different magnitudes at different points but is always opposite to vector $\vec{r}$ from sun S to planet.
- Hence $\vec{F}$ produces zero torque around sun.
- Hence angular momentum $\vec{L}$ of planet around sun is constant in both magnitude and direction.

Example:

A planet (P) is moving around the sun (S) in an elliptical orbit. As the planet moves from aphelion to perihelion, the planet's angular momentum

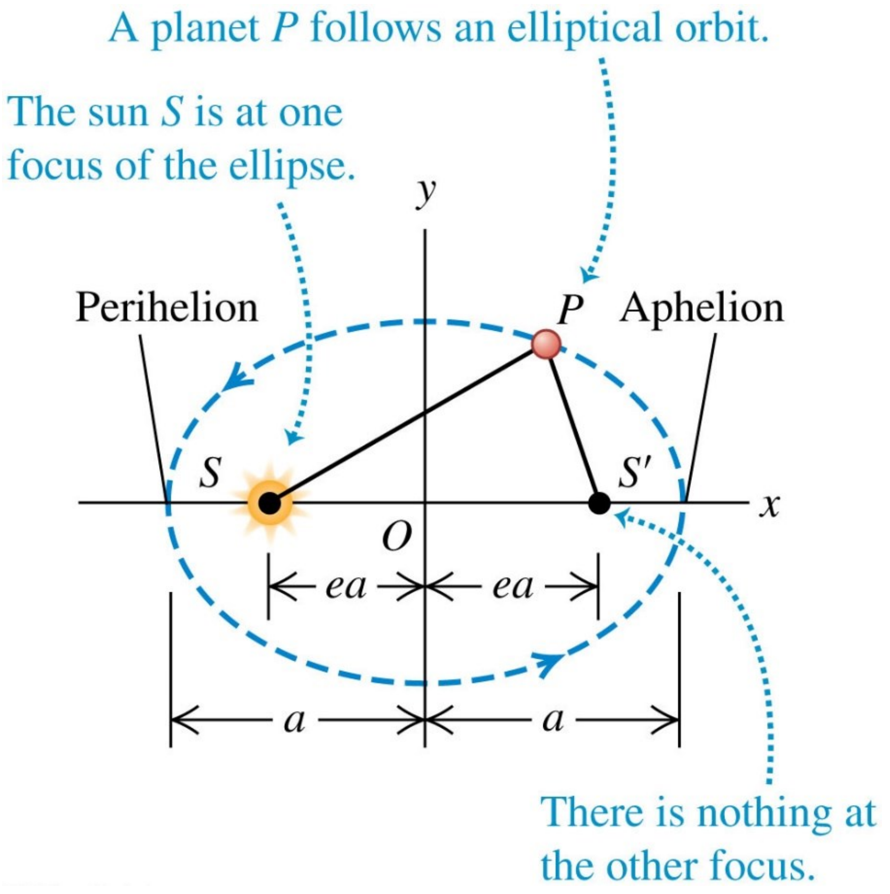
A. increases at all times.

B. decreases at all times.

C. decreases during part of the motion and increases during the other part.

D. increases, decreases, or remains the same during various parts of the motion.

E. remains the same at all points between aphelion and perihelion.



Kepler's Laws: Third Law

- The periods of the planets are proportional to the 3/2 powers of the semimajor axis lengths of their orbits.

$$T = \frac{2\pi a^{3/2}}{\sqrt{Gm_S}}$$

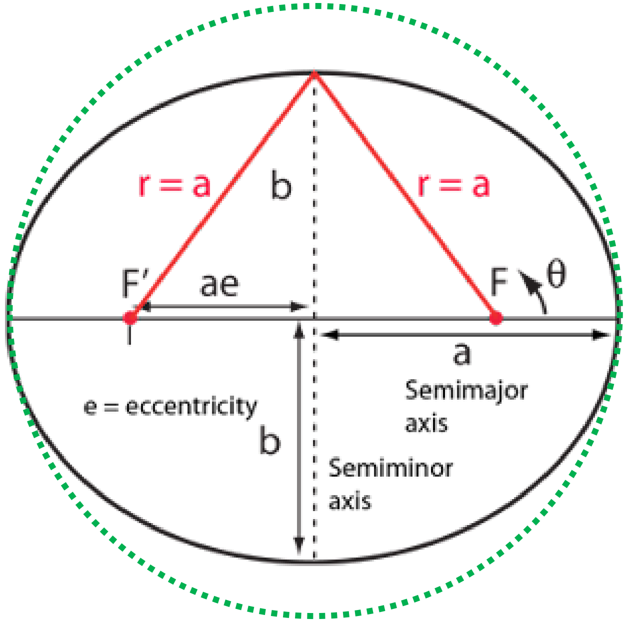
elliptical orbit around the sun

Sun's mass: m_s

- The period does not depend on the eccentricity *e*.
- An asteroid in an elongated elliptical orbit with semi-major axis *a* will have the same orbital period as a planet in a circular orbit of radius *a*.

$$v = \sqrt{Gm_s/a}$$

$$T = \frac{2\pi a}{v} = 2\pi a\sqrt{\frac{a}{Gm_s}} = \frac{2\pi a^{3/2}}{\sqrt{Gm_s}}$$



Eccentricity:

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

Examples: Kepler's Laws

- The asteroid Pallas has an orbital period of 4.62 years and an orbital eccentricity of 0.233, around sun. Find the semi-major axis of its orbit.

$$T = \frac{2\pi a^{3/2}}{\sqrt{Gm_S}} \qquad T = (4.62\ \mathrm{y})(3.156\times10^{7}\ \mathrm{s/y}) = 1.46\times10^{8}\ \mathrm{s}$$

$$m_S = 1.99\times10^{30}\ \mathrm{kg} \qquad a = \left(\frac{Gm_S T^2}{4\pi^2}\right)^{1/3} = 4.15\times10^{11}\ \mathrm{m}$$

Examples: Kepler's Laws

In 2004 astronomers reported the discovery of a large Jupiter-sized planet orbiting very close to the star HD-179949. The orbit radius is 6.43×10^9 m, and it takes the planet only 3.09 days to make one orbit (assumed to be circular). (a) what is the mass of the star HD-179949? (b) How fast (in km/s) is this planet moving?

$$T = \frac{2\pi r^{3/2}}{\sqrt{Gm_{\text{star}}}}, \qquad 3.09\text{ days} = 2.67\times10^5\text{ s.} \qquad T = 2.67\times10^5\text{ s.}$$

$$m_{\text{star}} = \frac{4\pi^2 r^3}{T^2 G} = \frac{4\pi^2(6.43\times10^9\text{ m})^3}{(2.67\times10^5\text{ s})^2(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)} = 2.21\times10^{30}\text{ kg.}$$

$$v = \frac{2\pi r}{T} = \frac{2\pi(6.43\times10^9\text{ m})}{2.67\times10^5\text{ s}} = 1.51\times10^5\text{ m/s} = 151\text{ km/s}$$

Example:

A planet (P) is moving around the sun (S) in an elliptical orbit. As the planet moves from aphelion to perihelion, the sun's gravitational force does

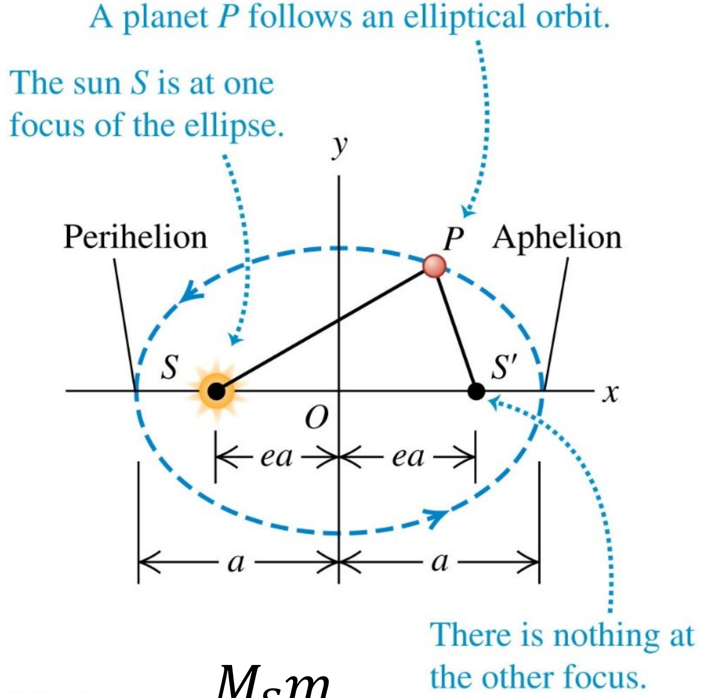
(A). positive work on the planet. ✓

(B). negative work on the planet.

(C). positive work on the planet during part of the motion and negative work on the planet during the other part.

(D). positive, negative, or zero work on the planet during various parts of the motion.

(E). zero work on the satellite at all points between aphelion and perihelion.

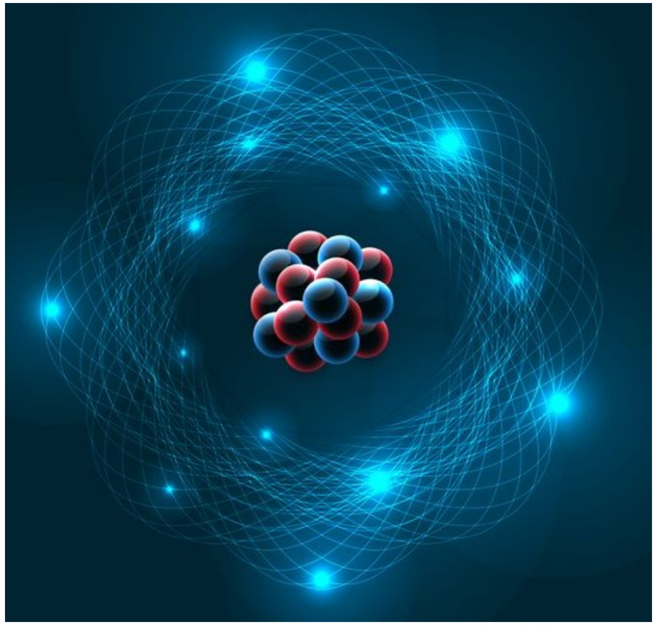


$$U = -G\frac{M_S m}{r}$$

$$E = K + U = \frac{1}{2}mv^2 - G\frac{M_S m}{r}$$

Physics 111: Mechanics
Final Review 1

Junjie Yang
Department of Physics, NJIT



1. A student throws a set of keys vertically upward to her sorority sister, who is in a window **4.0 m above**. The second student catches the keys **1.50 s** later. With what initial speed were the keys thrown?

A) 32 m/s
B) 41 m/s
C) 10 m/s
D) 22 m/s
E) 82 m/s

$$y = v_0 t - \tfrac{1}{2} g t^2$$

$$4.0 = v_0(1.50) - \frac{1}{2}(9.8)(1.50)^2$$

$$v_0 \approx 10\,\mathrm{m/s}$$

20. A proton moving along the x axis has an **initial speed** in the positive x direction of **4.0×10^6 m/s** and a constant **acceleration** in the positive x direction of **6.0×10^{12} m/s^2**. What is the velocity of the proton after it has traveled a **distance of 80 cm**?

A) **5.1×10^6** m/s
B) 6.3×10^6 m/s
C) 4.8×10^6 m/s
D) 3.9×10^6 m/s
E) 2.9×10^6 m/s

$$v_0 = 4.0\times10^6\ \mathrm{m/s}$$
$$a = 6.0\times10^{12}\ \mathrm{m/s^2}$$
$$x = 80\ \mathrm{cm} = 0.80\ \mathrm{m}$$

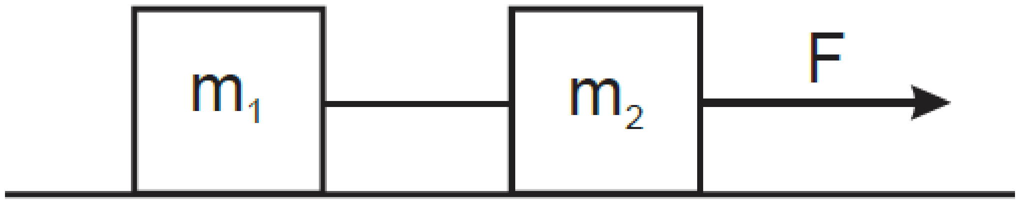
$$v^2 = v_0^2 + 2ax$$

$$v^2 = (4.0\times10^6)^2 + 2(6.0\times10^{12})(0.80)$$

$$v \approx 5.1\times10^6\ \mathrm{m/s}$$

a_x = constant
$v_x = v_{0x} + a_x t$
$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$
$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$
$x - x_0 = \frac{1}{2}(v_{0x} + v_x)t$

2. Two blocks (**m_1 = 2 kg, m_2 = 4 kg**) are tied together by a string and pulled across a horizontal frictionless surface as shown in the figure. The external force which pulls has a magnitude of **F = 10 N**. Calculate the magnitude of the tension in the string between the two blocks.



A) 10 N
B) 1.7 N
C) 6.7 N
D) 3.3 N
E) none of the above

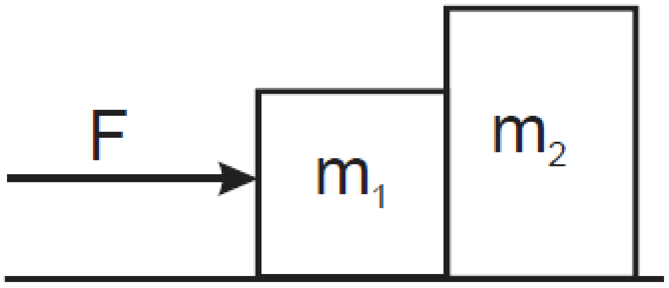
$$a = \frac{F}{m_1 + m_2}$$

$$T = m_1 a$$

$$a = \frac{10}{2+4} = 1.67\,\mathrm{m/s^2}$$

$$T = (2)(1.67) = 3.33\ \mathrm{N}$$

12. Two blocks (**m_1 = 2 kg, m_2 = 3 kg**) are pushed along a frictionless horizontal surface by an external force of magnitude **F = 10 N.** Calculate the magnitude of the force of m_1 on m_2.



A) 10 N
B) 5 N
C) 2 N
D) 4 N
E) none of the above (6 N)

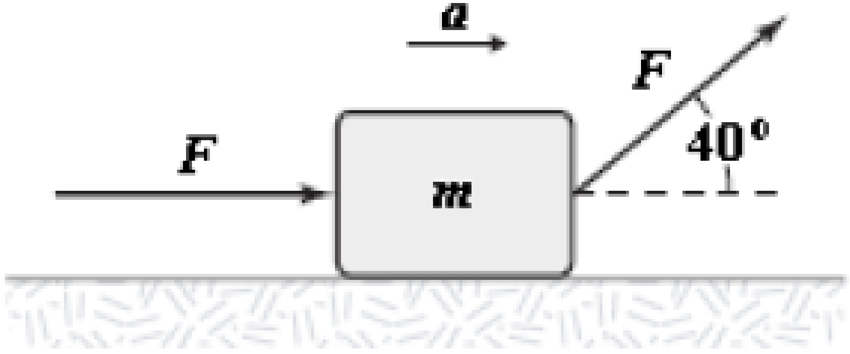
$$a = \frac{F}{m_1 + m_2}$$

$$F_{1\to 2} = m_2 a$$

$$a = \frac{10}{2+3} = 2.00\,\mathrm{m/s^2}$$

$$F_{1\to 2} = (3)(2.0) = 6.0\,\mathrm{N}$$

6. If ***F*** **= 4.0 N** and ***m*** **= 2.0 kg**, what is the magnitude ***a*** **of the acceleration** for the block shown below? The surface is frictionless.



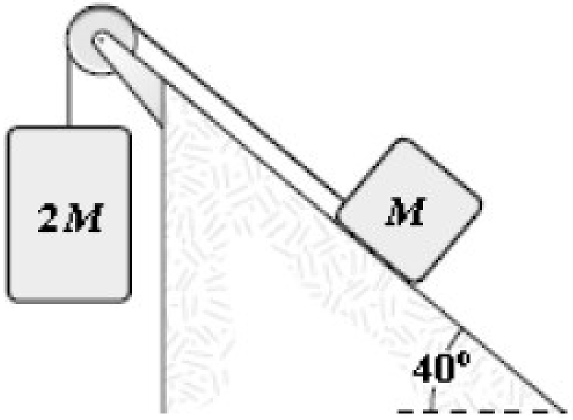
A). 5.3 m/s2
B). 4.4 m/s2
C). 3.5 m/s2
D). 6.2 m/s2
E). 8.4 m/s2

$$F_{\text{net}} = F + F\cos 40^\circ$$

$$a = \frac{F_{\text{net}}}{m} = \frac{F(1+\cos 40^\circ)}{m}$$

$$a = \frac{4(1+0.766)}{2} = \frac{4(1.766)}{2} = \frac{7.064}{2} \approx 3.53\ \text{m/s}^2$$

7. In the figure shown, the coefficient of **kinetic friction** between the block and the incline is **0.40**. What is the magnitude of the **acceleration** of the suspended block as it falls? Disregard any pulley mass or friction in the pulley.



A). 3.4 m/s²
B). 3.7 m/s²
C). 4.2 m/s²
D). 3.9 m/s²
E). 5.4 m/s²

$$2Mg - T = 2Ma$$

$$T - Mg\sin 40^\circ - \mu_k Mg\cos 40^\circ = Ma$$

$$2Mg - 2Ma - Mg\sin 40^\circ - \mu_k Mg\cos 40^\circ = Ma$$

$$a = \frac{g(2 - \sin 40^\circ - 0.40\cos 40^\circ)}{3}$$

$$a \approx 3.4\ \mathrm{m/s^2}$$

10. How much work is done by a person lifting a **2.0-kg object** from the bottom of a well at a **constant speed of 2.0 m/s for 5.0 s**?

A). 0.22 kJ
B). 0.20 kJ
C). 0.24 kJ
D). 0.27 kJ
E). 0.31 kJ

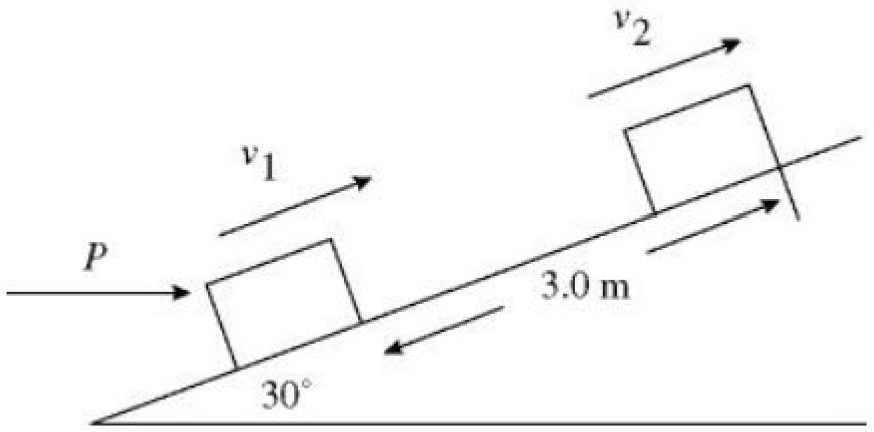
$$W = Fd = mgd$$

$$d = vt = (2.0)(5.0) = 10 \text{ m}$$

$$W = (2.0)(9.8)(10) = 196 \text{ J}$$

23. In the figure, a **700-kg crate** is on a rough surface **inclined at 30°.** A constant external force **P = 5600 N** is applied horizontally to the crate. As the force pushes the crate a distance of **3.00 m up** the incline, the speed changes from **1.40 m/s to 2.50 m/s**. How much **work does gravity** do on the crate during this process?

A) -10,300 J
B) -3,400 J
C) 10,300 J
D) 3,400 J
E) Zero



$$\Delta h = d\sin 30^\circ = 3.00(0.5) = 1.50 \text{ m}$$

$$W_g = -mg\Delta h$$

$$W_g = -(700)(9.8)(1.50)$$

$$W_g = -10290 \text{ J}$$

11. A **1.5-kg** block sliding on a rough horizontal surface is attached to one end of a horizontal spring (**k = 200 N/m**) which has its other end fixed. If this system is **displaced 20 cm** horizontally from the equilibrium position and released from rest, the block first reaches the **equilibrium position** with **a speed of 2.0 m/s**. What is the **coefficient of kinetic** friction between the block and the horizontal surface on which it slides?

A). 0.34
B). 0.24
C). 0.13
D). 0.44
E). 0.17

$$K_1 + U_1 + W_{other} = K_2 + U_2 \qquad K_1 = 0,\ U_2 = 0$$

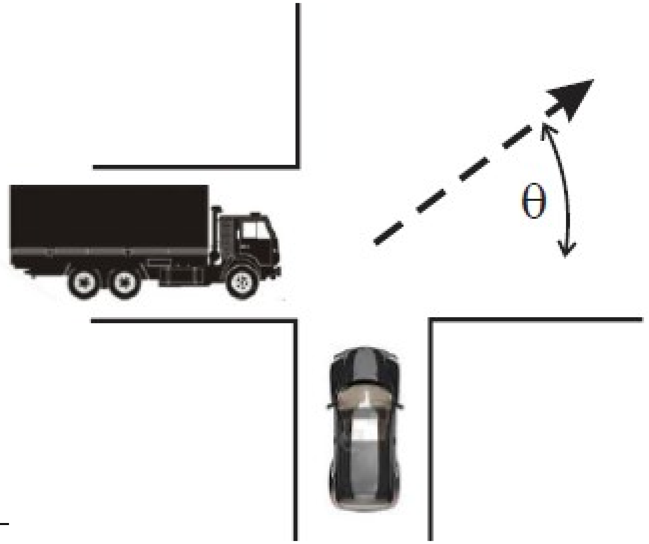
$$U_1 + W_{other} = K_2$$

$$U_1 = \frac{1}{2}kx^2 \qquad W_{\text{other}} = -\mu_k mgx \qquad K_2 = \frac{1}{2}mv^2$$

$$\frac{1}{2}kx^2 - \mu_k mgx = \frac{1}{2}mv^2 \qquad \frac{1}{2}(200)(0.20)^2 - \mu_k(1.5)(9.8)(0.20) = \frac{1}{2}(1.5)(2.0)^2$$

$$4.0 - 2.94\mu_k = 3.0$$

13. A car (**mass 1000 kg**) heading north runs a stop sign and collides with a truck (**mass 3000 kg**) which is heading **East at a speed of 30 m/s**. After the collision, the car and truck are **stuck together** and move at **an angle of 35 degrees** relative to the East direction (see figure). What is the speed of the car BEFORE the collision.



A) none of the other answers
B) 14 m/s
C) 21 m/s
D) 90 m/s
E) 63 m/s

$$p_x = m_t v_t = (3000)(30)$$

$$p_y = m_c v_c = (1000) v_c$$

$$\tan 35^\circ = \frac{p_y}{p_x} \qquad \tan 35^\circ = \frac{1000 v_c}{3000(30)}$$

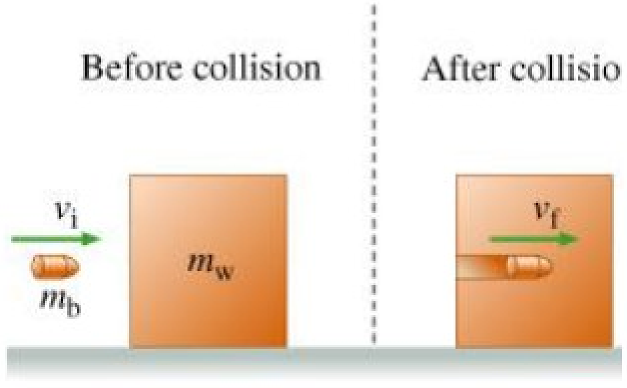
$$v_c = \frac{3000(30)\tan 35^\circ}{1000} \qquad v_c \approx 63 \text{ m/s}$$

In order to find the speed of a fast bullet, it is fired into a **4.0 kg wooden** block on a horizontal surface. The bullet gets stuck in the block which starts moving with the speed of $\boldsymbol{V_f}$ **= 2.5 m/s**. Find the original speed of the bullet V_i if its mass is **7.54 gram**.

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_f$$

$$(0.00754) v_1 + 0 = (0.00754 + 4.0)(2.5)$$

$$v_1 = 1330 \text{ m/s}$$



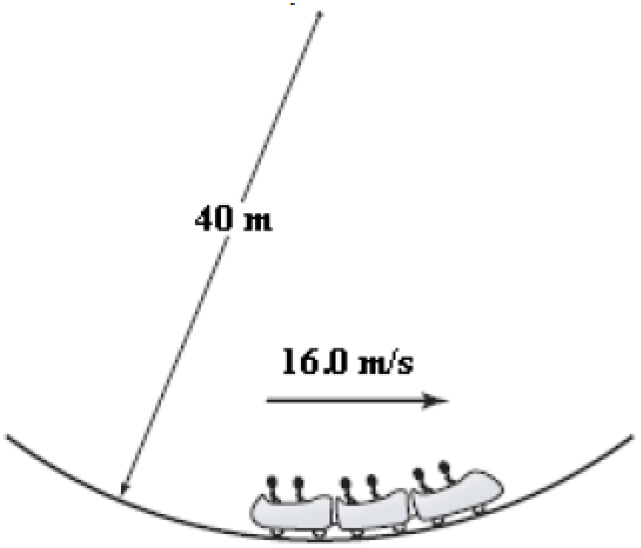
8. A roller-coaster car has a mass of **500 kg** when fully loaded with passengers. At the bottom of a circular dip of **radius 40 m** (as shown in the figure) the car has a **speed of 16 m/s**. What is the **magnitude of the force** of the track on the car at the bottom of the dip?

A). 3.2 kN
B). 8.1 kN
C). 4.9 kN
D). 1.7 kN
E). 5.3 kN

$$n - Mg = Ma_{rad}$$

$$a_{rad} = (v^2/r)$$

$$n - Mg = M(v^2/r)$$



$$n - (500\text{ kg} \times 9.8) = (500\text{ kg})[(16\text{ m/s})^2/(40\text{ m})]$$

$$n = 8100\text{ N}$$

14. A multi-level pulley of rotational inertia $\boldsymbol{I}$ **= 0.1 kg.m²** is pivoted about a frictionless axis through O and perpendicular to the pulley, as shown in the figure (not to scale). If **a = 10 cm and b = 15 cm** (see the figure) what is the **angular acceleration** of the pulley?

A) 7.0 rad/s² cw
B) 7.0 rad/s² ccw
C) 2.0 rad/s² ccw
D) 2.0 rad/s² cw
E) 0 rad/s² , equilibrium

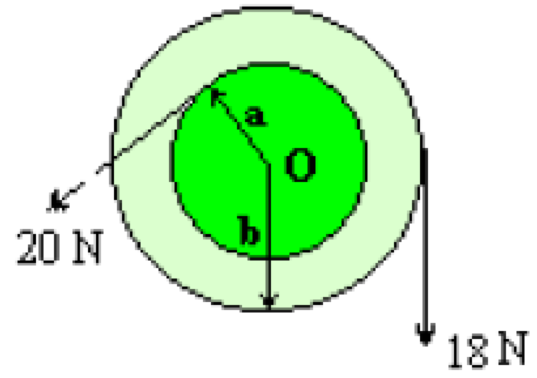
$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau_{20} = +(0.10)(20) = +2.0$$

$$\tau_{18} = -(0.15)(18) = -2.7$$

$$\tau_{\text{net}} = \tau_{20} + \tau_{18} = +2.0 - 2.7 = -0.7$$

$$\alpha = \frac{\tau_{\text{net}}}{I} = \frac{-0.7}{0.1} = -7.0\ \text{rad/s}^2$$



$\alpha_z =$ constant
$\omega_z = \omega_{0z} + \alpha_z t$
$\theta = \theta_0 + \omega_{0z} t + \frac{1}{2}\alpha_z t^2$
$\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$
$\theta - \theta_0 = \frac{1}{2}(\omega_{0z} + \omega_z)t$

The angular velocity after first 5 seconds, if it starts from rest is closest to:

$$\omega = 0 + \vec{\alpha}t = -0.7 \times 5 = -3.5$$

24. What is the magnitude of the free-fall acceleration at a point that is a **distance 2R above the surface** of the Earth, where **R is the radius** of the Earth?

A). 4.8 m/s^2
B). 1.1 m/s^2
C). 3.3 m/s^2
D). 2.5 m/s^2
E). 6.5 m/s^2

$$g = \frac{GM}{r^2}$$

$$g_0 = \frac{GM}{R^2}$$

$$r = R + 2R = 3R$$

$$g' = \frac{GM}{(3R)^2} = \frac{GM}{9R^2} = \frac{g_0}{9} \qquad g' = \frac{9.8}{9} \approx 1.1\ \mathrm{m/s^2}$$