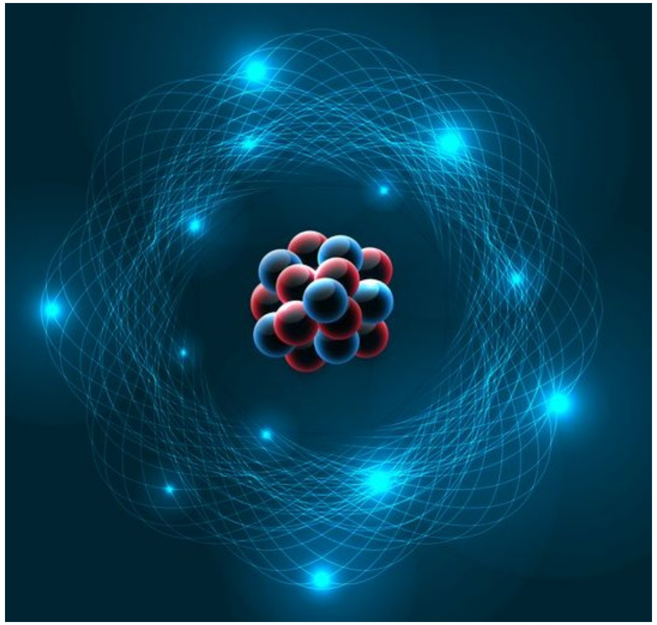
Physics 111: Mechanics
Final Review 2

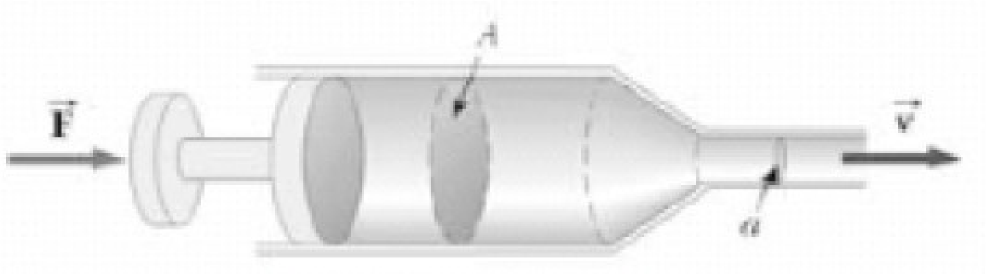


Junjie Yang

Department of Physics, NJIT

A hypodermic syringe contains water. The barrel of the syringe has a cross-sectional area ***A* = 2.5 × 10^{-5} m^2**, and the needle has a cross-sectional area ***a* =1.00 × 10^{-8} m^2**. In the absence of a force on the plunger, the pressure everywhere is **1.00 atm**. When a force of magnitude **2.00 N** acts on the plunger, what is the **speed of water** as it leaves the needle.

$$p + \frac{1}{2}\rho v^2 + \rho g y = constant \qquad A_1 v_1 = A_2 v_2$$



$p = 2\ \text{N}/2.5 \times 10^{-5}\ \text{m}^2 = 8 \times 10^4\ \text{Pa}$

$(2.5 \times 10^{-5}\ \text{m}^2)v_1 = (1.0 \times 10^{-8}\ \text{m}^2)v_2 \qquad v_1 = 0.4 \times 10^{-3}\ v_2$

$$(8 \times 104\ \text{Pa} + 1\ \text{atm}) + \frac{1}{2}\rho v_1^2 = (1\ \text{atm}) + \frac{1}{2}\rho v_2^2$$

$(8 \times 104\ \text{Pa}) + 0 = \frac{1}{2}\rho v_2^2 \qquad 16 \times 10 = v_2^2 \qquad v_2 = 12.65\ \text{m/s}$

4. A ball is thrown **horizontally** from the top of a building **0.10 km high**. The ball strikes the ground at a point **65 m horizontally away** from and below the point of release. What is the **initial speed** of the ball?

A). 43 m/s
B). 47 m/s
C). 39 m/s
D). 36 m/s
E). 14 m/s

$$y = \frac{1}{2}gt^2$$

$$h = 0.10 \text{ km} = 100 \text{ m}$$

$$100 = \frac{1}{2}(9.8)t^2$$

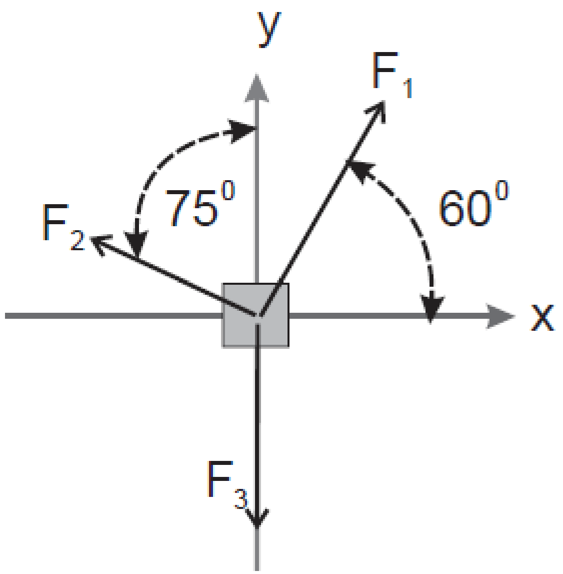
$$t \approx 4.52 \text{ s}$$

$$x = v_0 t$$

$$65 = v_0(4.52)$$

$$v_0 \approx \frac{65}{4.52} \approx 14.4 \text{ m/s}$$

11. Three forces are acting on a mass of **2 kg** as shown in the figure. The magnitudes of the forces are $\mathbf{F_1 = 10\ N}$, $\mathbf{F_2 = 5\ N}$, and $\mathbf{F_3 = 7\ N}$. What is the magnitude of the net **acceleration** of the mass?



A) none of the other answers

B) 4.5 m/s^2

C) 1.5 m/s^2

D) 3.7 m/s^2

E) 3.0 m/s^2

$$F_{1x} = 10\cos 60^\circ = 5, F_{1y} = 10\sin 60^\circ = 8.66$$

$$F_{2x} = 5\cos 165^\circ = -4.83, F_{2y} = 5\sin 165^\circ = 1.29$$

$$F_{3x} = 0, F_{3y} = -7$$

$$F_x = 5 - 4.83 = 0.17, F_y = 8.66 + 1.29 - 7 = 2.95$$

$$F_{\text{net}} = \sqrt{(0.17)^2 + (2.95)^2} = 2.96\ \text{N}$$

$$a = \frac{F_{\text{net}}}{m} = \frac{2.96}{2} = 1.48\ \text{m/s}^2$$

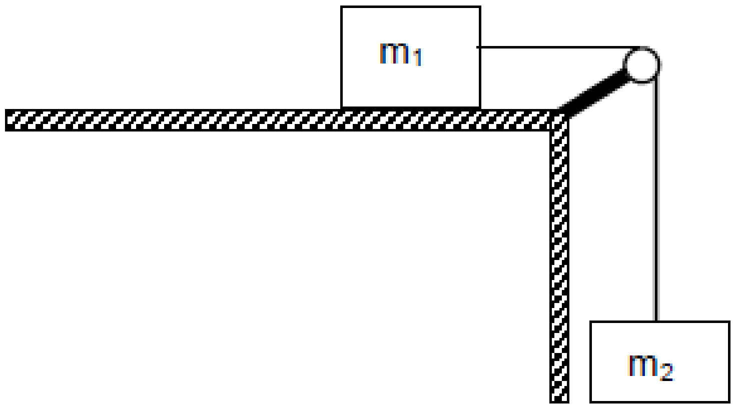
In the sketch at right, assume that the pulley is massless and all surfaces are **frictionless**. What is the magnitude of the **acceleration** of mass $\mathbf{m_1}$ in terms of m_1, m_2, and g?

$$T = m_1 a$$

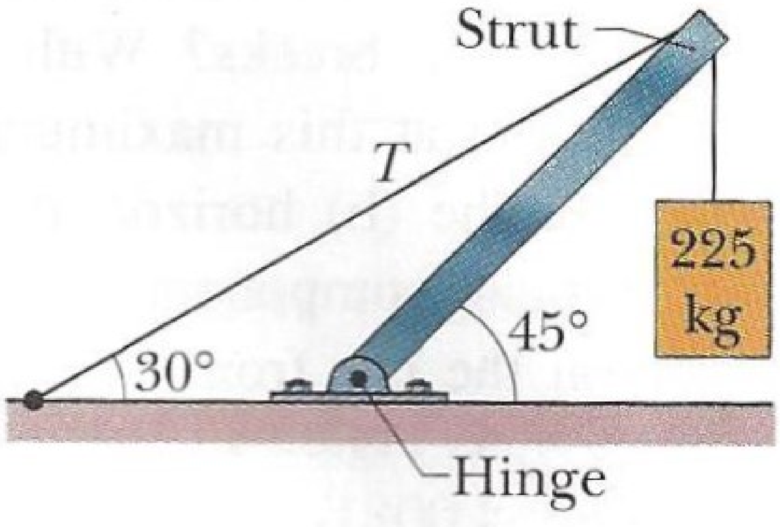
$$m_2 g - T = m_2 a$$

$$m_2 g - m_1 a = m_2 a$$

$$a = \frac{m_2 g}{m_1 + m_2}$$



The system in the figure is in equilibrium. A concrete block of mass **225 kg** hangs from the end of the uniform strut whose mass is **46.0 kg**. What is the magnitude of the **tension *T*** in the cable?



$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = -LW_B\sin(135°) - (0.5L)W_S\sin(135°) + LT\sin(165°)$$

$$= -L(225\times9.8)\sin(135°) - (0.5L)(46\times9.8)\sin(135°) + LT\sin(165°)$$

$$= 0$$

$$T\sin(165°) = (225\times9.8)\sin(135°) + (0.5)(46\times9.8)\sin(135°)$$

$$T = 6.64 \text{ kN}$$

A) 2.43 kN
B) 3.44 kN
C) 4.86 kN
D) 6.64 kN
E) 7.26 kN

5. A **1.5-kg** mass has an acceleration of **(4.0i - 3.0j) m/s²**. Only **two forces** act on the mass. If one of the forces is **(2.0i - 1.4j) N**, what is the magnitude of the **other force**?

A). 4.1 N
B). 6.1 N
C). 5.1 N
D). 7.1 N
E). 2.4 N

$$\vec{F}_{\text{net}} = (1.5)(4.0\hat{\imath} - 3.0\hat{\jmath}) = (6.0\hat{\imath} - 4.5\hat{\jmath})\ \text{N}$$

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2$$

$$\vec{F}_2 = \vec{F}_{\text{net}} - \vec{F}_1$$

$$\vec{F}_2 = (6.0\hat{\imath} - 4.5\hat{\jmath}) - (2.0\hat{\imath} - 1.4\hat{\jmath})$$

$$\vec{F}_2 = (4.0\hat{\imath} - 3.1\hat{\jmath})$$

$$|\vec{F}_2| = \sqrt{4.0^2 + (-3.1)^2} = \sqrt{16 + 9.61} = \sqrt{25.61} \approx 5.1\ \text{N}$$

7. A **4.0-kg** block slides down a **35° incline** at a **constant speed**. What is the **coefficient of kinetic friction** between the block and the surface of the incline?

A). 0.20
B). 0.33
C). 0.46
D). 0.53
E). 0.70

along the incline:

Component of gravity $F_{g,x} = mg\sin\theta$

Kinetic friction $f_k = \mu_k N = \mu_k mg\cos\theta$

$\sum F_x = ma$ $mg\sin\theta - \mu_k mg\cos\theta = 0$

$$\mu_k = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\mu_k = \tan 35^\circ \approx 0.70$$

A block is placed on a inclined plane which makes **27 degrees** with the horizontal. The **kinetic friction** coefficient between the block and the incline is **μ = 0.4**. Find the **acceleration** of the block in m/s^2.

$$F_{g,x} = mg\sin\theta \qquad f_k = \mu_k N = \mu_k mg\cos\theta$$

$$\sum F_x = mg\sin\theta - \mu_k mg\cos\theta = ma$$

$$a = g(\sin\theta - \mu_k\cos\theta) \qquad a = 9.8(\sin 27^\circ - 0.40\cos 27^\circ)$$

$$a = 9.8(0.454 - 0.40\times 0.891)$$

$$a = 9.8(0.098)$$

$$a \approx 0.96\ \mathrm{m/s^2}$$

8. A **2.0-kg** projectile moves from its initial position to a point that is **displaced 20 m horizontally** and **15 m above** its initial position. How much **work** is done by the **gravitational force** on the projectile?

A). +0.29 kJ
B). -0.29 kJ
C). +30 J
D). -30 J
E). -50 J

$$W_g = -mg\Delta y$$

$$W_g = -(2.0)(9.8)(15)$$

$$W_g = -294\text{ J}$$

$$W_g \approx -0.29\text{ kJ}$$

25. A **20-N** crate starting at rest slides down a rough **5.0-m long ramp**, inclined at **25° with** the horizontal. **20 J of energy's** lost to friction. What will be the **speed of the crate** at the bottom of the incline?

A) 0.98 m/s
B) 1.9 m/s
C) 3.2 m/s
D) 4.7 m/s
E) 0.7 m/s

$$K_1 + U_1 + W_{\text{other}} = K_2 + U_2 \qquad K_1 = 0, \qquad U_2 = 0$$

$$U_1 + W_{\text{other}} = K_2 \qquad W_{\text{other}} = -20\ \text{J}$$

$$h = L\sin\theta = 5.0\sin 25^\circ \qquad U_1 = Wh = (20)(2.11) = 42.2\ \text{J}$$

$$h \approx 2.11\ \text{m}$$

$$K_2 = \frac{1}{2}mv^2$$

$$K_2 = 42.2 - 20 = 22.2\ \text{J} \qquad m = \frac{W}{g} = \frac{20}{9.8} = 2.04\ \text{kg} \qquad 22.2 = \frac{1}{2}(2.04)v^2$$

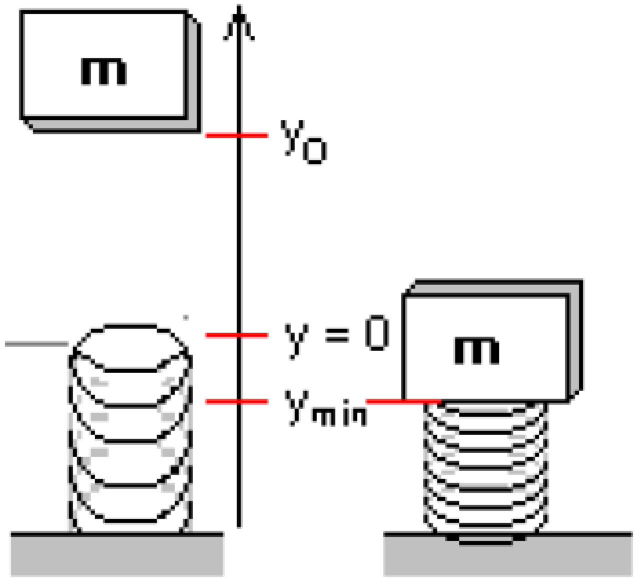
$$v \approx 4.7\ \text{m/s}$$

A **5-kg ball** is dropped from a height $\mathbf{y_0}$ above the top of a vertical spring whose spring constant is **2000 N/m**. The spring is **compressed 0.4 m**. The **height $\mathbf{y_0}$** is closest to

$$K_1 + U_{s1} + U_{g1} = K_2 + U_{s2} + U_{g2}$$

$$K_1 = 0, \qquad U_{s1} = 0, \qquad U_{g1} = mgy_0$$

$$K_2 = 0, \qquad U_{s2} = \frac{1}{2}kx^2$$



But the block also drops an additional distance $x = 0.40\ m$, while compressing the spring, so gravity does extra work:

$$mg(y_0 + x) = \frac{1}{2}kx^2$$

$$y_0 = \frac{\frac{1}{2}kx^2}{mg} - x \qquad y_0 = \frac{\frac{1}{2}(2000)(0.40)^2}{(5)(9.8)} - 0.40 \qquad y_0 = 3.27 - 0.40 = 2.87\ \text{m}$$

2. A **5.0-kg** mass with an **initial velocity of 4.0 m/s, east** collides with a **4.0-kg** mass with an initial velocity of **3.0 m/s, west**. After the collision the **5.0-kg** mass has a velocity of **1.2 m/s, south**. What is the magnitude of the **velocity of the 4.0-kg** mass after the collision?

A) 2.0 m/s
B) 1.5 m/s
C) 1.0 m/s
D) 2.5 m/s
E) 3.0 m/s

Before collision: $p_{x,i} = (5)(4.0) + (4)(-3.0) = 20 - 12 = 8$

$$p_{y,i} = 0$$

After collision:

5-kg mass moves south: $\vec{v}_1 = (0, -1.2) \Rightarrow \vec{p}_1 = (0, -6)$

Let 4-kg mass velocity be (v_x, v_y) : $\vec{p}_2 = (4v_x, 4v_y)$

Conservation of linear moment:

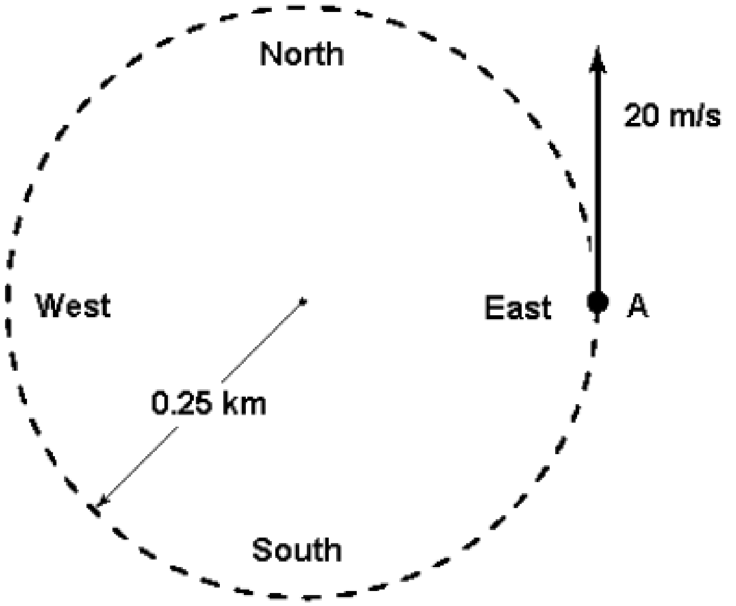
The x direction: $8 = 4v_x \Rightarrow v_x = 2.0$

The y direction: $0 = -6 + 4v_y \Rightarrow v_y = 1.5$

$$v = \sqrt{(2.0)^2 + (1.5)^2} = 2.5 \text{ m/s}$$

22. A car travels **counterclockwise** around a flat circle of **radius 0.25 km** at a constant speed of **20 m/s.** When the car is at point A as shown in the figure, what is the car's **acceleration**?

A) 1.6 m/s², south
B) Zero
C) 1.6 m/s², east
D) 1.6 m/s², north
E) 1.6 m/s², west



$$a_c = \frac{v^2}{r}$$

$$r = 0.25 \text{ km} = 250 \text{ m}$$

$$a_c = \frac{(20)^2}{250} = \frac{400}{250} = 1.6 \text{ m/s}^2$$

17. A frictionless pulley free to rotate about a frictionless axle has a radius **R = 0.12** m and a moment of **inertia I = 0.050 kg×m²**. A **1.5-kg** object is attached to a very light wire that is wrapped around the rim of the pulley. The system is **released from rest** and mass m moves **downward** a distance of **63.7 cm**. Find the **angular velocity** of the pulley at this instant.

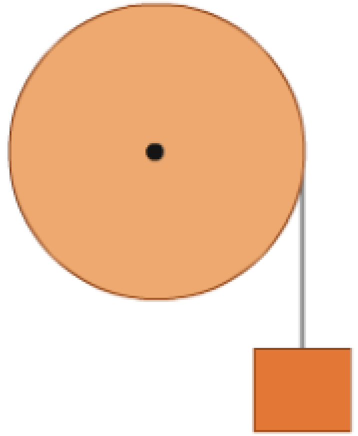
A) 28.4 rad/s
B) 5.75 rad/s
C) 16.2 rad/s
D) 0.25 rad/s
E) 32.2 rad/s

$$W = mgh = (1.5\text{ kg})(9.8)(0.637\text{ m}) = 9.3639\text{ J}$$

$$K = W = 9.3639\text{ J} = (1/2)mv^2 + (1/2)I\omega^2$$

$$v = r\omega = (0.12)(\omega)$$

$$9.3639\text{ J} = 0.0108\omega^2 + 0.025\omega^2$$



A body oscillates with simple harmonic motion along the x axis. Its displacement varies with time according to the equation

$$x(t) = (5.0\ m)\sin[(\pi\ s^{-1})t + \pi/3].$$

The velocity (in m/s) of the body at $t = 1.0\ s$ is

A) +7.9
B) −7.9
C) −14
D) +14
E) −5.0

$$v(t) = \frac{dx}{dt} = 5\pi\cos(\pi t + \tfrac{\pi}{3})$$

$$v(1) = 5\pi\cos(\pi + \tfrac{\pi}{3}) = 5\pi\cos\left(\tfrac{4\pi}{3}\right)$$

$$\cos\left(\tfrac{4\pi}{3}\right) = -\tfrac{1}{2}$$

$$v = 5\pi\left(-\tfrac{1}{2}\right) = -2.5\pi$$

$$v \approx -7.9\ \mathrm{m/s}$$

28. A solid rock, suspended in air by a spring scale, has a measured mass of **9.00 kg**. When the rock is submerged in water, the scale reads **3.30 kg**. What is the density of the rock? (water density = 1,000 kg/m³).

A) 4.55×10^3 kg/m³
B) 3.50×10^3 kg/m³
C) 1.20×10^3 kg/m³
D) 1.58×10^3 kg/m³
E) 1.35×10^3 kg/m³

Weight in air: $W = mg = (9.00)g$

Forces in water $W_{\text{app}} = mg - B$ $\quad W_{\text{app}} = (3.30)g$

$mg - B = 3.30g \qquad B = (9.00 - 3.30)g = 5.70g$

$B = \rho_{\text{water}} V g$

$\rho_{\text{water}} V g = 5.70g$

$\rho_{\text{water}} V = 5.70$

$V = \frac{5.70}{1000} = 5.70 \times 10^{-3}\ \text{m}^3$

$\rho_{\text{rock}} = \frac{m}{V} = \frac{9.00}{5.70 \times 10^{-3}}$

$\rho_{\text{rock}} \approx 1.58 \times 10^3\ \text{kg/m}^3$

29. An ideal fluid flows through a pipe made of two sections with **diameters of 1.0 and 3.0 inches**, respectively. The speed of the fluid flow through the 3.0-inch section will be what factor times that through the 1.0-inch section?

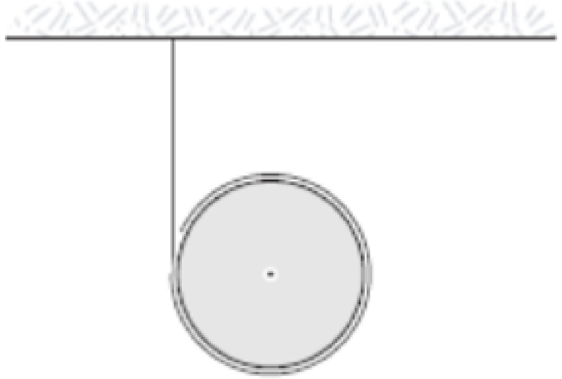
A) 6.0
B) 9.0
C) 1/3
D) 1/9
E) 1/2

$$A_1 v_1 = A_2 v_2 \qquad A = \frac{\pi d^2}{4}$$

$$\frac{A_{3\text{-inch}}}{A_{1\text{-inch}}} = \left(\frac{3}{1}\right)^2 = 9$$

$$v_2 = \frac{A_1}{A_2} v_1 = \frac{1}{9} v_1$$

A massless rope is wrapped around a disk that has radius $\boldsymbol{R} = \mathbf{0.2\ m}$ and mass $\boldsymbol{M} = \mathbf{0.5\ kg}$, as shown in the figure. The disk is **released from rest**. Find the **angular speed** of the disk after it **descends 2 m**.



A. 9.5 rad/s

B. 14.2 rad/s

C. 25.6 rad/s ✔

D. 44.8 rad/s

E. 65.8 rad/s

$$W = mgh \qquad K = W$$

$$K = \tfrac{1}{2}(I\omega^2) + \tfrac{1}{2}(Mv^2), \qquad \omega R = v$$

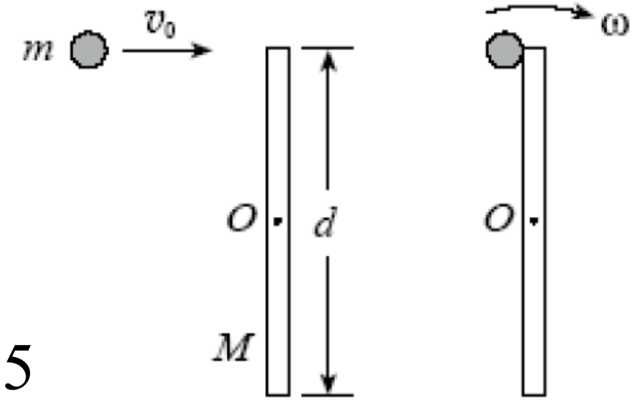
$$I = 1/2(MR^2) = 0.01,$$

$$K = \tfrac{1}{2}(I\omega^2) + \tfrac{1}{2}M(\omega R)^2 = 0.015\omega^2 = mgh$$

$$0.015\omega^2 = 0.5\times 2\times 9.8 = 9.8$$

$$\omega = 25.6 \text{ rad/s}$$

A thin rod of mass M **= 1.2 kg** and length d **= 2 m** is struck at one end by a ball of clay of mass m **= 0.050 kg**, moving with speed v_0 **= 7.5 m/s** as shown in the figure. The ball sticks to the rod. After the collision, the **angular velocity** of the clay-rod system about O, the midpoint of the rod, is



A. 0.5 rad/s
B. 1.1 rad/s
C. 0.83 rad/s ✓
D. 2.8 rad/s
E. 4.0 rad/s

Conservation of angular moment

Before collision:
$L = r(mv_0) = (0.05\ \text{kg})(7.5\ \text{m/s})(1\ \text{m}) = 0.375$

After collision: $L = \omega I = 0.375$

$$I = (1/12)ML^2 + mr^2 = 0.45$$

$$\omega = L/I = 0.83\ \text{rad/s}$$

An asteroid, headed directly toward Earth, has a **speed of 12 km/s** relative to the planet when it is at a **distance of 10 Earth radii** from Earth's center. Neglecting the effect of Earth's atmosphere on the asteroid, find the asteroid's **speed** when it reaches Earth's surface. The Earth's radius and mass are $R_E = 6.38\times10^6$ m, and $M_E = 5.97\times10^{24}$ kg.

$G = 6.673 \times 10^{-11}$ N m^2/kg^2

$$E = K + U = \frac{1}{2}mv^2 - G\frac{M_E m}{r}$$

$$E_1 = \frac{1}{2}m(12\ km/s)^2 - G\frac{M_E m}{10R_E}$$

$$E_2 = \frac{1}{2}mv^2 - G\frac{M_E m}{R_E} = E_1$$

$$E_1 = E_2$$

$$\frac{1}{2}mv^2 - G\frac{M_E m}{R_E} = \frac{1}{2}m(12\ km/s)^2 - G\frac{M_E m}{10R_E}$$

$$\frac{1}{2}v^2 = \frac{1}{2}(12\,\mathrm{km/s})^2 + G\frac{9M_E}{10R_E}$$

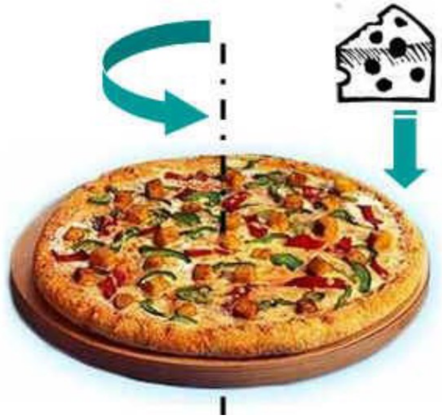
$$v = 16\ \mathrm{km/s}$$

The planet Venus requires 225 days to orbit the sun, which has a mass $M = 1.99 \text{ x } 10^{30}$ kg, in an almost circular trajectory. Calculate the radius of the orbit as it circles the sun.

$$T = \frac{2\pi r^{3/2}}{\sqrt{Gm_{\text{star}}}}, \qquad 225\ days = 1.9\times10^{7}\ \text{s}$$

$$r = \sqrt[3]{\frac{T^2 Gm_{\text{star}}}{4\pi^2}} = \sqrt[3]{\frac{(1.9\times10^{7}\ \text{s})^2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\ kg)}{4\pi^2}} = 1.1\times10^{11}\text{m}$$

A pizza together with its pan is rotating about a vertical axis through its center. Its rotational inertia **I = 2.0 kg.m²** and radius **r = 50 cm.** The initial angular speed is **4.0 rad/s**. A **1.0 kg** chunk of Mozzarella cheese is initially at rest over the disk. It drops vertically onto the pizza from above and sticks to the edge. What is the angular speed of the pizza after the cheese becomes stuck to it?



Conservation of angular moment

$$L_1 = I_1\omega_1 = 2.0 \times 4.0 = 8$$

$$\omega_1 = 4.0 \text{ rad/s}$$

$$L_2 = I_2\omega_2 = L_1 = 8$$

$$I_2 = I_1 + mR^2 = 2.0 + (1.0)\left(0.5^2\right) = 2.25$$

$$8 = 2.25\omega_2 \qquad \omega_2 = 3.56 \text{ rad/s}$$