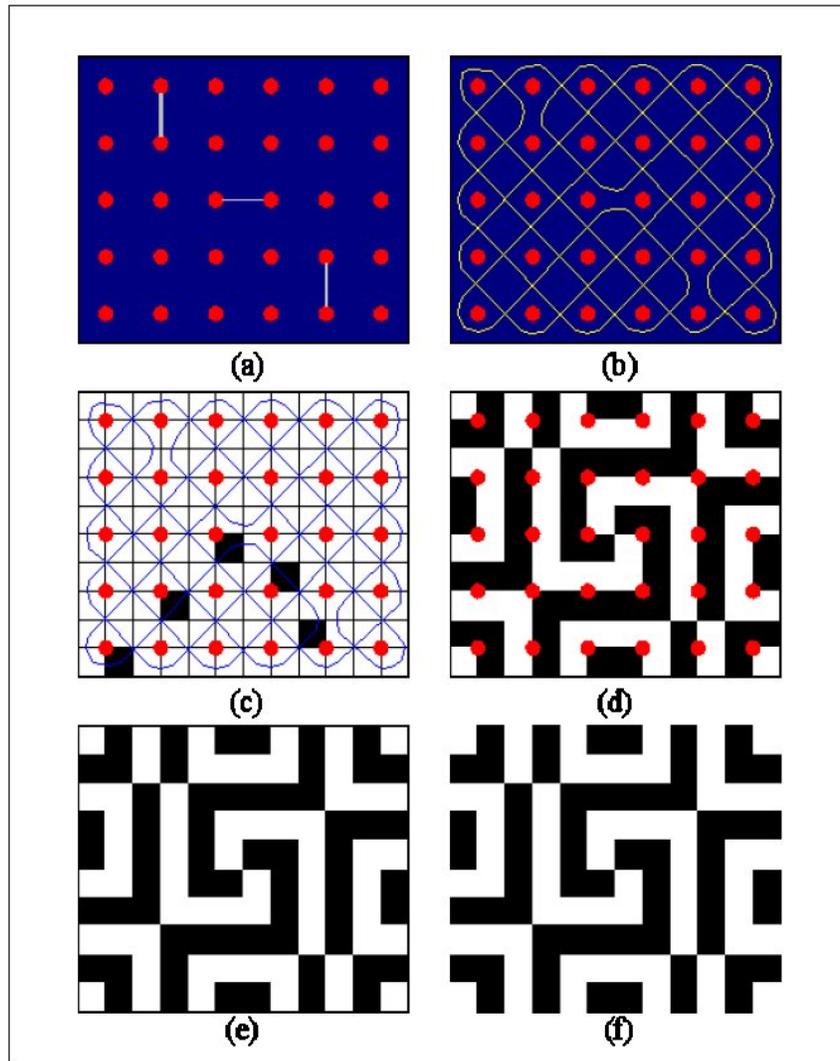


LUNDA DESIGNS

by Ljiljana Radovic

After learning how to draw mirror curves, we consider designs called *Lunda designs*, based on monolinear mirror curves. Every red dot in $RG[a,b]$ is the common vertex of four small squares (called “fields”) surrounding it. Every mirror curve contains diagonals of these small squares. If we trace a monolinear mirror curve and follow the sequence of adjacent diagonals which it contains (called *steps*), after coloring small squares corresponding to the successive steps in the alternating manner (black-white-black-white...), we obtain a “black-white” design, named by P. Gerdes “Lunda designs” (Fig. 1).



In order to draw a Lunda design, we first choose an arrangement of mirrors that results in a monolinear mirror curve. The method of construction is described in the exercise “Mirror curves”. Then we trace the mirror curve and color the small squares (Fig. 2). In our drawing we started coloring from the second small square in the bottom row (the first square colored black), then followed the mirror curve alternating colors, black-white-black.... After closing the curve, we remove the red dots and obtain a black-white design much like Neolithic, black-white antisymmetrical patterns.

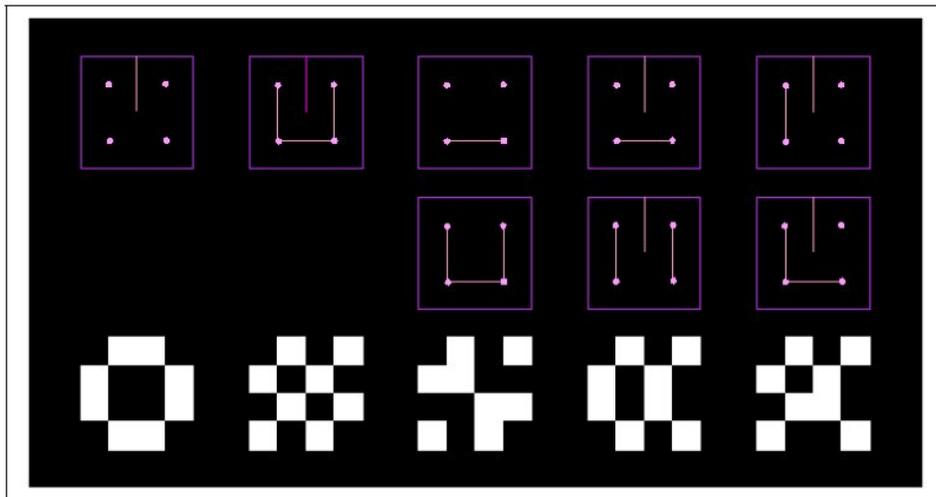
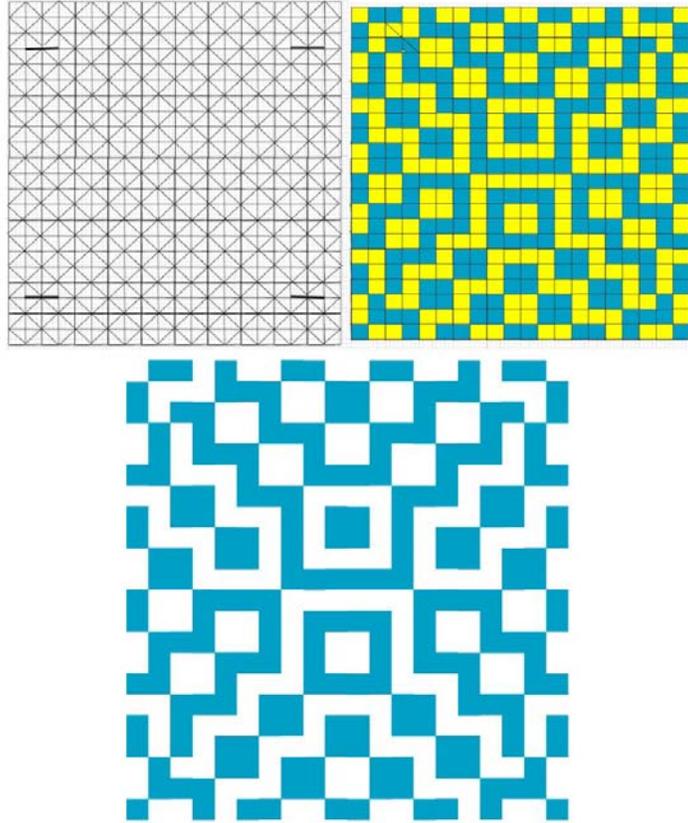
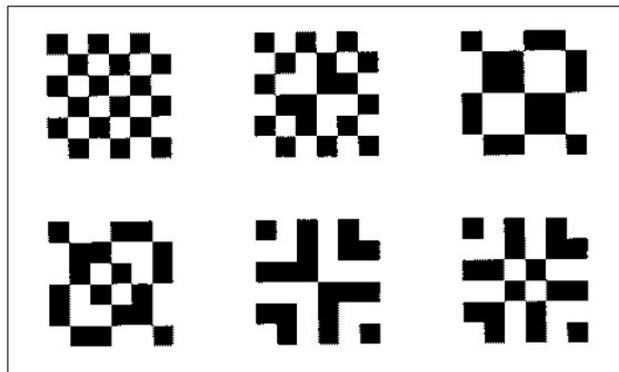


Figure 3 shows different arrangements of two-sided mirrors in the square grid 2×2 , and their corresponding Lunda designs. Notice that the correspondence between mirror curves (i.e., mirror arrangements) and Lunda designs is many-to-one: different mirror curves can give the same Lunda design. Try to find the criterion in which two or more different (up to isometry) mirror arrangements result in the same Lunda pattern. If two mirror arrangements M_1 and M_2 giving the same Lunda design, find the transformation rules (moves of mirrors) converting M_1 to M_2 .



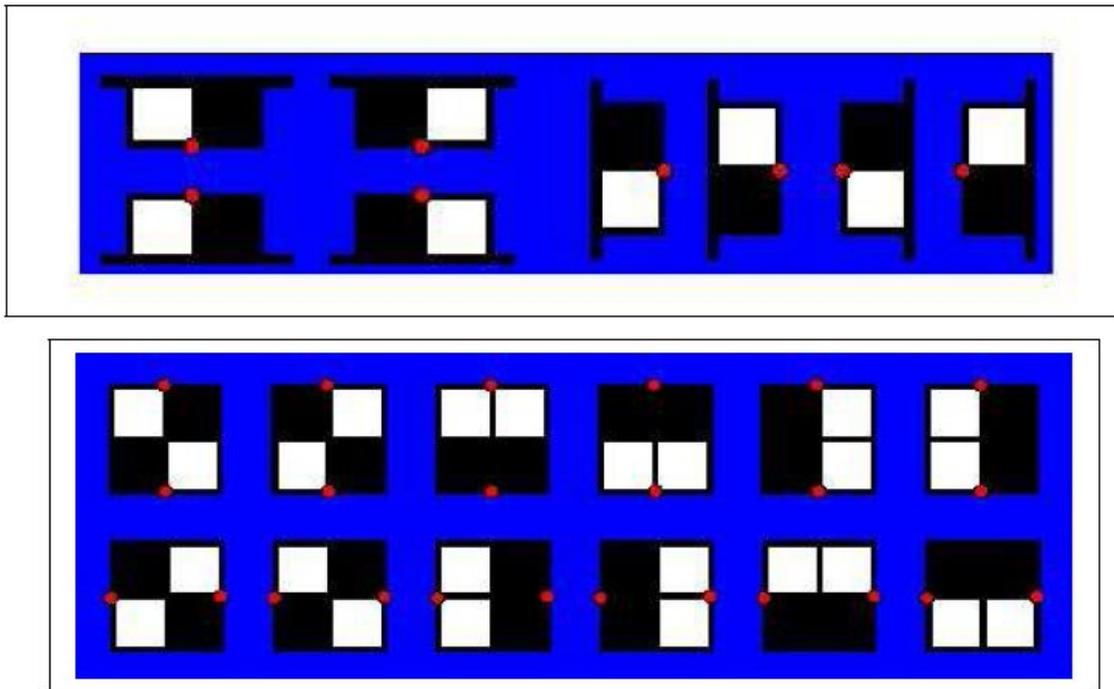
In the 10×11 grid, place four mirrors as shown in Figure 4; draw the mirror curve; follow the steps; and color small squares (fields) by two colors (yellow and light blue) in an alternating manner. The result is an antisymmetrical design which has vertical mirror reflection and horizontal anti-reflection (reflection in horizontal mirror which changes colors).



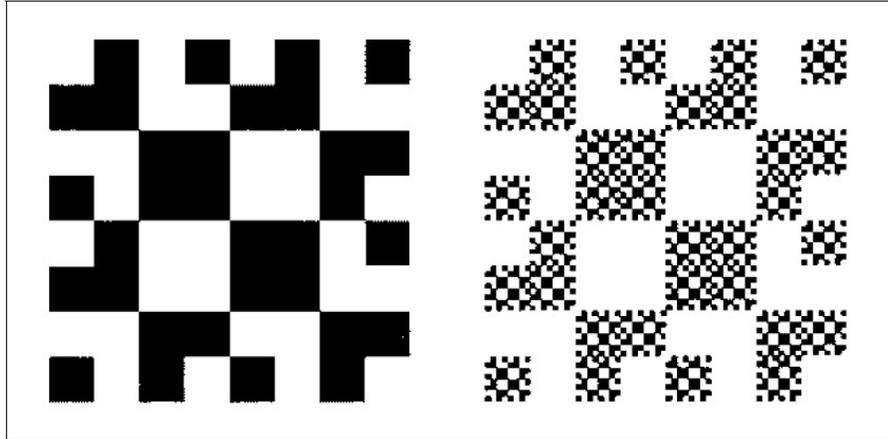
The same construction can be applied to an arbitrary mirror curve. If we count the steps along a mirror curve and numerate them by $1, 2, 3, 4, \dots$, by taking all numbers modulo 2, we obtain the sequence $1, 0, 1, 0, \dots$ which corresponds to the alternating coloring of steps “black”, “white”, “black”, “white”,... By taking all numbers in the sequence $1, 2, 3, 4, \dots$ modulo 4, we obtain the sequence $1, 2, 3, 0, 1, 2, 3, 0, \dots$. After coloring all fields denoted by

the same numbers, we obtain a 4-colored Lunda design. In the same way, we can work with 8, 16,... colors. With two colors we obtain antisymmetrical rosettes with one or two antisymmetry axes, and if we use more colors (i.e., 2^n colors, $n>1$) we obtain so-called multiple antisymmetry.

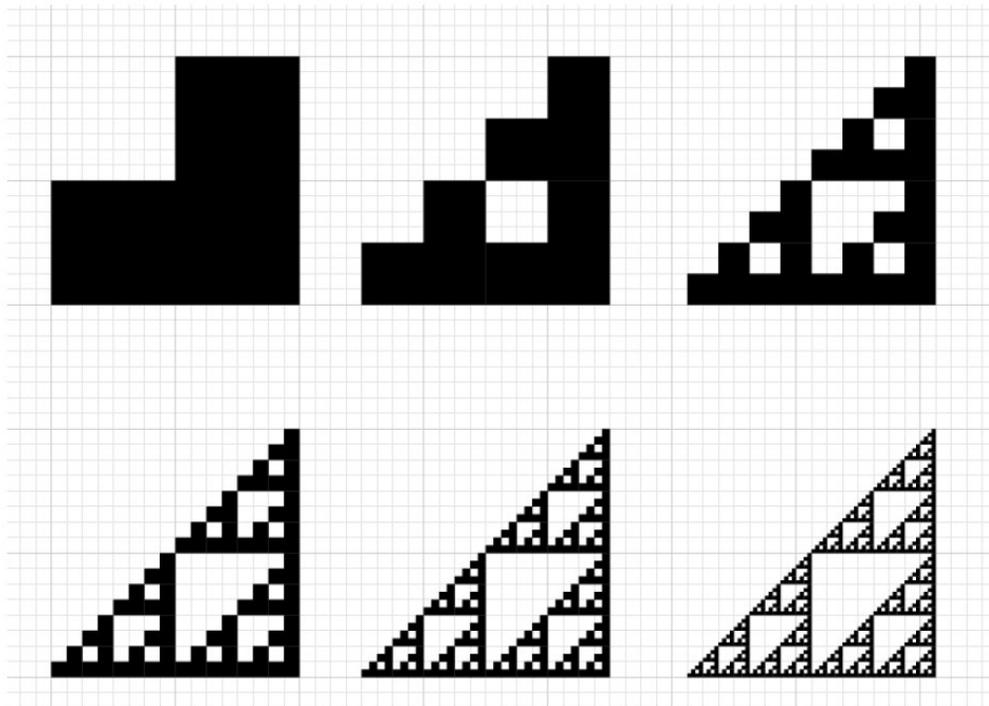
On the border of $RG[a,b]$, every red dot is the common vertex for two adjacent small squares (fields), and in the interior of $RG[a,b]$ every square of dimensions 2×2 can be divided in two rectangles of dimensions 2×1 or 1×2 , placed between two red dots, each of them containing one black and one white field. Hence, from this local arrangement results the global arrangement: the number of black fields in any row or column is equal to the number of white fields (Fig. 5). Translated into the language of numbers, if we denote black fields by 1, and white squares by 0, and consider Lunda designs as matrices of ones and zeros, we obtain so-called Lunda matrices with the amazing property: sum of the numbers in every row is a , and the sum of numbers in every column is b . In particular, for $a=b$, we obtain square 0-1 matrices giving the same sum of numbers in each row and column.



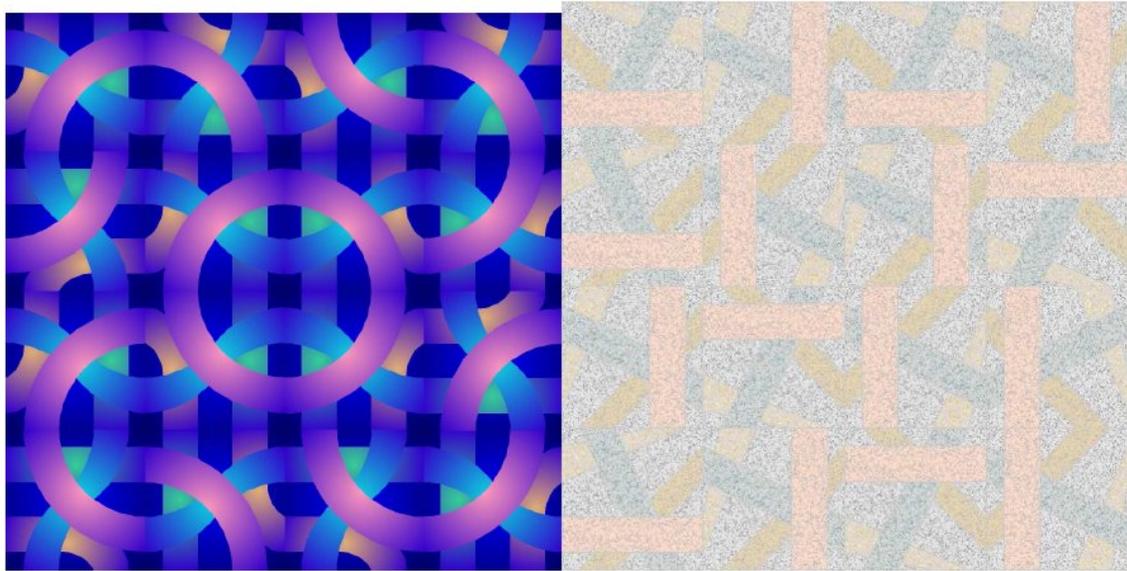
Another possibility for creating visually interesting designs is the derivation of fractals from Lunda designs. As a result, we obtain so-called Lunda fractals (Fig. 6). If the same algorithm used for the creation of initial Lunda design is applied again at a smaller scale, we obtain self-referential Lunda designs, i.e., Lunda fractals. Lunda fractals can be easily drawn in any drawing computer program. After making an initial Lunda design, we can scale it (in ratio 1:4), and then apply the same construction rule used for the creation of the initial design to the scaled parts.



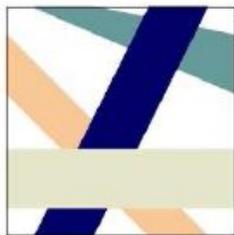
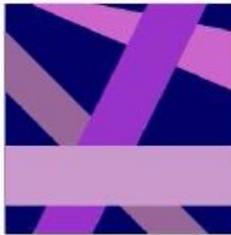
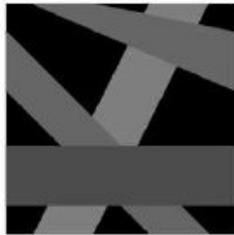
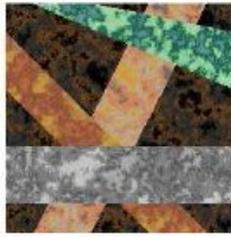
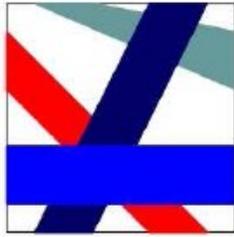
In *Inkscape* you can do a similar thing (Fig. 6): beginning from a black square of dimensions 2×2 with deleted upper left field (which is not a Lunda design, since the number of black squares in each row or column is not the same), you can scale it (in the ratio $1:2$) and then organize the three scaled images, into the same pattern as the initial motif was made from three black squares. Continuing to apply the same algorithm, we obtain one of the most famous fractals called the Sierpinski triangle. We will learn more about fractals in future.



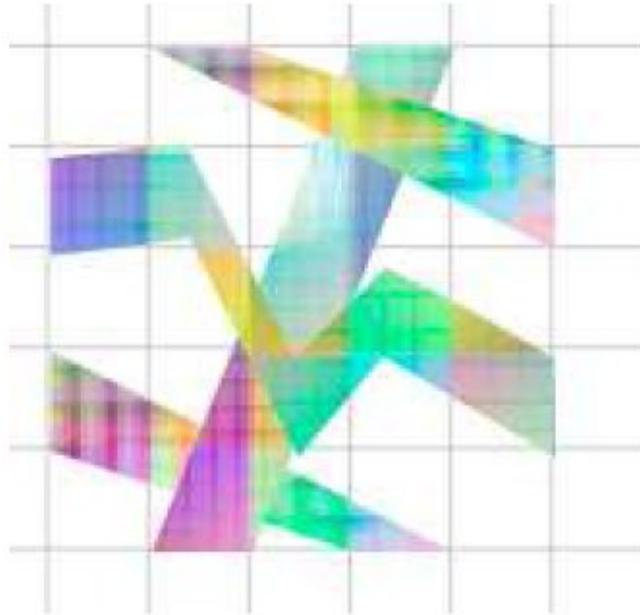
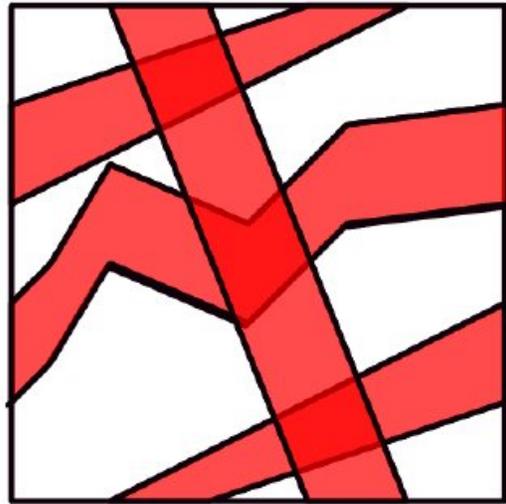
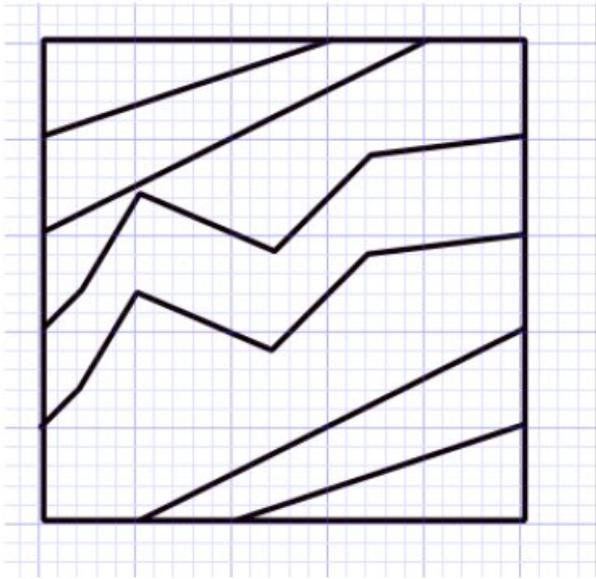
At the end of this exercise you can reconsider modular games “OpTiles”, “OrnTiles” and “KnotTiles” created by S. Jablan (<http://www.mi.sanu.ac.rs/vismath/op/tiles/index.html>), trying to create your own sets of modular elements.

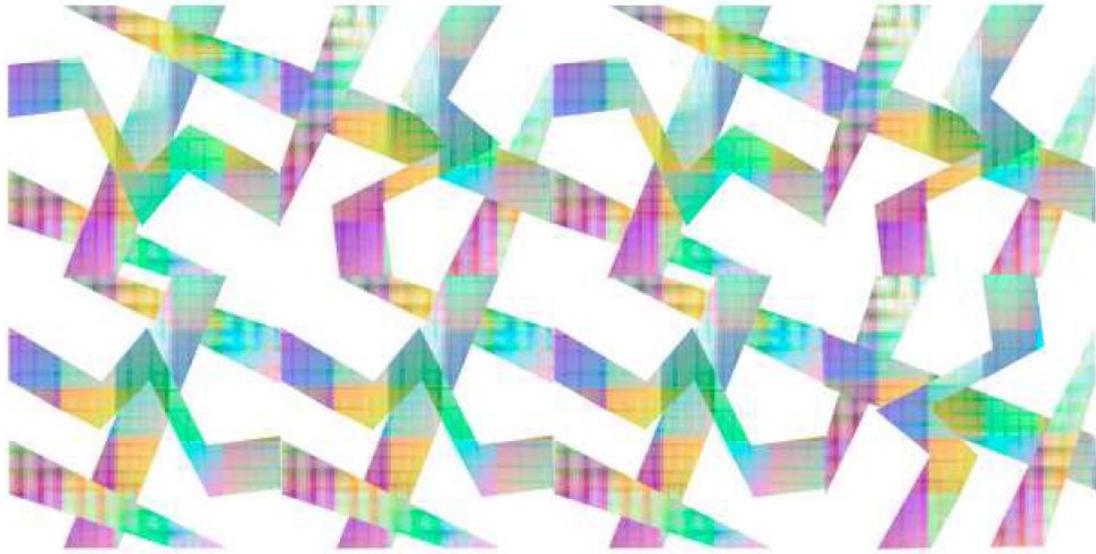


Inspired by potato prints by M.C. Escher and their basic tile called the “Escher tile”, “OrnTiles” is a modular game created from only one basic element (tile) and its transforms obtained by rotations and reflections in the vertical and horizontal mirror axis. As a result, every “OrnTile” fits perfectly (edge-to-edge) with all of its neighbors, and independent of the positions of tiles, always gives an interlacing pattern (periodic or not), placed in the rectangular grid. This is made possible by the appropriate choice of an initial tile which acts like a tangle with two loose ends at each side, permitting different topological variations (use of curved lines and arcs for the borders of colored regions instead of straight lines). Try to make similar tiles with more than two loose ends at each side. Can you make them from polygons other than squares? Which polygons can you use? Can you compose interlacing patterns beginning with different polygonal basic tiles?



In order to make your first “OrnTile”, take a square of dimensions 5×5 and join the edges by “strips” (not necessarily rectilinear), as shown on the left side of the Figure 7. Then join two opposite edges, top and the bottom, by the strip crossing the others. If you like, you can make your strips alternating (going “over-under”), or curvilinear. At the end of the construction you can make the edges of the basic square invisible. Instead of coloring the strips, you can use different textures (e.g., marble, recycled paper, *etc.*) from the texture library (e.g., in *Corel Draw* by the command Fill Tool>Texture Fill Dialog). Tiles constructed in this way always fit perfectly with their neighbors, transforms of the initial tile (obtained in *Corel Draw* by rotating or adding mirrors to the initial tile by the commands: Transform>Rotate or Transform>Scale and Mirror). What will happen if you shift your tiles along the vertical or horizontal line?





Homework: Lunda fractal

Construct a Lunda design of your choice, and then apply self-similarity by copying and scaling the initial tile, and then arranging the resulting tiles following the same rule according to which the initial Lunda design is made. You can work in any graphical computer program (*Inkscape, Adobe Illustrator, Corel Draw*).

As an alternative homework you can construct “OpTiles” and make your own designs from them. In this case, interesting patterns will be obtained if you shift your tiles along the vertical and horizontal line (i.e., you obtain patterns that are not trivial “edge-to-edge” patterns).