CHAPTER 6

BASICS OF STRUCTURES

6.1 INTRODUCTION

The disciplines of Physics, engineering, architecture, and mathematics come together in the study of structure. The physicist thinks in the language of equilibrium of forces and applications of Newton's three laws. The engineer wishes to apply basic physics to create designs that satisfy an assortment of constraints. The architect wishes to apply these principles to a study of why buildings stand up. Mathematicians, on the other hand, see a common language that spans all of these fields and tries to find principles that can be abstracted and generalized. In this course the mathematician will use calculus to shine a light on the study of structures. In this chapter we consider the basics of structures which we will then follow through the remainder of the book..

6.2 EQUILIBRIUM OF FORCES AT A POINT

Consider five forces acting at a single point as shown in Fig 1.

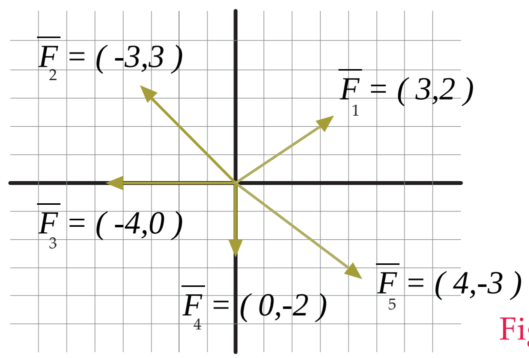


Fig. 1

A force $\overline{F}$ can be represented by a *vector*. Vectors will always be written with superlines. Vectors are quantities with magnitude and direction. If a vector is moved without changing its magnitude and direction, it is considered to be the same vector. Any force (vector) $\overline{F}$ acting in 2- dimensional space can be broken down into the sum of a vertical component $\overline{V}$ and a horizontal component $\overline{H}$. For example in Fig. 2, $\overline{F} = \overline{V} + \overline{H}$

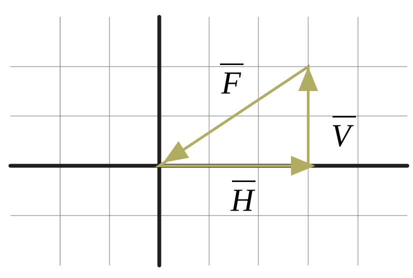


Fig. 2

where, $\overline{F}$ = (3,2) = (3,0) + (0,2) = $\overline{H} + \overline{V}$. Here the vector is named by the point in a cartesian coordinate system that the point intersects when the tail of the vector is at the origin of the coordinates. You can also name a vector by starting at the tail and walking to the tip. The first component is the number of units that you go right (positive) or left (negative) and the second component is the number units that you go up (positive) or down (negative).

Notice that this triangle is a right triangle so that we can use the *Pythagorean theorem* to find the magnitude of the force, $|\overline{F}|$, i.e.,

$$|\overline{F}| = \sqrt{3^2 + 2^2} = \sqrt{13}$$

Let's move the vectors in Fig. 1 to the ones in Fig. 3.

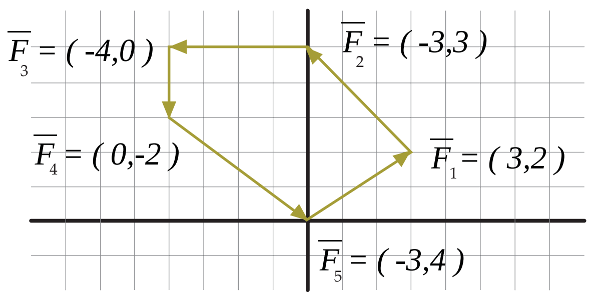


Fig. 3

Notice that the vectors (forces) form a closed polygon. This means that the original forces are at *equilibrium* which means that the point upon which they are acting does not move. Add up the horizontal and vertical components to get:

$$\overline{H_1} + \overline{H_2} + \overline{H_3} + \overline{H_4} + \overline{H_5} = (3,0) + (-3,0) + (-4,0) + (0,0) + (4,0) = (0,0)$$

and,

$$\overline{V_1} + \overline{V_2} + \overline{V_3} + \overline{V_4} + \overline{V_5} = (0,2) + (0,3) + (0,0) = (0,-2) + (0,-3) = (0,0)$$

Now add up all the forces,

$$\overline{F_1} + \overline{F_2} + \overline{F_3} + \overline{F_4} + \overline{F_5} = (3,2) + (-3,3) + (-4,0) + (0,-2) + (4,-3) = (0,0)$$

So forces at equilibrium (no movement) have the property that the forces form a closed polygon, and the sums of the vertical and horizontal components equal zero.

Problem 1: On a piece of graph paper draw a polygon of five forces, name the vectors, find their lengths and check to see that the sum of the horizontal and vertical components are zero**.**

The vertical and horizontal components of a force can also be expressed in terms of sines and cosines. For example, for the vectors in Fig. 2 shown again in Fig. 4,

$\overline{F} = (3,2)$

$\overline{H} = (3,0)$

$\overline{V} = (0,2)$

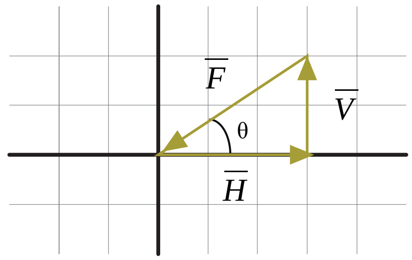


Fig. 4

$$|\overline{F}| = \sqrt{13}, \ |\overline{H}| = 2, \ |\overline{V}| = 3$$

From the trigonometry of a right triangle,

$$|\overline{H}| = |\overline{F}| \cos\theta$$
$$|\overline{V}| = |\overline{F}| \sin\theta$$

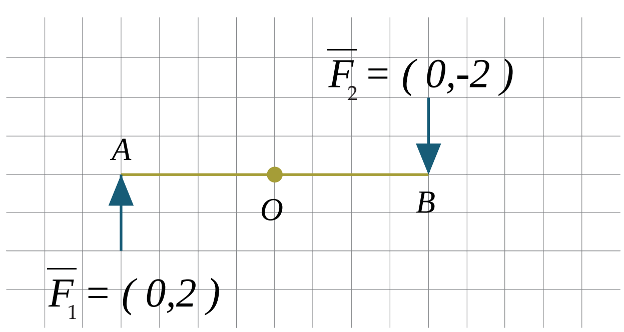
So if you know the angle at which the force is acting and its magnitude, you can find the magnitude of its horizontal and vertical components.

Problem 2: If $|\overline{F}| = 10$ and $\theta = 45\text{deg.}$, find $|\overline{H}|$ and $|\overline{V}|$

Problem 3: If $|\overline{F}| = 10$ and $\theta = 60\text{deg.}$, find $|\overline{H}|$ and $|\overline{V}|$

6.3 FORCES NOT ACTING A POINT

What if the forces do not act at a single point as in Fig. 5 where AB is a beam and O is a point on the beam.



Components of

Fig. 5

Clearly the horizontal and vertical components of forces, $\overline{F_1}$ and $\overline{F_2}$, balance (their sum equals (0,0)). But notice that if you place your finger at O the beam rotates clockwise around point O. So now we have to also consider a quantity known as the *moment* $\overline{M}$ which is the tendency to rotate around a point.

Consider a force $\overline{F}$ acting on a particle at point P causing the particle to rotate about an axis through point O in either a clockwise or counterclockwise direction in the plane defined by the line of action of the force and point O (see Fig. 6).

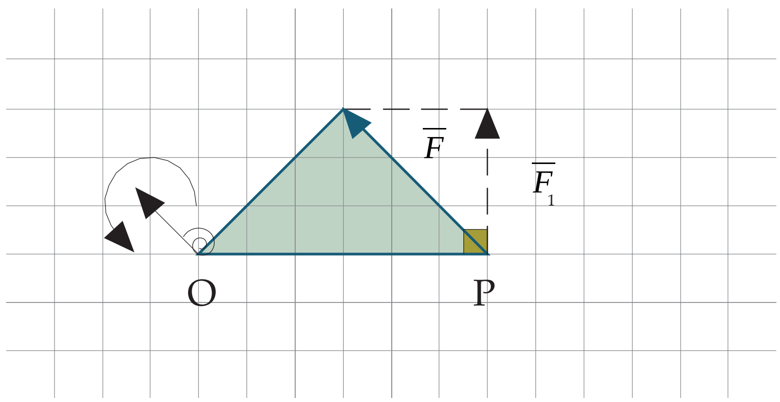


Fig. 6

The magnitude of the moment is defined as the magnitude of the component of the force that is perpendicular to the line segment OP multiplied by the distance |OP| or, what amounts to the same thing, it is the magnitude of the force $|\overline{F}|$ multiplied by the perpendicular distance from point O to the line of action of the force as shown in Fig. 6. For our purposes the particle at P will usually be a point on a beam, and if the force is acting perpendicular to the beam (see Fig. 7) then we can say that the magnitude of the moment $|\overline{M}|$ about O is computed as follows,

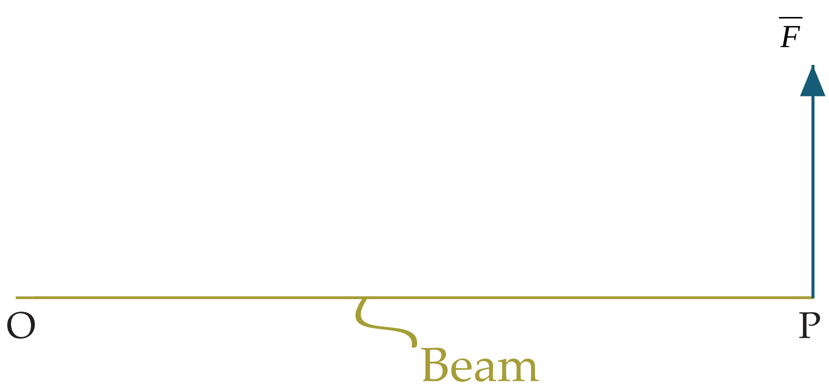


Fig. 7

$|\overline{M}|$ = (Magnitude of the force) x (perpendicular distance from the center of rotation to the line of action of the force) $= |\overline{F}|$ x |OP|

Notice that the moment has been written as a vector (i.e., it has a line on top). We establish the convention that the moment is positive if the rotation is counterclockwise and negative if the rotation is clockwise about O. We could have chosen the opposite convention that the moment is positive (negative) if the rotation is clockwise (counterclockwise) about O as some books do, but for reasons that will become evident in the next chapter, we will consistently stick to our convention. The moment can be represented by a vector that has a direction perpendicular to the plane of the rotation. If you take your right hand with thumb pointed in the direction of the axis of rotation then your fingers will curl in the direction of the rotation. So if the vector comes out of the paper the rotation will be counterclockwise (positive moment) , whereas a vector into the paper represents a clockwise rotation (negative moment).

The conditions for equilibrium are now that:

1) the sum of the horizontal components of the forces equal zero;
2) the sum of the vertical components of the forces equal zero;
3) the sum of the moments equal zero.

All three conditions must hold.

Problem 4: For the external forces acting on a beam in Figure 8a, b, and c determine the moment of these forces about point O.

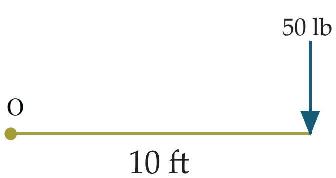


Fig. 8a

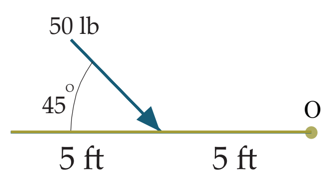


Fig. 8b

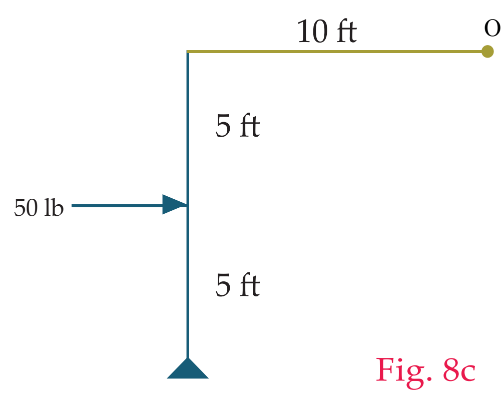


Fig. 8c

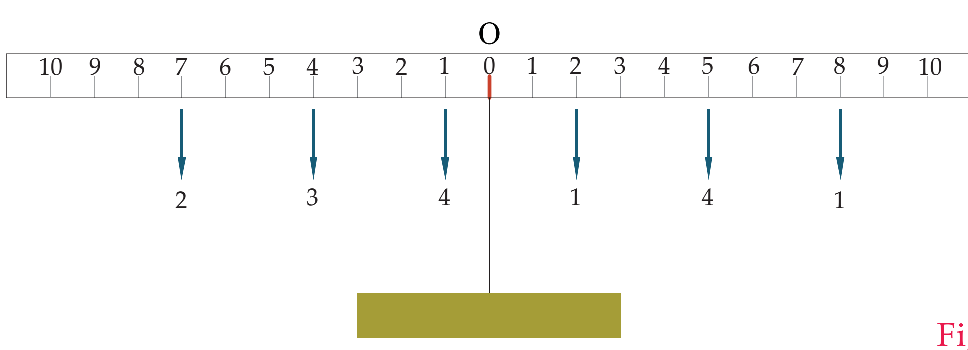


Fig. 8d

Consider the beam balance in Fig. 8d. Weights can be placed on hooks located at integral distances of between 1 and 10 units in either direction from the center post.. When the beam is loaded as it is in Fig. 8d, the moments about O located at the center post, the moment equation balances so the beam is at equilibrium. Show that the total moment equals zero.

6.4 THE BEAM

Generally, we will be considering a beam with one or several external forces, or loads as they are called, acting on it. The loads can either be acting at discrete points (i.e., *concentrated loads*) along the beam or acting *continuously* as would the gravity force be distributed due to the weight of the material that makes up the beam. The beam could also be supported by one or more supports not connected to a wall in which case we say the beam is *free-standing*. (See Fig. 9a). The beam might also have one end attached to a wall with the other end free which is referred to as a *cantilever* (see Fig. 9b).

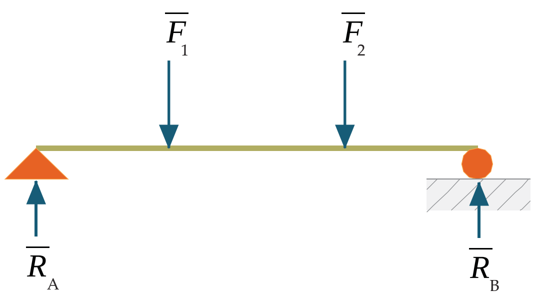


Fig. 9a

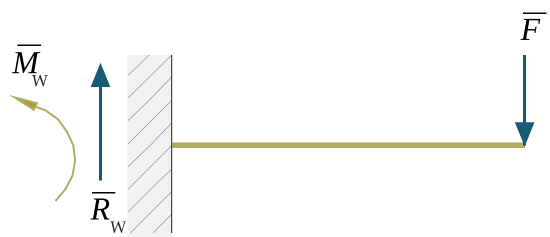


Fig. 9b

In response to the external forces $\overline{F}_1$ and $\overline{F}_2$ acting on the free standing beam in Fig. 9a and $\overline{F}$ for the cantilever beam in Fig. 9b there will be *reaction forces* due to the supports, as shown by $\overline{R}_1$, $\overline{R}_2$, and $\overline{R}_w$ due to the wall. The wall can also exert a reaction moment $\overline{M}_w$ on the beam.

For the remainder of these notes we will represent forces by their magnitudes and their directions. We adopt the convention that the vertical component of a downward pointed force is positive while the vertical component of an upward force is negative. This is opposite of the convention that we used at the beginning of this section. The reason for the change will become evident in the next chapter.

Forces pointed to the right have horizontal components that are positive while forces pointed to the left have horizontal components that are negative.

Using this convention, the force in Fig. 10a is $\overline{F} = (0,100)$ with magnitude $|\overline{F}|=100$ while each of the reaction forces is $\overline{R} = (0,-50)$ with magnitude $|\overline{R}|=50$ (magnitudes are always positive numbers). Since there is only a vertical component to the force, we represent the force by its vertical component only, i.e., instead of $\overline{F} = (0,100)$ we use the shorthand $\overline{F} = 100$ for the force. For the magnitude $|\overline{F}|=100$ of the force we eliminate the absolute value signs and the and simply write F=100 (no line on top). Instead of $\overline{R} = (0,-50)$ we say $\overline{R} = -50$ with magnitude R = 50. The external and reaction forces and moments for the cantilever beam are shown in Fig. 10b.

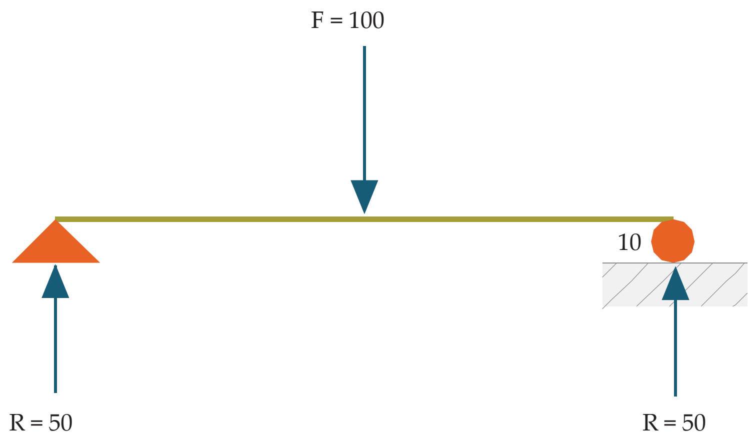


Fig. 10a

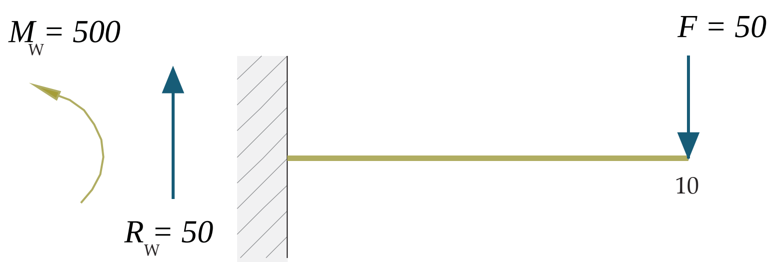


Fig. 10b

This can be confusing. In our diagrams we will always specify only the magnitude of the force so you will only see forces represented by positive numbers. If you want to find the vector value of the force you have to look at the direction of the force. For example, in Fig. 10a the force, $\overline{R}$, has a magnitude of R = 50 lb.and since it is represented by an upward pointed arrow, $\overline{R} = -50$. Strictly speaking it should be represented by (0, - 50) but we use this shorthand form. Likewise the force in Fig. 10b has a magnitude of $|\overline{F}| = 100$ represented by F = 100, and since the arrow points down, $\overline{F} = 100$ instead of the standard vector notation $\overline{F} = (0,100)$.

When summing the vertical components of the forces in Fig. 10a we have:

$$\overline{F} + \overline{R_1} + \overline{R_2} = 100 - 50 - 50 = 0$$

Problem 5: The magnitudes are given for the forces shown in Fig. 11. By observing the directions of the arrows, determine the values of the corresponding forces $\overline{F}$.

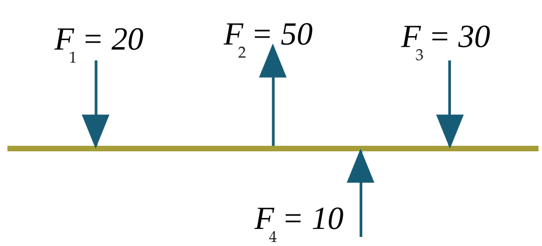


Fig. 11

The same convention is used for moments. The magnitude of the moment M is given in the force diagram along with an indication as to whether the moment is acting clockwise or counterclockwise. To get the vector moment $\overline{M}$ you use the convention that counterclockwise rotations give positive moments while clockwise rotations give negative moments.

6.5 DETERMINATION OF THE REACTION FORCES ON A FREE-STANDING BEAM

We are now in a position to determine the reaction forces of a free standing beam once the external forces on the beam are given. However, since there are three equations of equilibrium, we will only be able to solve uniquely for three reaction forces for any given beam loading. For example, consider the beam in Fig. 12.

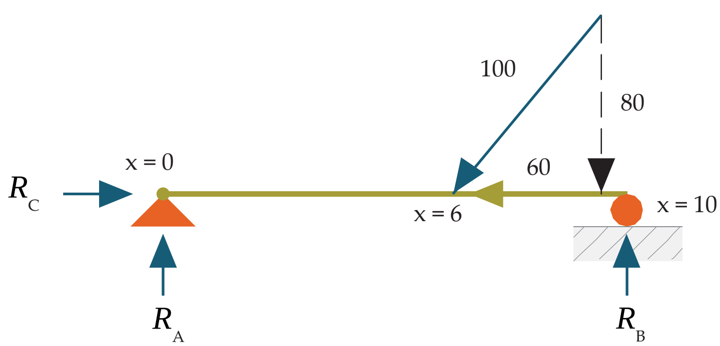


Fig. 12

Notice that there are two components of the reaction force, R_1 and R_3 acting on the left hinge The hinge at the left can exert both vertical and horizontal forces but cannot sustain moments. However, the roller support at the right can only exert a vertical force resulting in three unknown reaction forces, R_1, R_2, and R_3. These can be determined from the three equilibrium equations, and therefore this beam is called *determinate*. If we had two hinged supports then there would be four reaction forces to compute, and the system would be underdetermined. Such a beam and its loading is called *indeterminate*. There are more advanced ways of getting around this problem which we will not discuss in this book.

To calculate R_1, R_2, and R_3 for the beam in Fig. 12, we use the three equations of equilibrium.

1. Balance of horizontal forces: $R_3 - 60 = 0$ or $R_3 = 60$
2. Balance of vertical forces: $- R_1 - R_2 + 80 = 0$ or $R_1 + R_2 = 80$
3. Balance of moments about point O: $-(80)(6) + 10\, R_2 = 0$ or $R_2 = 48$.

As a result of the Equilibrium Equations 2 and 3, $R_1 = 32$. The fact that the magnitudes R_1, R_2, and R_3 came out positive means that the directions that we assumed for the forces were correct. If one of the reaction forces came out negative we would have to reverse its direction in the original diagram.

Remark 1: R_3 does not enter into the moment equation because its line of action goes through the point O of rotation.

Remark 2:

We have one opportunity to find the moment about any point on the beam. By choosing point O about which to evaluate the moments we eliminate the unknown R_1 from the moment equation (why?) and therefore we are able to solve directly for R_2. However, we would have gotten the same result by choosing O to be any other point at which to evaluate the moments. It would just have taken a little more math to solve for the reaction forces.

Problems: Compute the reaction forces shown in Fig. 13..

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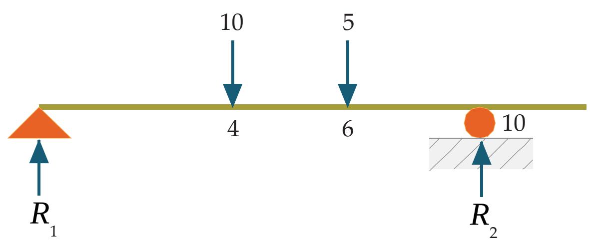

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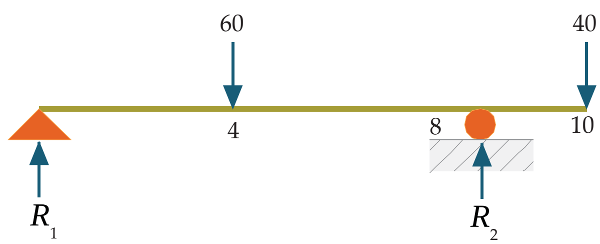

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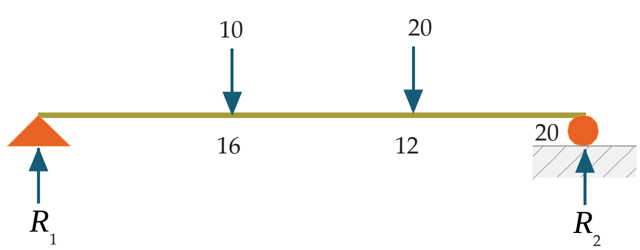

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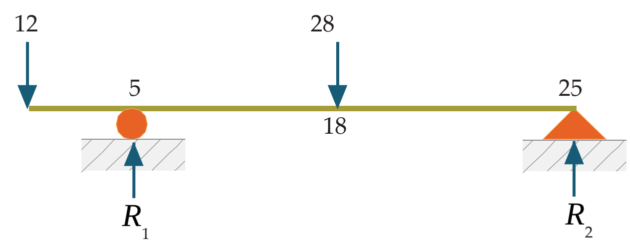

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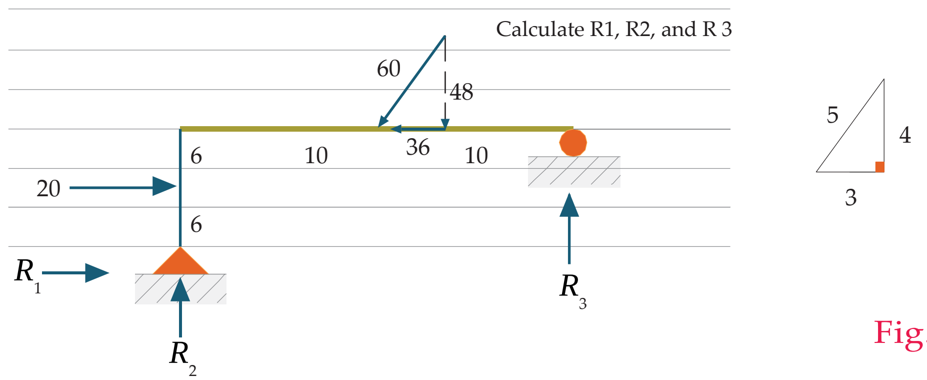

Fig. 13

6.6 STRUCTURAL SYSTEMS

The problem of evaluating the behavior of a structure can best be understood in the context of a system. How we define a system can change depending on the objective of the designer. We could consider the structural system defining a building to be either the entire building or some part of it, for example a ceiling beam in one of the rooms. Whatever we define as the system, there will be three kinds of forces acting on it: 1) external forces and moments due to the outside world acting on the structure such as forces of gravity, wind, or forces due to activities of people moving about within the building (live loads) or furniture within the room (dead loads); 2) reaction forces due to external supports of the earth or supporting walls or adjacent portions of the structure acting on the portion of the structure being considered; 3) internal forces and moments acting within the elements of the structure (these will be discussed in succeeding chapters). It is important to understand that however the system is defined, it must be in equilibrium with regard to these three kinds of forces.

In Fig. 14 a systems diagram is introduced showing that, in response to input forces and moments, the system reacts with certain outputs. The system itself is characterized by certain parameters that describe the nature of the material from which it is constructed and its inherent geometry. The system responds to the inputs by outputs which may include the amount of deflection of beams or information as to whether the structure is able to sustain the loads placed on it without failing.

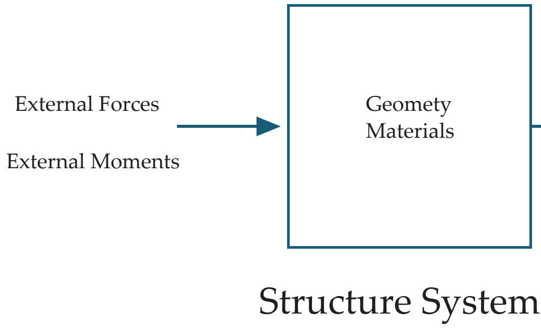

Fig. 14

Problem 11

In Fig. 15a, an indeterminate system is shown in which four reaction forces are acting on the structure. Since only three reaction forces can be computed from the three equations of equilibrium, we are at an impass. However, this situation is remedied in Fig. 15b by introducing a hinge at point A and replacing the beam system by a pair of systems separated at the hinge. (see Fig.15c). Determine the reaction forces for the hinged system. Hint: For the first subsystem take moments about O_1 and for the second subsystem take moments about O_2.

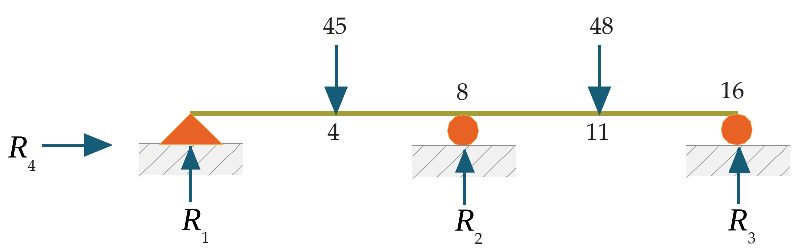

Fig. 15a

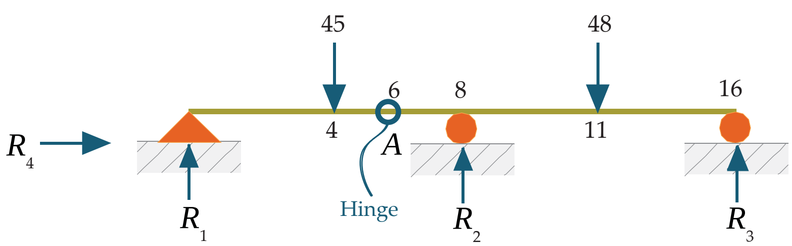

Fig. 15b

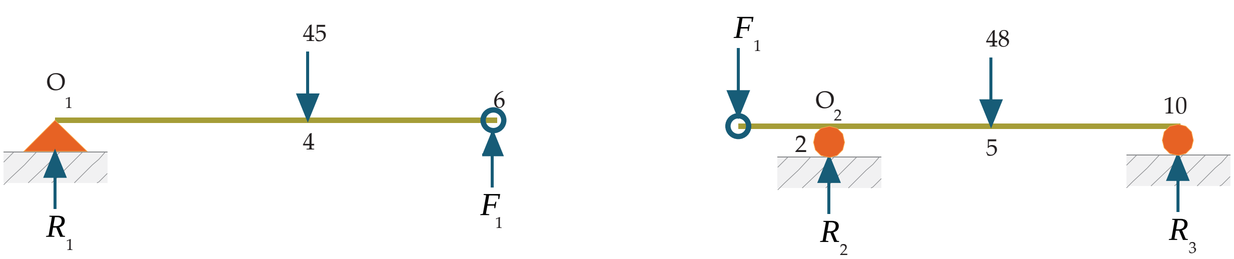

Fig. 15c

Problem 12: The beam system in Fig. 16a is indeterminate. A hinge is introduced at point A in Fig. 16a and the system is then subdivided into two subsystems in Fig. 16b each of which is now determinate. In this way an indeterminate system can be made determinate. Try your hand at computing the reaction forces for this pair of subsystems.

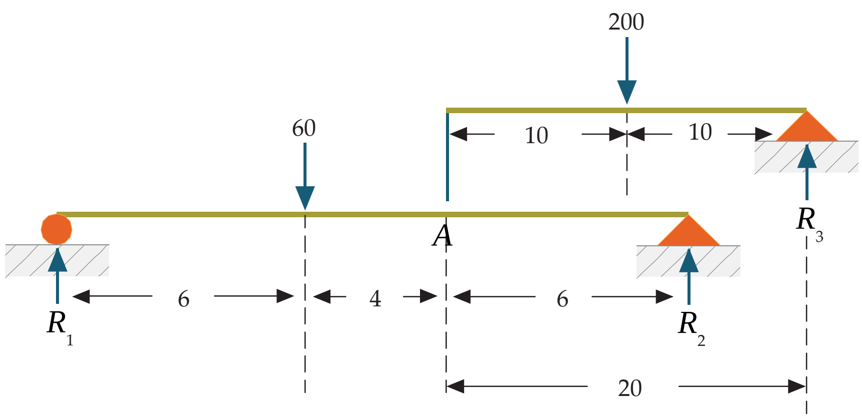

Fig. 16a

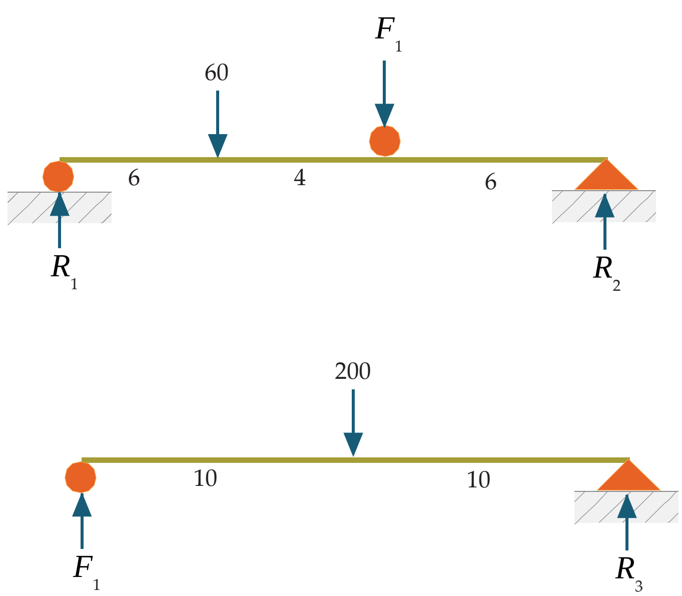

Fig. 16b

Problem 13:

The right angle bar structure labeled ABC is shown in Fig. 17a. The hinges at A and C support two vertical and two horizontal reaction forces; therefore the system is indeterminate. In Fig. 17b, the structure is divided into two subsystems at point B. A pair of undetermined forces F_1 and F_2 act at the point B where the system has been divided into two subsystems. If you count the undetermined forces on the pair of subsystems you will see that there are six. In addition, an unknown moment acts at B making seven unknowns but only six equations (three equilibrium equations for each subsystem). This indeterminacy is finally resolved in Fig. 17c where a hinge is placed at B and the system is divided into a pair of subsystems. Since hinges do not support moments, there are now six unknowns and six equations. Your job is to compute these six unknown forces.

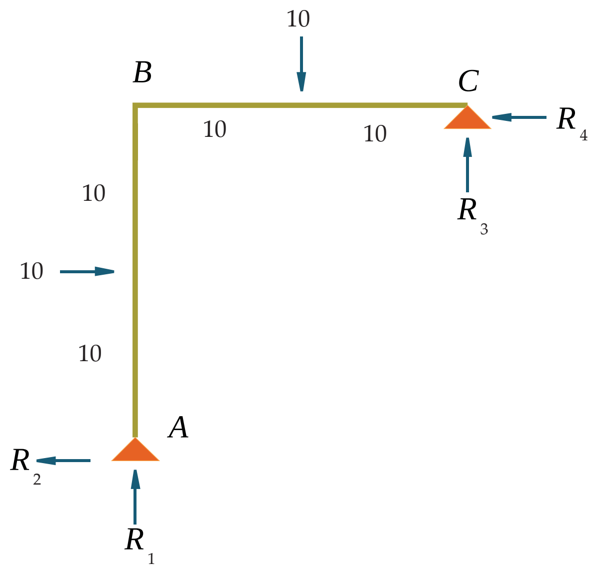

Fig. 17a

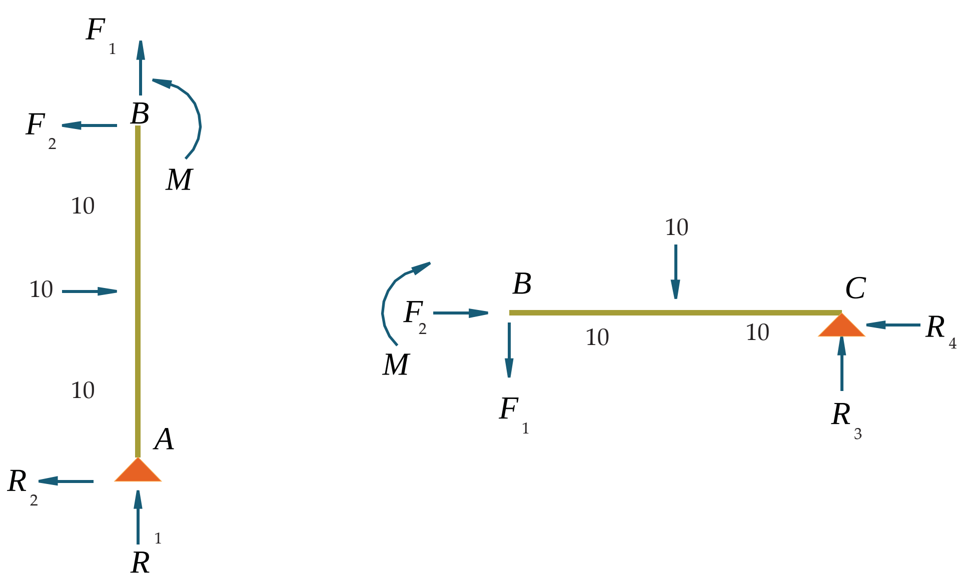

Fig. 17b

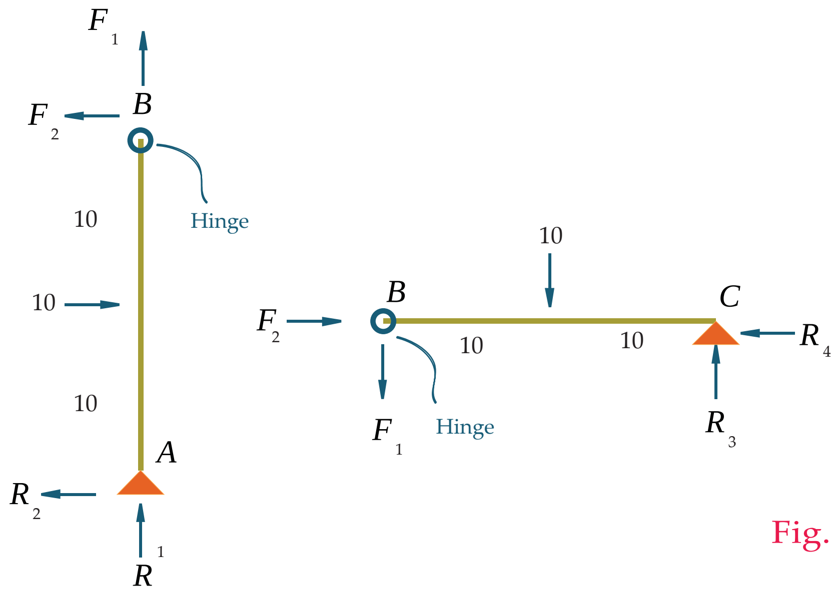

Fig. 17c