CHAPTER 20

DETERMINING MAXIMUM STRESS ON A BEAM

20.1 MAXIMUM STRESS ON A BEAM

If a beam is subjected to a stress beyond its capabilities, it will rupture. By stress we mean an internal force per square foot of the beam cross section. Clearly steel can endure stresses far beyond wood. The maximum stress that beams made of various materials can withstand is shown in Table 1. In this lesson we will show how to compute the maximum stress on a beam due to the external forces imposed on it.

Table 1

Material	Maximum stress (psi)
Wood	1500 - 1600
Steel	24,000 – 33,00
Concrete	1350 – 1800 (in compression only)

For beams with small deflections, it can be shown that the stress varies linearly across the cross section of the beam with zero stress on the neutral axis at the midline of the beam with half of the beam under tension (with negative stresses) and the other half under compression (with positive stresses) and with the maximum stress occurring at the upper and lower surfaces of the beam. The more the bream bends the greater is the stress. Therefore it is not surprising that the maximum stress is directly proportional to the bending moment. The geometry of the beam is also important, in particular the maximum stress will depend on the width and depth of the beam. We will now derive the formula for maximum stress.

20.2 A FORMULA TO DETERMINE MAXIMUM STRESS

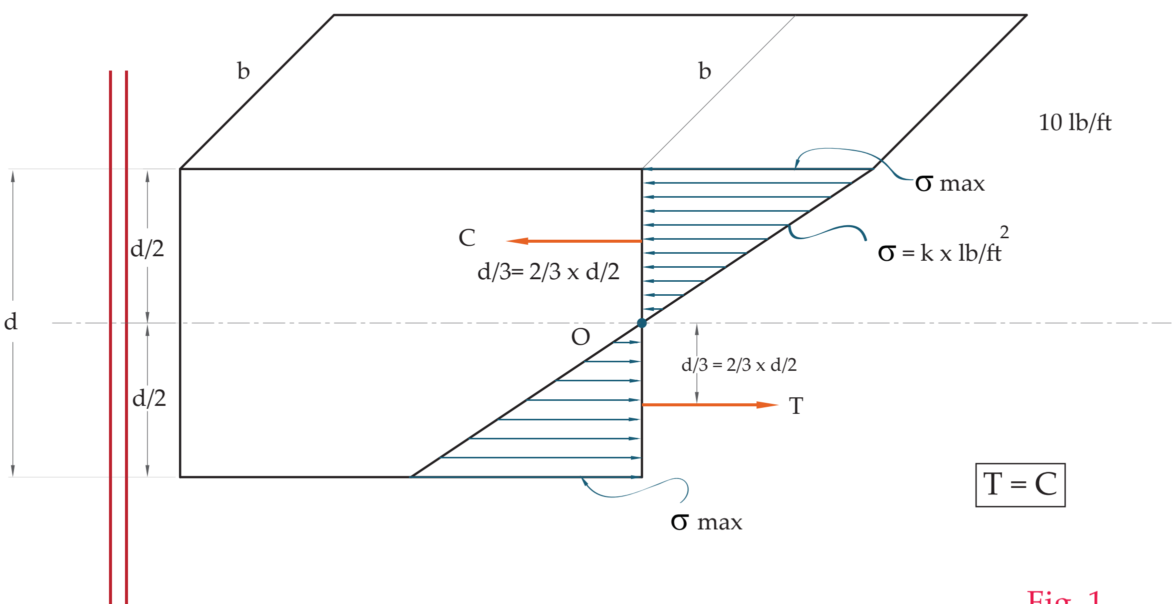

Fig. 1

We denote the stress by σ in units of pounds per square inch (psi or lbs/in^2), the width of the beam by b in, the depth by d in, and as usual the bending moment by $\overline{M}$ lb-in. The cross section of the beam is shown in Fig. 1 where the beam is shown in a state of bending where the compressive forces are in the upper half and tension forces in the lower half of the beam. The origin at O is chosen to be the center of the beam on the front face. In this coordinate system the linear stress is then given by $\sigma = kx$ in its variation across the depth of the beam while it is constant across the width of the beam.. Stress can be thought of as a new kind of force density. Just as the continuous force density ρ in Chapter 8 extended across the length of the beam, and the total force resulting from this density is the area under the ρ vs x curve, the total compressive force C is the volume of the triangular prism that makes up the upper half of the beam in Fig. 1. The total tension force T is the volume of the triangular prism in the lower half and equals C in magnitude but is oppositely directed. C can be thought of as acting at the center of gravity x_c of the upper triangle which is 2/3 the value of half the depth, i.e.,

$$x_c = (\frac{2}{3})(\frac{d}{2}) = \frac{d}{3}$$

Since the volume of the triangular prism is (area of the triangle) x (width),

$$C = \frac{1}{2} \times \frac{d}{2} \times \frac{kd}{2} \times b = \frac{db\sigma_{\max}}{4} \qquad (1)$$

where, since the maximum stress can be seen to take place at the upper edge of the prism,

$$\sigma_{\max} = k(\frac{d}{2})$$

Since $\overline{C}$ acts in the negative direction, $\overline{C} = -C$. Also clearly the magnitude of the compressive forces C equals the tensile forces T, but $\overline{T}$ is oppositely directed from $\overline{C}$ so that $\overline{T} = T$

Taking moments about O with the direction of the moment being clearly
10 counterclockwise,

$$M = \frac{d}{3}T + \frac{d}{3}C$$

and since C = T,

$$M = \frac{2d}{3}T \qquad (2)$$

Remark: The moment that we just computed can clearly be identified as the bending moment as defined in Chapter 7.

Replacing Eq. 1 in Eq. 2 and simplifying,

$$M_{\max} = \frac{d^2 b}{6}\sigma_{\max}$$

or solving for $\sigma_{\max}$,

$$\sigma_{\max} = \frac{6M_{\max}}{d^2 b} \qquad (3)$$

From Eq. 3 we see that the maximum stress is directly proportional to the bending moment and inversely proportional to the width and the square of the depth. For a given beam, the maximum stress can be computed from Eq. 3; if it exceeds the maximum allowable stress for the material of the beam as stated in Table 1, then the beam will rupture.

20. 3 AN EXAMPLE TO DETERMINE MAXIMUM STRESS ON A BEAM

Consider the following beam and predict whether is will or will not rupture.

Example:

A 4 inch by 10 inch wood beam spans a distance of 16 ft and carries a uniform load of 200 lb/inch including its own weight (see Fig. 2). Find the maximum stress in bending.

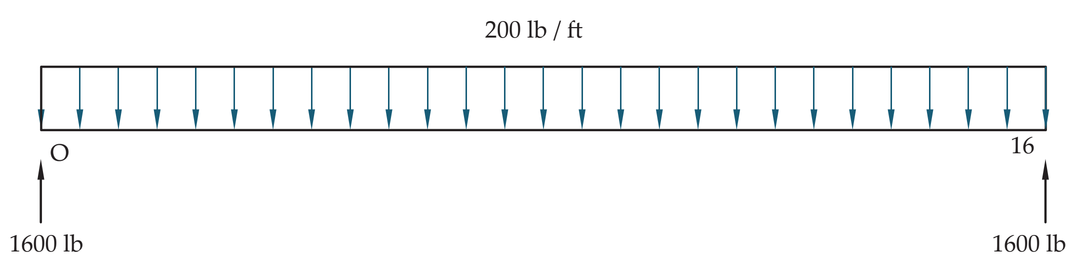

Fig. 2

The total force is 3200 lb. and each reaction force must be 1600 lb. The maximum bending clearly occurs in the middle of the beam so we will calculate $\overline{M}_{\max}$ directly. Consider the portion of the beam on the interval [0,8] shown in Fig. 3. The total force on this interval is 1600 lb. acting at the center of gravity, $x_c = 4$. Since $\overline{M}$ has a maximum at x = 8, it follows that $\overline{V} = 0$ (why?) Therefore the balance of moments about O (x = 0) is,

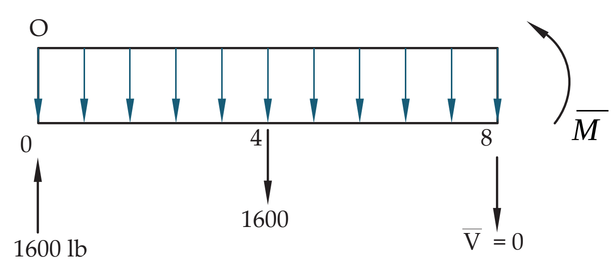

Fig. 3

$$- (1600)(4) + M = 0 \quad \text{or} \quad \overline{M} = 6400 \text{ lb-inches.}$$

Note: We are using inches as the unit of length rather than feet.

Replacing M in Eq. 3,

$$\sigma_{\max} = \frac{6(6400)}{(4)(10^2)} = 1152 \text{ psi (pounds per square inch)}$$

From Table 1, maximum stress that can be sustained by wood is 1500 – 1600 psi. Therefore this beam will not rupture.