

Module 7: A Golden Mean Tiling Design

We have seen in Module 4 that the golden mean relates a pentagon (see Fig. 1) where the diagonal of the pentagon is ϕ units when the edge is 1 unit where $\phi = \frac{1+\sqrt{5}}{2}$. Note that we are using the Greek symbol phi (ϕ) to represent the golden mean since that symbol is popular among artists and architects, but sometimes we will use the Greek symbol tau (τ) which is popular among mathematicians. We also find that if a square (S) is removed from a golden mean rectangle (GM) the result is another GM at a smaller scale, i.e., If $U = GM$ then $G = S$ (see Fig. 2). In this way, a sequence of squares can be removed from GM to result in a system of “whirling squares” as shown in Fig. 3.

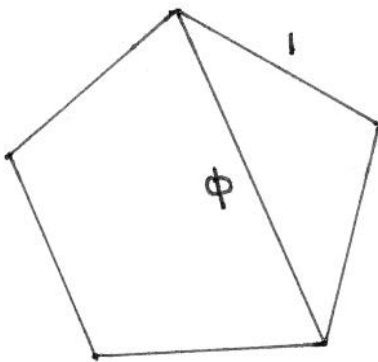


FIG. 1

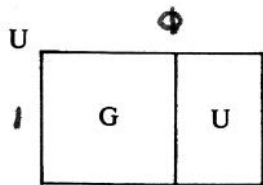


FIG. 2

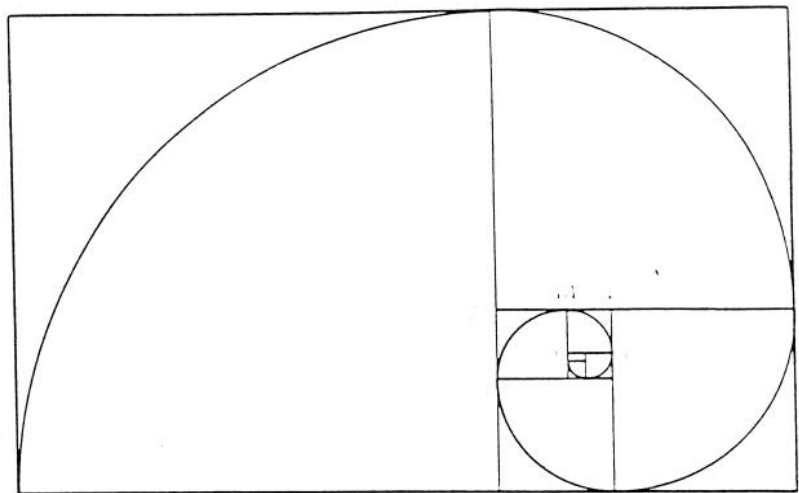


FIG. 3

Because the diagonal of a pentagon with edge 1 unit is ϕ units, the isosceles triangle in Fig. 4 has a base of 1 unit and sides of ϕ units and is referred to as golden triangle 1 or GT_1 . If the 72 deg. base angle of a golden triangle is bisected, a pair of golden triangles is the result. One of them is a GT_1 at a smaller scale and the other is GT_2 , an isosceles triangle with base ϕ units and side 1 unit. This construction can be repeated to result in golden triangles at smaller and smaller scales.

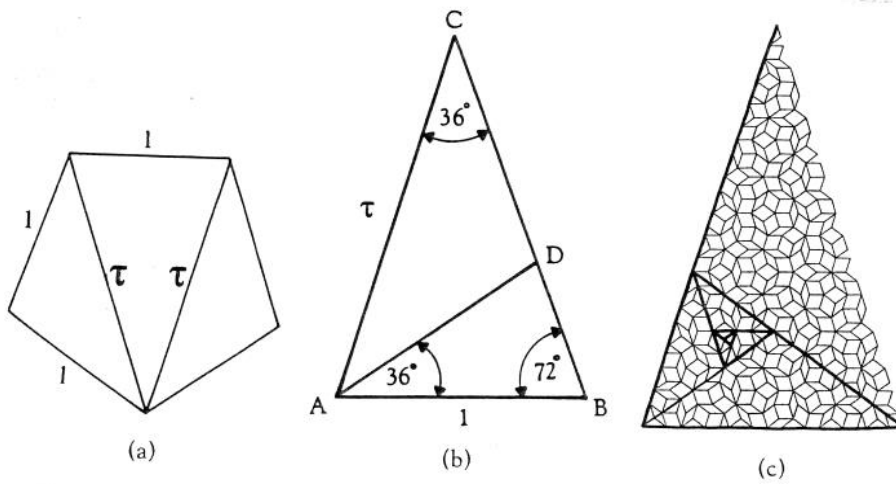
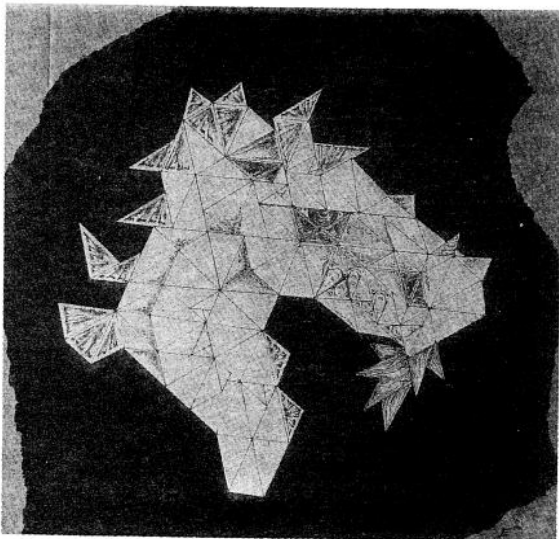


FIG. 4 (a) A pentagon subdivided into one type 1 and two type 2 golden triangles; (b) the base angle of a golden isosceles triangle of type 1 is bisected to form a type 1 and type 2 golden triangle; (c) a pattern of "whirling" golden triangles with mirror symmetry formed by Penrose rhombuses.

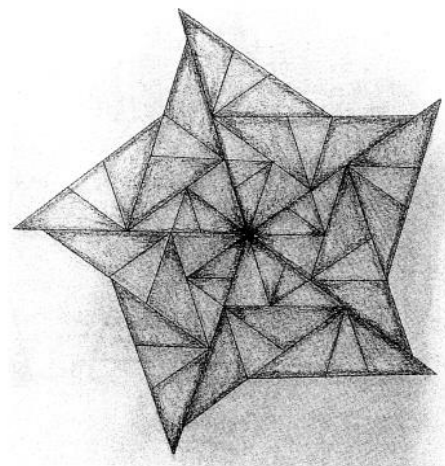
Remark: Because of these geometric properties, golden triangles and rectangles are said to be self-similar, and the golden mean is said to be the essence of self-similarity in geometry and nature.

Construction: Create a number of GT_1 's and GT_2 's at three different scales and use them to create a tiling design. Two such designs are shown in Fig. 5: Elyse's Dragon by Elyse Grady and the other in Fig. 6 by Eileen Domonkos.



(a)

FIG. 5



(b)

FIG. 6

FIG. 5

Two designs created from golden triangles of type 1 and 2. (a) "Elyse's Dragon" (b) "Fivefold-Symmetry".