

DRAWING KNOTS BY THE PROGRAM *KnotPlot*

Today you will learn about the main properties and use of the program *KnotPlot* by Rob Scharein (<http://www.knotplot.com/>), which can be downloaded from the address: (<http://www.knotplot.com/download/>). The instructions given in this file need to be updated for the most recent version (*KnotPlot* 1.0) of the program. You can find more information in the *KnotPlot* manual (<http://knotplot.com/KPmanual/KnotPlotManual.html>, <http://knotplot.com/KPmanual/KPman.pdf>), which is, unfortunately, also out of date.

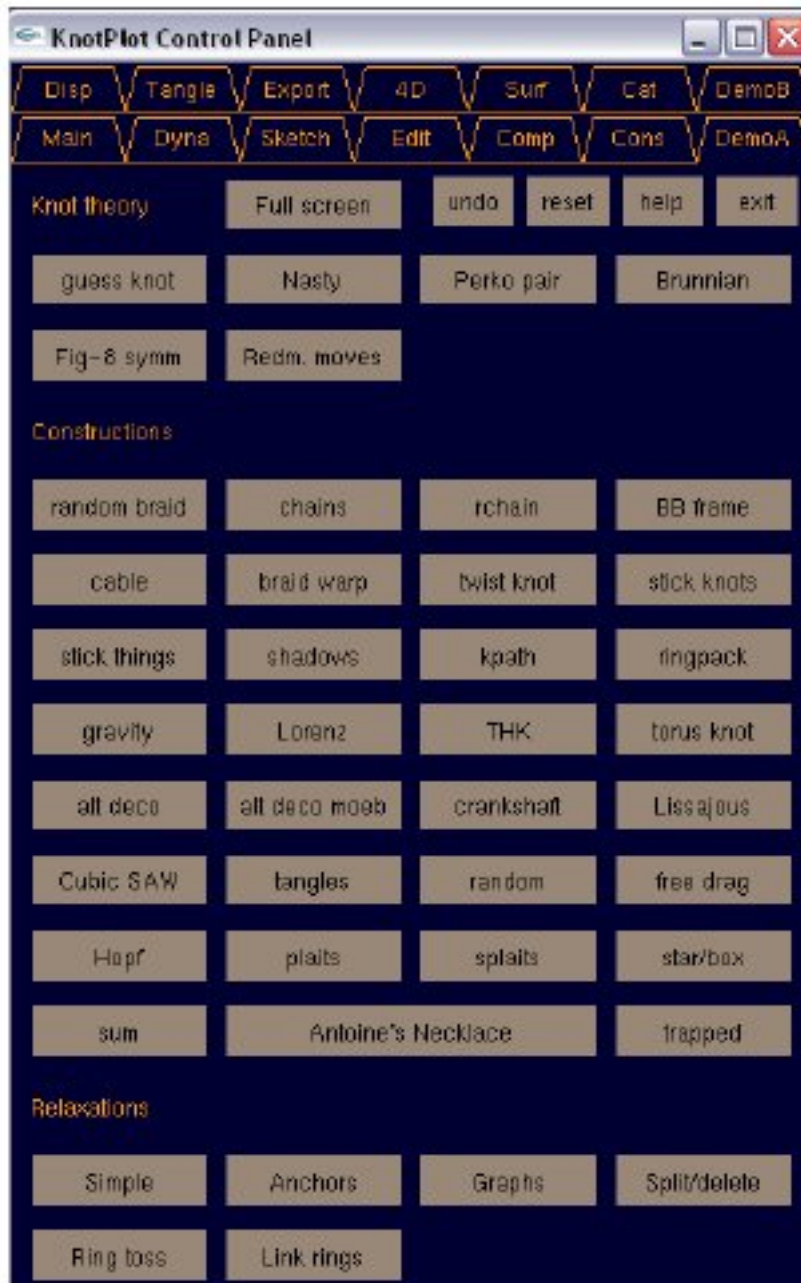
After installing *KnotPlot*, you can run it by clicking on its icon or directly from the Start menu.

A few windows will open, and the active window will be the welcome screen (Fig. 1).

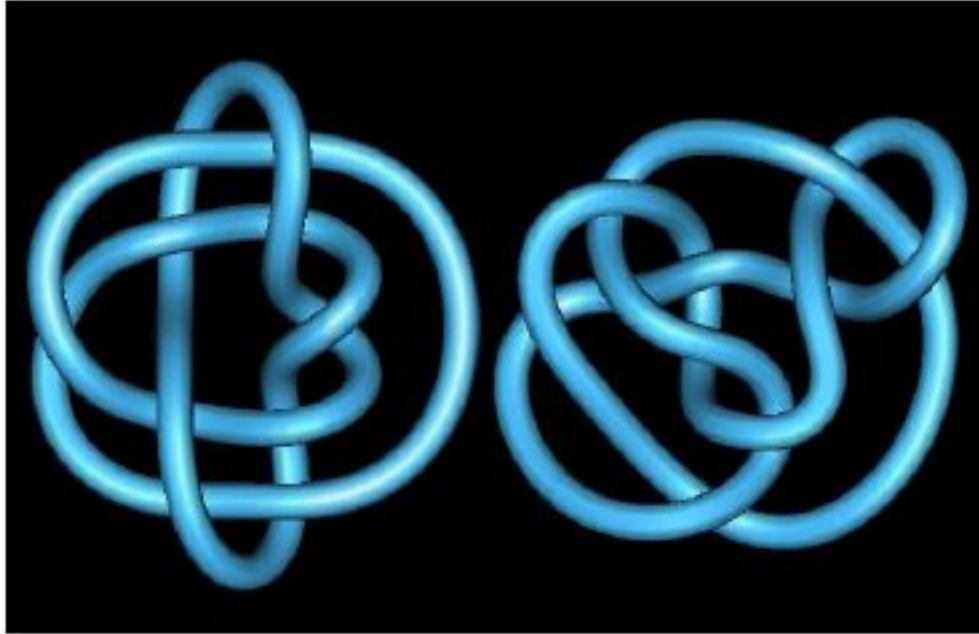


Read the basic instructions and note the important remark about closing the program: DO NOT USE THE CLASSIC "Close" BUTTON, i.e. cross, but instead use the "KnotPlot" button "Close window"; otherwise, the whole program closes.

On the right side of the screen you will see *KnotPlot ControlPanel* with many options, among which is an option that automatically opens **MainControlPanel**. Choose option *DemoA*. In this panel there are again many different options, so you can try them all.



For example, *guess knot* opens two sample knots, where one of them is knotted, and the other is unknotted. The unknotting process is animated. You can also check the Perko pair of knots and see that the knots belonging to this pair are the same. This is shown by a continuous transformation (ambient isotopy) turning one into another.



You can also see how to obtain different knots and links as the closures of braids, use random braids, and obtain images of knots represented by chains. Try to experiment and examine as many different examples as is possible. In particular, try to relax different knots (using the command **Relaxation**, i.e. *go* into the **Main** menu) and follow the reduction of a knot. By rotating the relaxed knot try to find the position in which the diagram of the knot exhibits the minimal number of crossings.

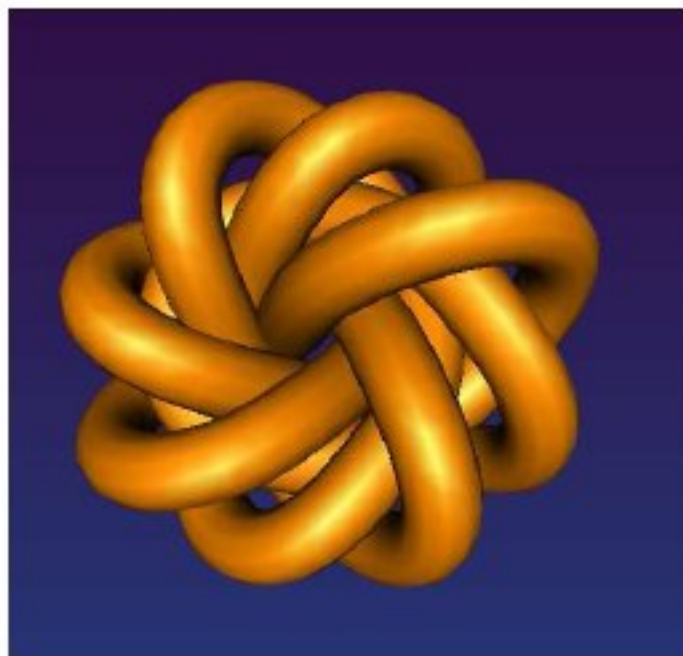
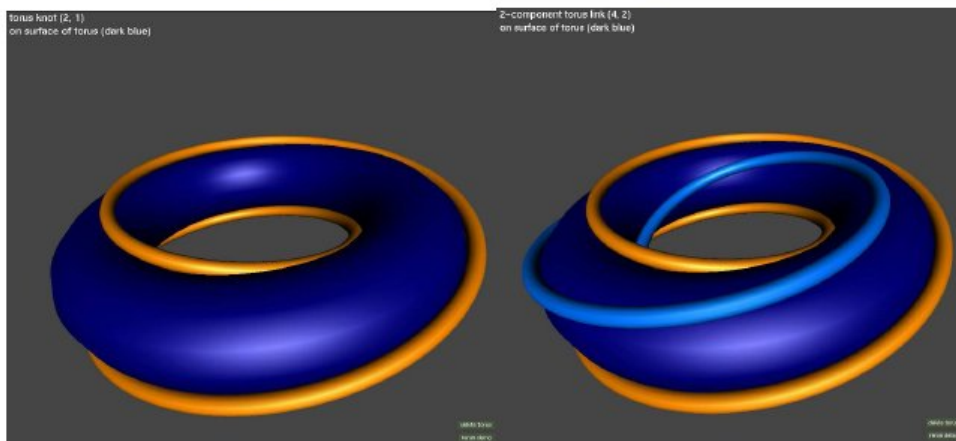
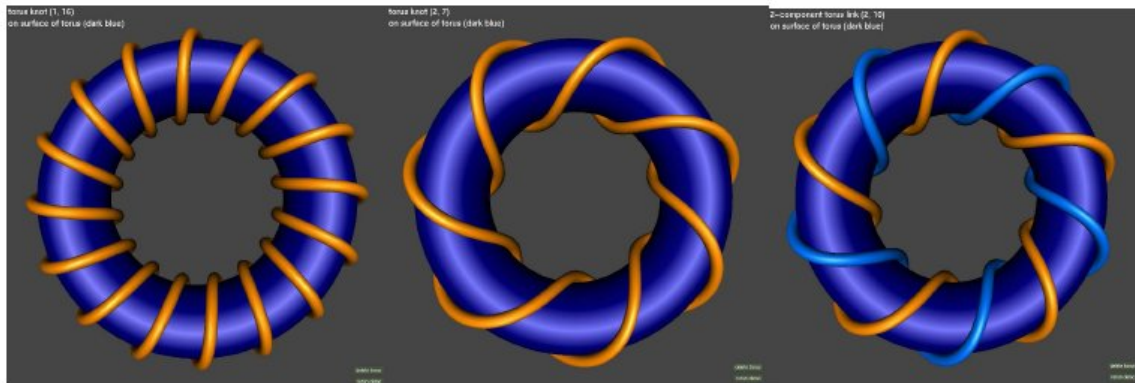
Among different notations for knots and links, we prefer the Conway notation (<http://math.ict.edu.rs:8080/webMathematica/LinkSL/kn033.htm>), a human-comprehensive notation suitable for classification of knots and links and their division into knot and link families (<http://math.ict.edu.rs:8080/webMathematica/LinkSL/kn037.htm>). First we consider torus knots (<http://math.ict.edu.rs:8080/webMathematica/LinkSL/kn115.htm>). A torus knot or link $[m,n]$ is a simple closed curve on a torus which wraps around m times meridionally and n times longitudinally. If the integers m and n are relatively prime, $[m,n]$ is a torus knot; otherwise, we have a torus link.

Press the button *torus knot* in **DemoA**, and then, using the option *rerun demos* in the lower right corner, examine the gallery of torus knots. Looking at the code $[m,n]$ that follows every torus knot or link, try to calculate the number of components of a torus link from its symbol $[m,n]$.

Try to construct some torus knots and links. This can be done in several different ways: you can run the command *torus* from the command line, press **Enter**, and you will obtain a torus knot randomly chosen by the program and followed by its symbol.

A trefoil knot can be obtained by typing *torus 2 3*, meaning that the trefoil belongs to the class of torus knots, and that it is a $[2,3]$ torus knot. Try with *torus 5 11*, *torus 2 8*, or

torus 2 4. Before typing the numbers, try to predict the number of components. What will happen if instead **torus 2 3** you type the command **torus 3 2**?

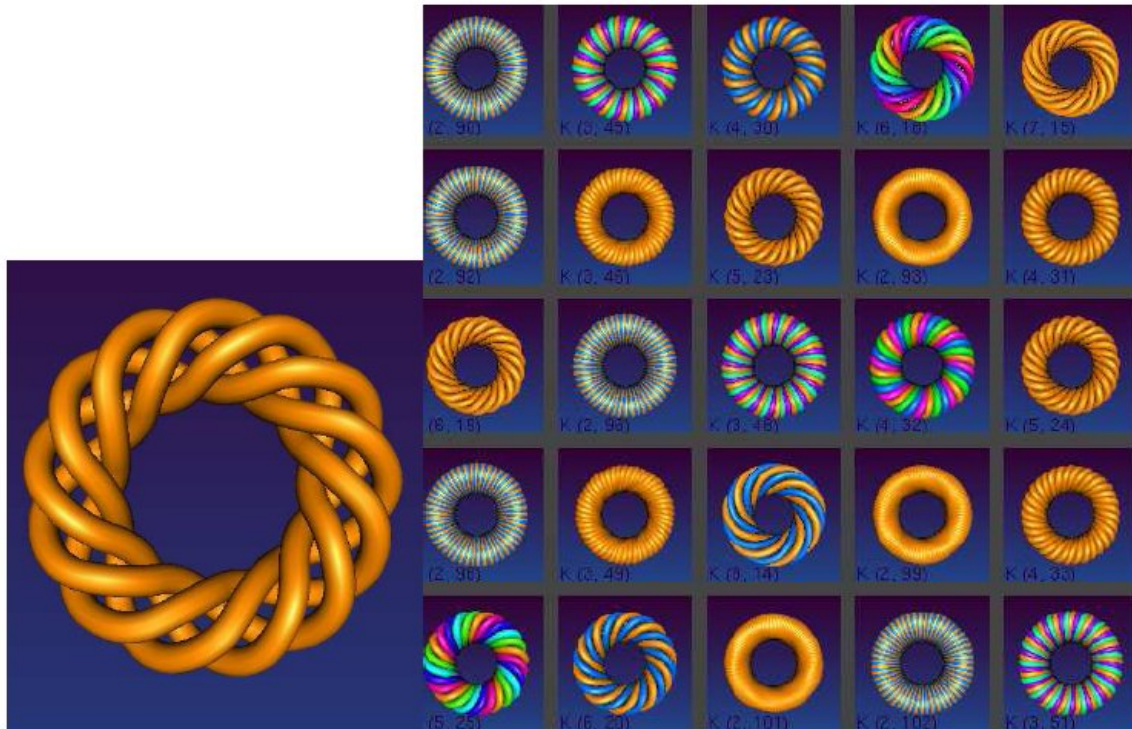


Now return to the **MainControl** Panel. For each option, you can get help by pressing on the **Help** button in the upper right corner. Every time when you start with a new function, open its corresponding help menu and try to learn more about the function. On the left side you will find the button **reset**, which you can use every time you wish to return to the default settings of the functions or parameters.



On the top of this screen you will find the tags leading to catalogues of different interesting knots. From the first row you can load the catalogues of 1-component, 2-component, 3-component links, etc.

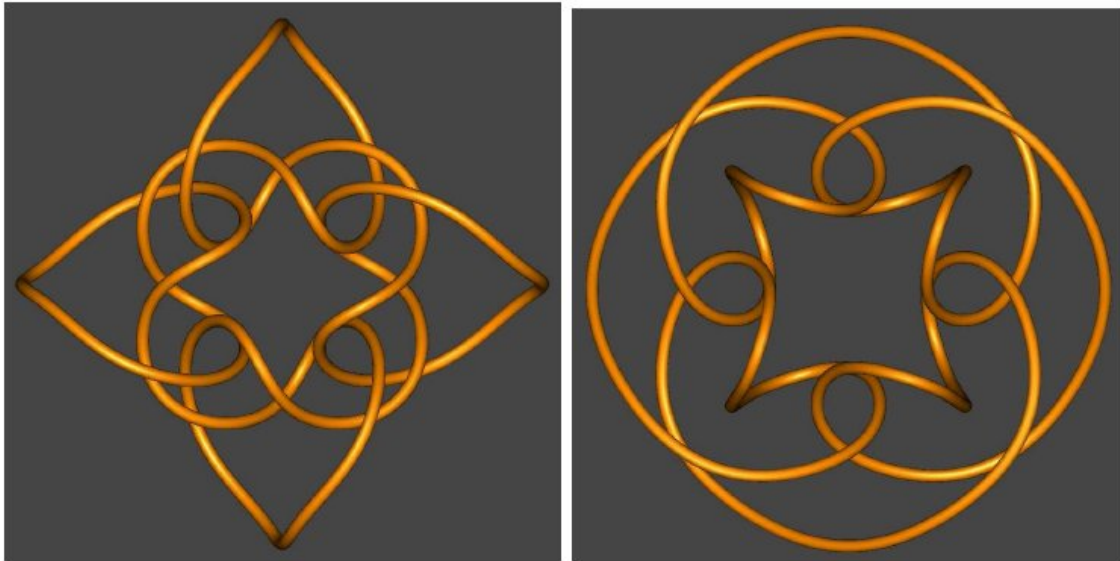
From the second row you can load catalogues of knot and link families. The same holds for the **Torus** button which leads to the catalogue of torus knots and links.



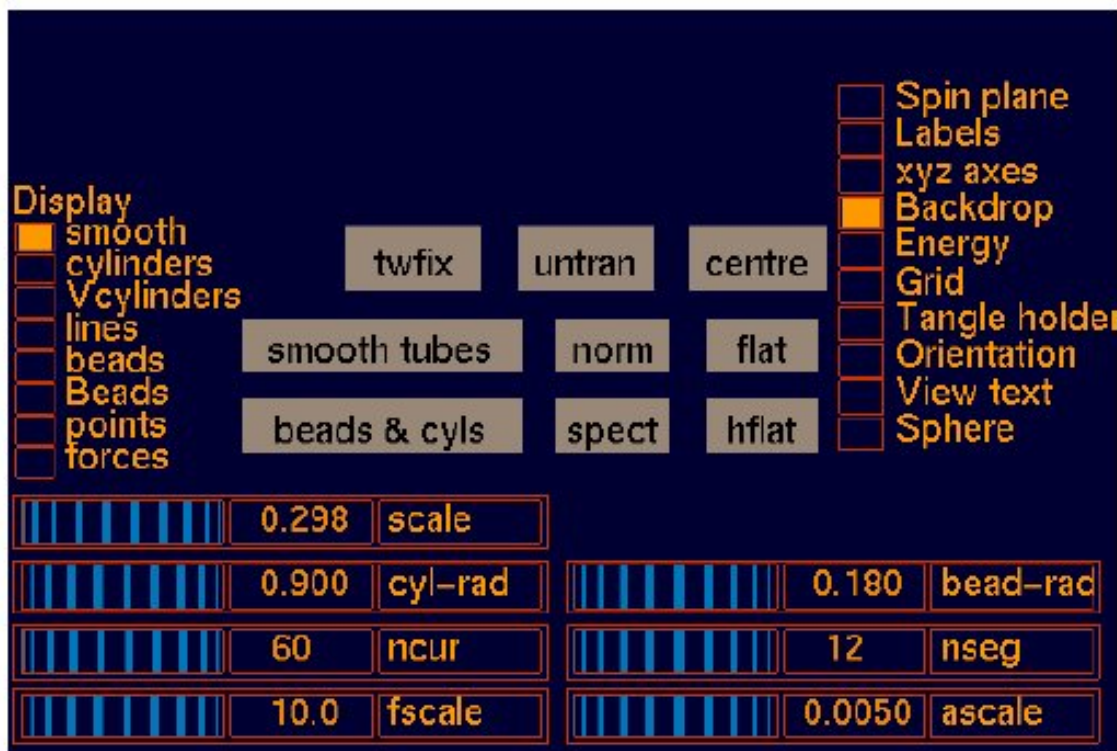
In the main menu you will find the button *Cat*, meaning catalogue. From this file, *Decorative knots* are particularly interesting. The letters c and d correspond to the cyclic and dihedral symmetry group of a periodic knot. You can press the same button several times.

Decorative knots						
d3-d8	d3	d4	d5	d6	d7	d8
c3-c8	c3	c4	c5	c6	c7	c8
all	c3/d3	c4/d4	c5/d5	c6/d6	c7/d7	c8/d8
<input type="checkbox"/> set ortho	<input type="checkbox"/> set cyl-rad	<input type="checkbox"/> untran	<input type="checkbox"/> fitto 16			

Here you can see some examples from this wonderful gallery.



The middle part contains commands connected with smoothness and approximation of the linear segments of knots. Usually, we use the command *smooth*, and after drawing a knot we use the option *smooth tubes*.



In order to choose the size of a knot image we use scaling (*scale*), and the same can be done by using the right mouse button. When using the option *smooth*, we need to fix *cyl-rad*, i.e., the radius of the cylindrical portion of the knot surface. Using the button *spectrum* you can choose different colors for your knot. Many coloring options can be

changed by using the scrolling buttons from the same menu, and the background color can be changed by using *Backdrop*.

The lower part of the screen contains commands connected with dynamics and the options placed in the *Dyna* palette. From this part, we mainly use the command *go* for the relaxation and simplification of knots.



When this palette is still active, type torus knots **torus 5 9** and **torus 9 5** again in the *KnotPlot Command Window*. Relax both of them by using the command *go*, and you will see that they represent the same knot. In general, the equality $[m,n]=[n,m]$ holds for torus knots and links.

We will not use the panel *Dyna (Dynamic)* in the next step, and we will not change its parameters.



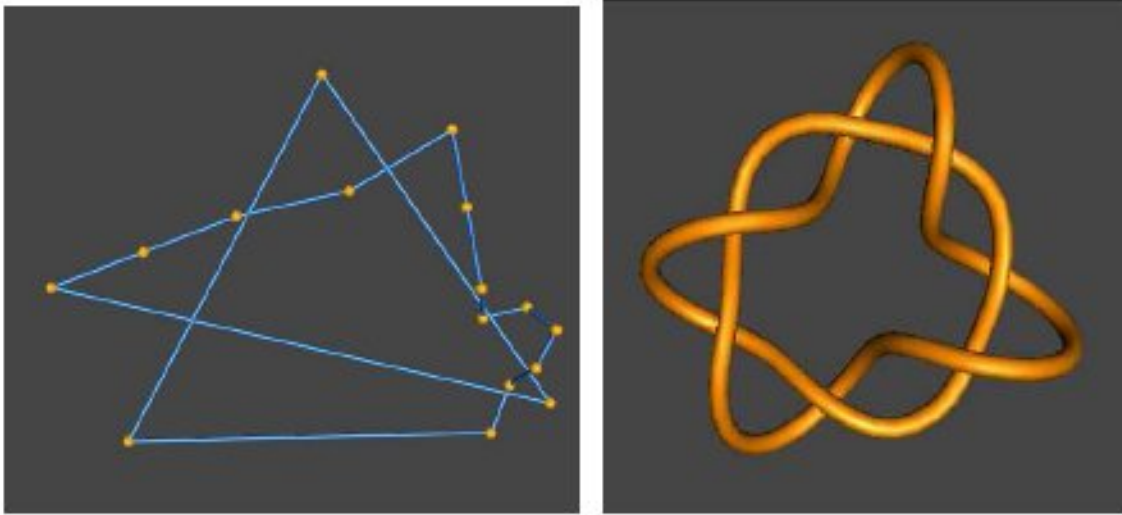
From the *Sketch* palette you can use the command *Delete all*.



You can also draw your own knot. With the mouse go to the left window. Using the right and left mouse button you can add the beads and trace your knot, where by using the

left mouse button we are not changing the relation over-under between two strands, and by pressing the right button we are changing this relation.

After making a sketch of the knot, we can close the component or leave it open, or simply complete our work by the command finish. After that we can open the *Main* menu and finalize the drawing (by smoothing, etc.).



At the bottom of this panel you can find the button for drawing *Celtic knots*.

--> Celtic knotting

Celtic knotting undo reset help exit

Show:

- Primary grid diagram auto diagram
- Secondary grid
- Tertiary grid
- Breaklines
- Knot path centreline
- Knot interlace

Clear breaklines:

all primary secondary

Apply symmetry:

- left/right
- top/bottom
- C4
- D4

Regular grid create auto create

4 ncols 2 nrows

Circular grid create auto create

12 ncircum 3 nradial

3.00 innerR

copy to arena

After copying to arena:

- set cyl-rad to -->
- set ortho
- show arena

Example grids

josephine gbain-p46a gbain-p46b ibain-p49 ibain-p57a

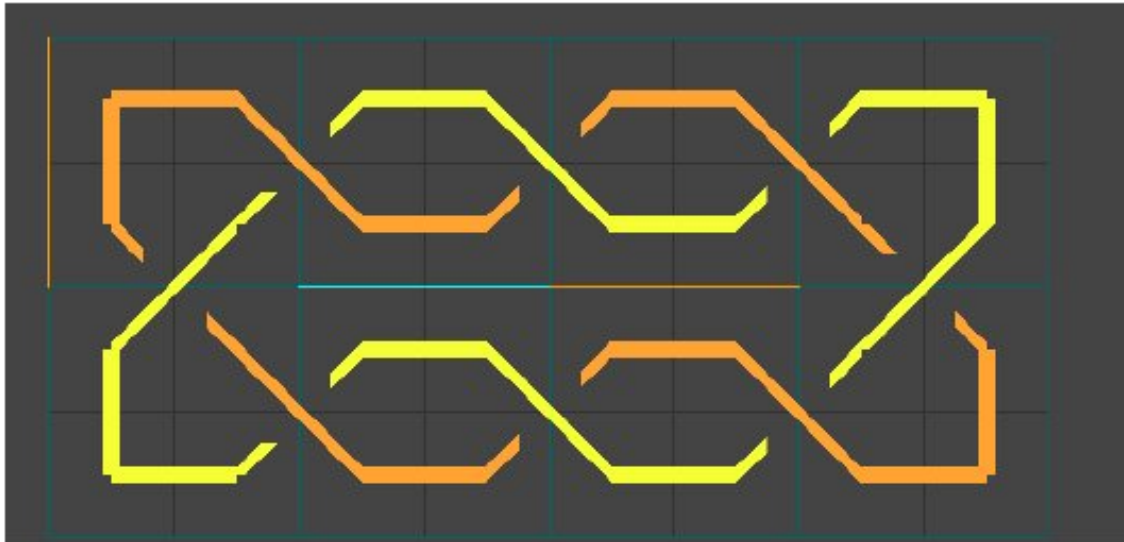
ibain-p64 ibain-p73 ibain-p74 ibain-p75d am-p130

am-p87 amck104a asckp60 rsck1 celtic1

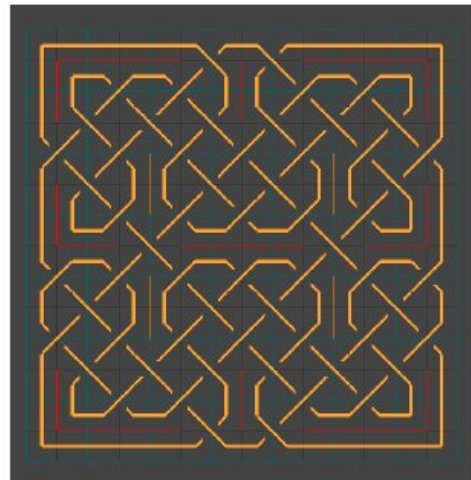
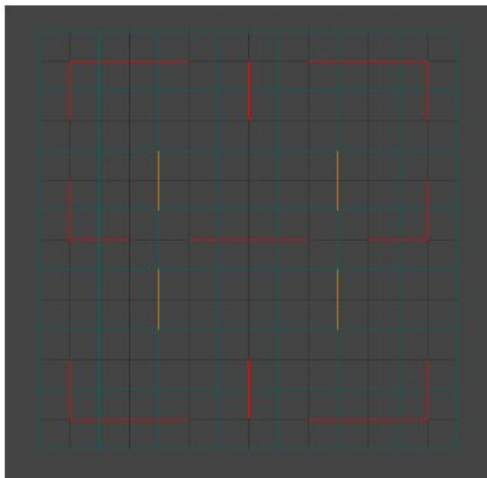
endless

--> Normal sketching

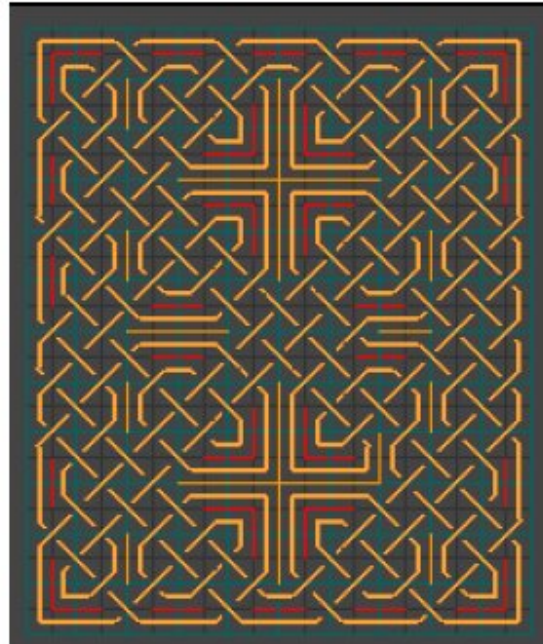
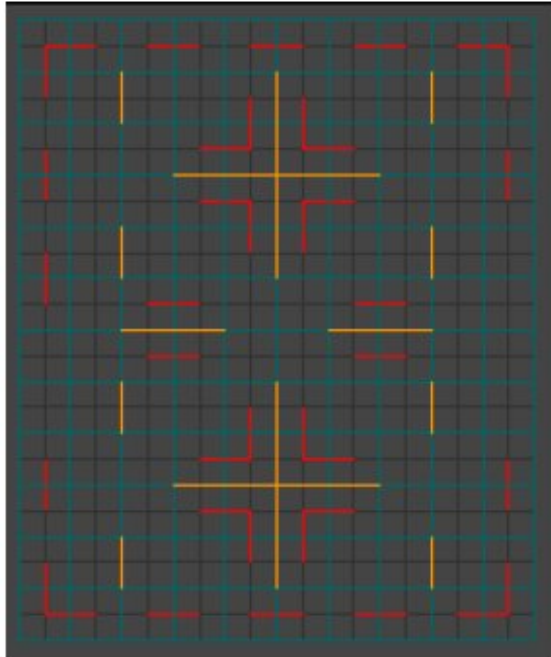
First you can choose the grid. Instead of a rectangular grid you can use a *Circular grid*. You will obtain the grid by pressing the button *create*. You can choose the parameters of your grid: number of columns and rows. Then, with the mouse, you can place internal mirrors in desired positions within the grid (the program shows automatically the options, where you approach by mouse to the corresponding edge, so you need only to click). It is also possible to introduce additional mirrors. Press the button **diagram** in order to obtain the mirror-curve.



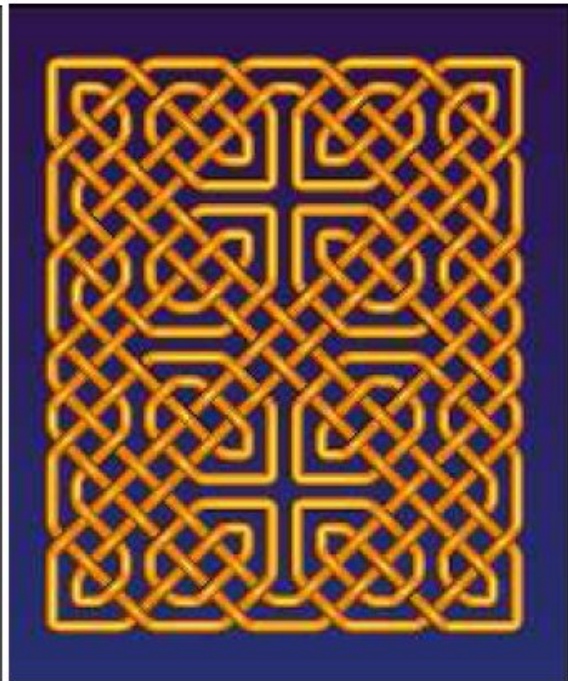
Try to use *KnotPlot* to construct some of your mirror-curve exercises, or repeat some of the mirror-curve drawings that you already produced in some other program. From Celtic knots (mirror-curves) you can obtain 3D-knots. Just press the button *copy to arena* and you will transfer automatically to the *MainControl* panel. Here you can fix the parameters for smoothness (*smooth*) and thickness (*cyl-rad*) and obtain a 3D image of the Celtic knot with desired properties.



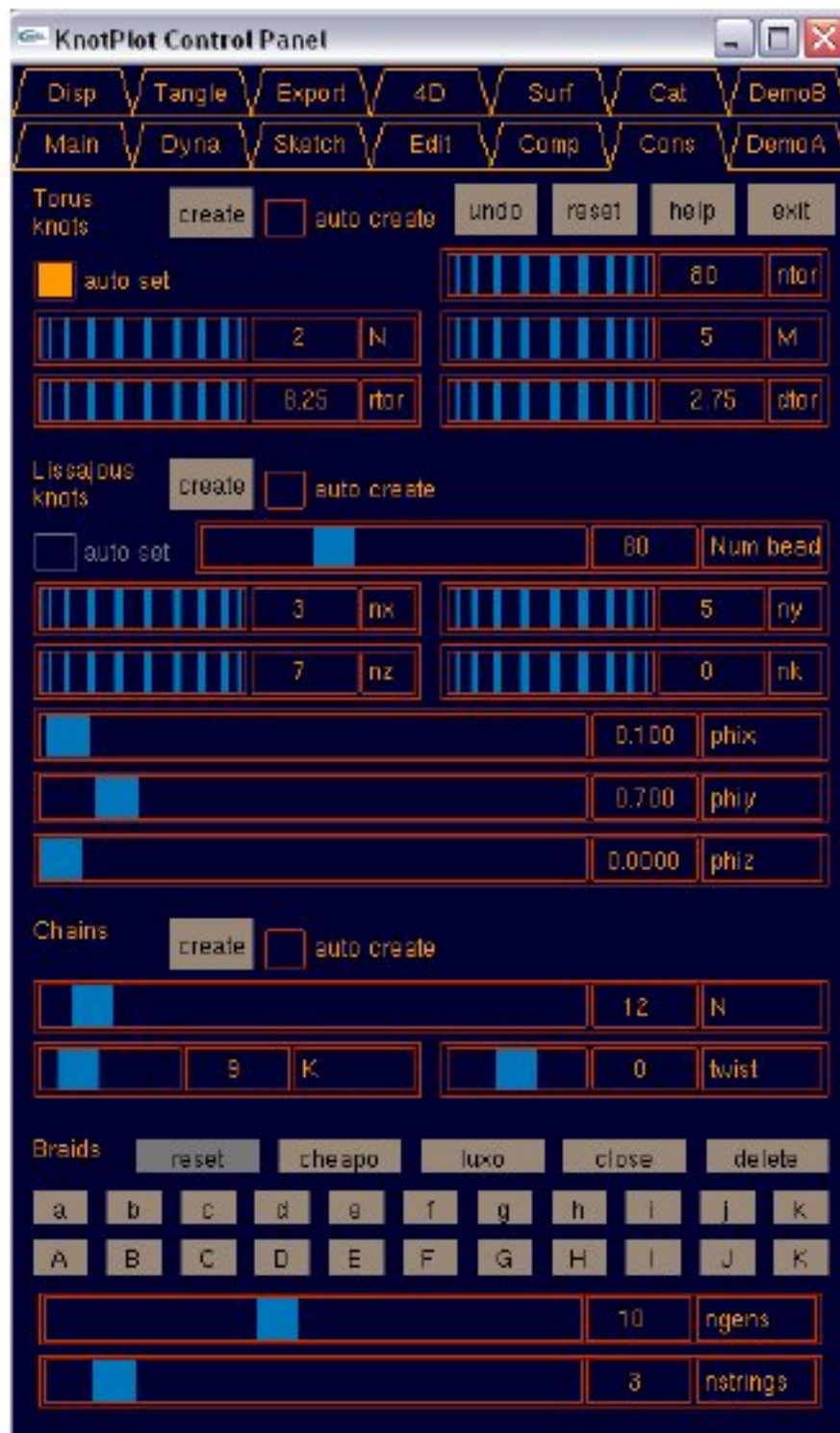
Pressing to the button *gbain-p46b* you obtain the grid and the knot from the book *Celtic Art – the Methods of Construction* (Dover, New York, 1973) by G. Bain.



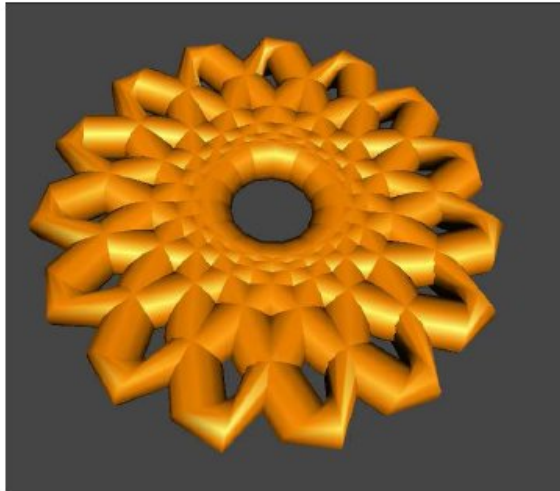
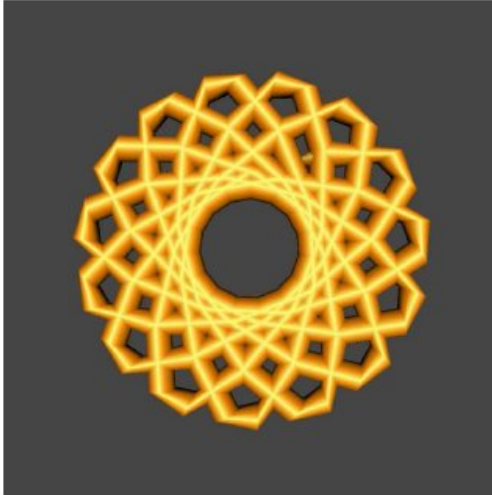
By using the command *copy to arena* and going to the *MainControl* we obtain the following Celtic knot.



By using the palette *Cons (Construction)* we can draw, with ease, some classes of knots. Here we have (again) torus knot parameters m and n , but now we are able to change certain parameters, and then the knot. Here is the example obtained by changing the parameters for torus knots.

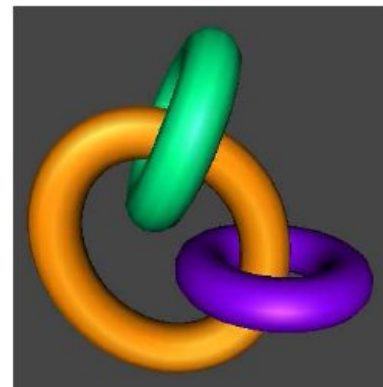
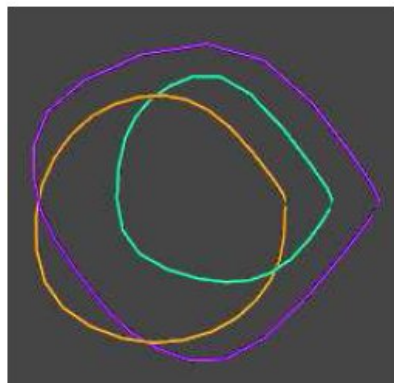


In the same way, we can draw interesting chains (*Chains*) and knots and links obtained as the closures of braids (*Braids*).



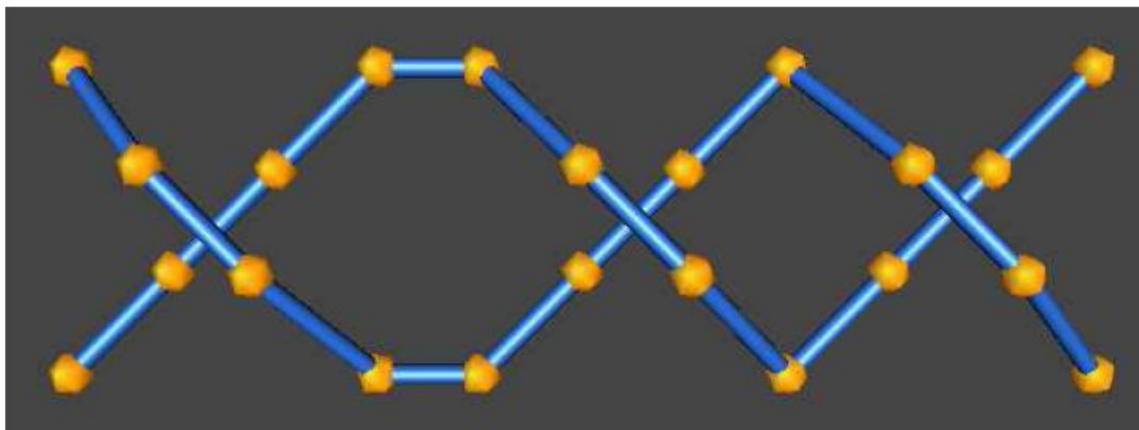
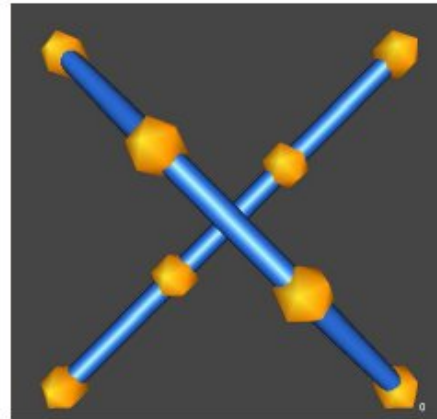
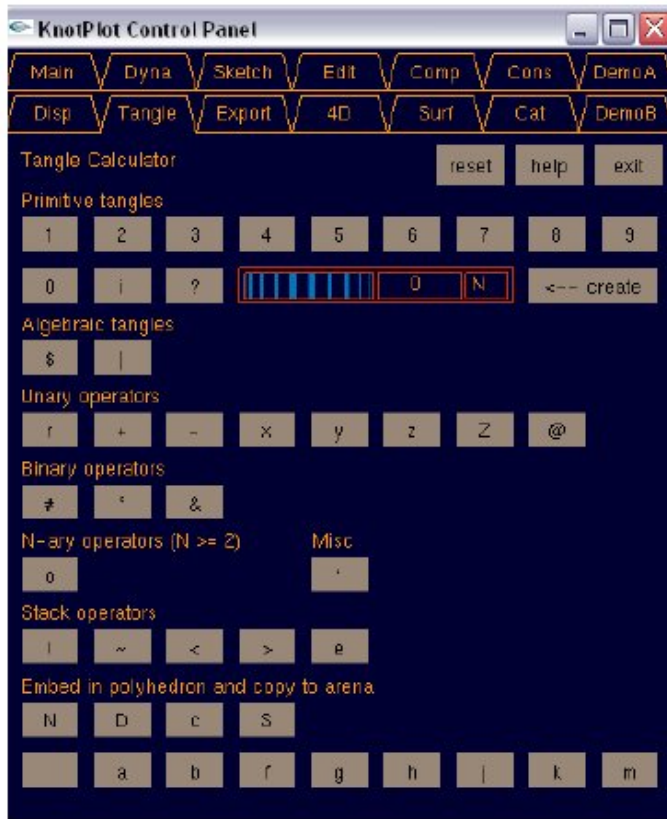
An n -strand consists of n disjoint arcs running vertically in R^3 space, in which the starting points lie on the same line. Every open braid is given by a braidword, e.g., $AaBbCc\dots$ (<http://math.ict.edu.rs:8080/webMathematica/LinkSL/kn079.htm>), where by the i -th capital (small) letter is denoted the situation in which strand s_{i+1} overcrosses (undercrosses) s_i ($i=1,2,\dots,n-1$).

The following example shows a 3-stranded braid obtained by the command ***braid aBBa*** written in the command line. After clicking the button ***Close*** and closing the braid, the knot or link obtained as the braid closure can be relaxed in the ***Main*** menu by the command ***go***.



Finally, we look at the palette **TANGLE**. Tangles are the basis of the Conway notation for knots and links.

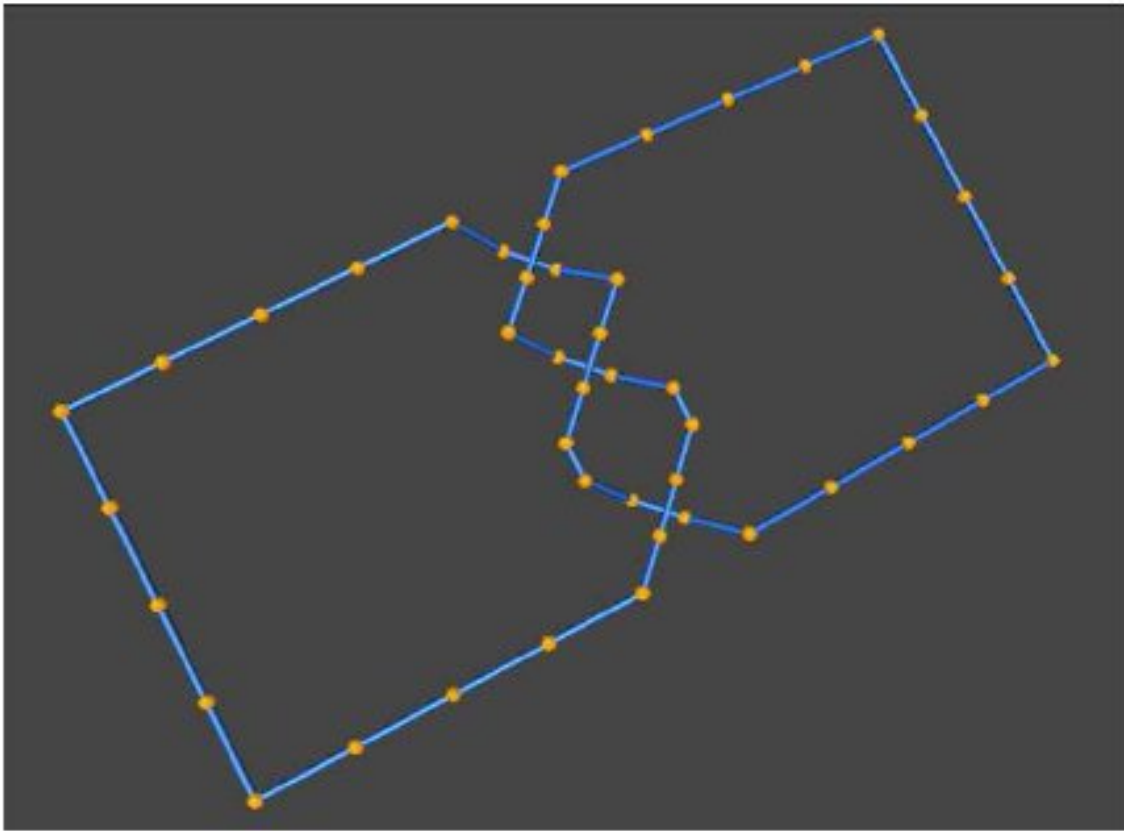
As *Primitive tangles* we are using n -twists (chains of bigons). By choosing tangle 1 we obtain the elementary tangle 1. Then we can choose tangle 2 (horizontal bigon). If we would like to join them we use the binary tangle operators.



The first operator # is the sum of tangles. From the open-ended tangle $1\#2$ we obtain the corresponding knot as a numerator (**N**) or denominator (**D**) closure, by clicking on the **N** or **D** button.



After this the image will disappear, but we can open it in the *Main* menu.



As before, we can smooth the knot and then relax it by the command *go*.



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r      Conway's reflect and rotate operator
+ -    Positive and negative twists on bottom of tangle
x y z  Reflections in one direction or another
Z      Alternate version of z command
@      CCW rotation by 90 degrees

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#      Direct sum (tangle 1 is the left summand)
*      Conway's multiplication operator, equivalent to r#
&      Vertical sum, equivalent to @e@e#@@@

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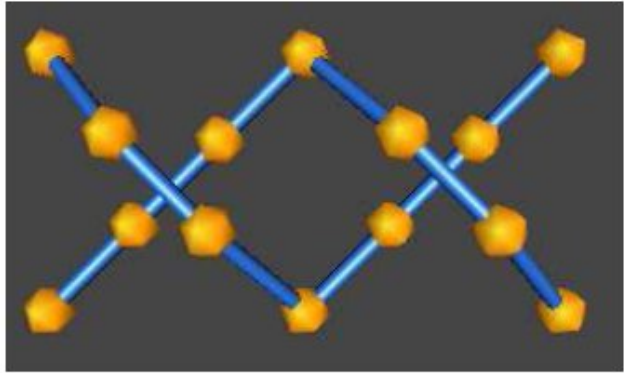
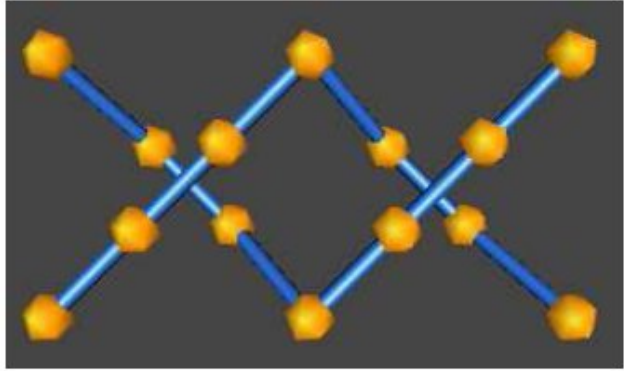
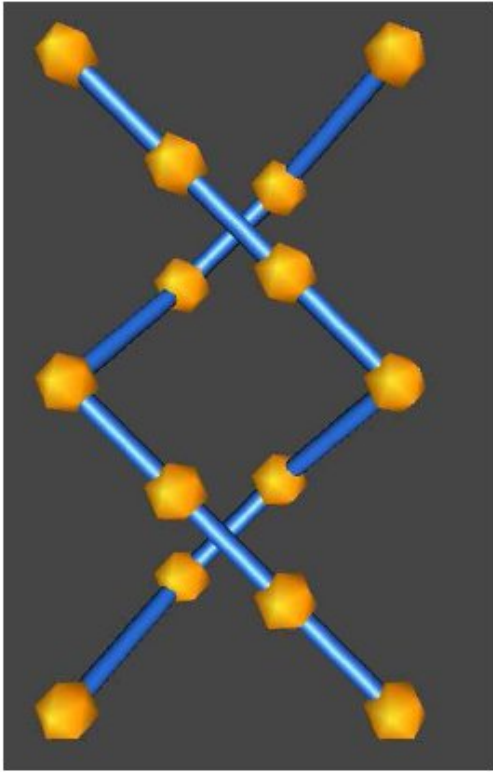
!      Duplicate bottom tangle (tangle 0)
~      Delete tangle 0
<      Shift up (or left)
>      Shift down (or right)
e      Exchange tangles 1 and 0

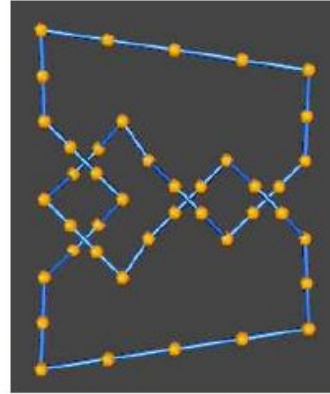
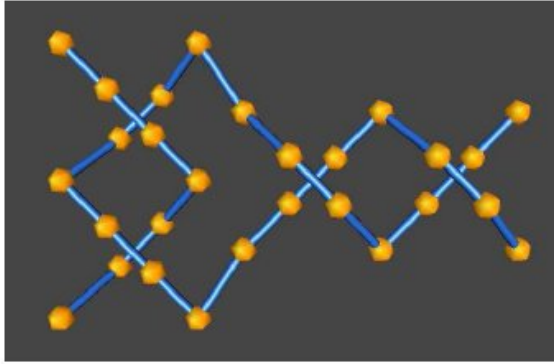
```

To obtain a figure-eight knot, given in Conway notation, you will input two tangle 2s. After drawing the first, input the second, rotate it by a 90° angle using the unary operator @ from the palette of unary operators, change its overcrossings to undercrossings by the unary operator Z, join the tangles obtained into one tangle by the binary operator #, and then make the numerator closure of this tangle. The sequence of commands necessary for obtaining a figure-eight knot (denoted as 2 2 in Conway notation) can be written in one line as 2#Z@2.

Unary operators

r	+	-	x	y	z	Z	@
---	---	---	---	---	---	---	---





Homework: Knots in KnotPlot

By using the program KnotPlot draw three knots of your own choice. Describe the construction for at least one of them.