Common Exam 3, tomorrow
8:30 – 9:45 A.M. at KUPF 205 (Arrive by 8:15)
Chaps. 6, 7, 8
Bring scientific calculators

Today....
Chapter 9
Review for Chaps. 6, 7, 8
Quiz #11

Last class, we learned...
Work and energy for System of objects

External forces: Forces from outside the system
Internal forces: Forces between objects within the system
→ Apply Newton’s 3rd law
→ Action & reaction forces have equal magnitude & opposite directions

Analysis of the system as a whole
→ Consider external forces only
Analysis of each object within the system
→ Consider both external and internal forces

Horizontal frictionless surface
Chapter 9. Center of Mass and Linear Momentum

Section 9-1, Motivation

Section 9-2, Center of Mass

Section 9-3, Newton's second law for a system of particles

Data: Fatality of a driver in head-on collision

(With a passenger) < (Without a passenger)
Section 9-2, Center of Mass

How should we define the position of the moving body?

What is $h$ for $U = mgh$?

Take the average Position of mass!

Call “Center of Mass” (com)

Center of Mass for fun: Equilibrium

$F_{\text{net}} = \frac{d\vec{P}}{dt} = 0$

Unstable Equil.

Stable Equil.
Center of Mass for a system of particles

2 bodies, 1 dimension

Center of Mass for a system of particles

2 bodies, 1 dimension

$$x_{\text{com}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$
Center of Mass and Uniform Gravity

The CM of an object or a system is the point, where the object or the system can be balanced in the uniform gravitational field.

How can we find Center of Mass for many particles in 3D?
Center of Mass for a System of Particles

\[ x_{\text{com}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \]  
2 bodies, 1 dimension

General case: \( n \) bodies, 3 dimensions

\[ x_{\text{com}} = \frac{1}{M} \sum_{i=1}^{n} m_i x_i, \quad y_{\text{com}} = \frac{1}{M} \sum_{i=1}^{n} m_i y_i, \quad z_{\text{com}} = \frac{1}{M} \sum_{i=1}^{n} m_i z_i \]

\[ \vec{r}_{\text{com}} = \frac{1}{M} \sum_{i=1}^{n} m_i \vec{r}_i \]  
\( n \) bodies, 3 dimensions

Example: Center of Mass for 2 bodies in 1 dimension

Where is the center of mass?
Sample Problem 9-1

Three particles of masses \( m_1 = 1.2 \text{ kg} \), \( m_2 = 2.5 \text{ kg} \), and \( m_3 = 3.4 \text{ kg} \) form an equilateral triangle of edge length \( a = 140 \text{ cm} \). Where is the center of mass of this system? (Hint: \( m_1 \) is at \((0,0)\), \( m_2 \) is at \((140 \text{ cm}, 0)\), and \( m_3 \) is at \((70 \text{ cm}, 120 \text{ cm})\), as shown in the figure below.)

Section 9-3, Newton's second law for a system of particles

System of particles  A firework rocket explodes

\[ \vec{F}_{\text{net}} = M \vec{a}_{\text{com}} \]

\( \vec{F}_{\text{net}} \): Net force of all external forces that act on the system

\( M \): Total mass of the system

\( \vec{a}_{\text{com}} \): Acceleration of the center of mass
Proof of \( \vec{F}_{\text{net}} = M\vec{a}_{\text{com}} \)

\[
M \vec{r}_{CM} = m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3 + \ldots + m_n \vec{r}_n
\]

\[
M \vec{v}_{CM} = m_1 \vec{v}_1 + m_2 \vec{v}_2 + m_3 \vec{v}_3 + \ldots + m_n \vec{v}_n
\]

\[
M \vec{a}_{CM} = m_1 \vec{a}_1 + m_2 \vec{a}_2 + m_3 \vec{a}_3 + \ldots + m_n \vec{a}_n
\]

\[
M \vec{a}_{CM} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots + \vec{F}_n
\]

Review for Chaps. 6, 7, 8

Ch. 6, Force and Motion-II

Friction and Uniform Circular Motion

Ch. 7, Kinetic Energy and Work

Ch. 8, Potential Energy and Conservation of Energy
Units

SI Unit (MKS Unit)

Length: m
Mass : kg
Time : s
Force : N
Work, Energy : J
Power : W

Other relations

$2\pi$ (radian) = 360°

1 T = 1000 kg

HW9 002 (part 1 of 2) 10 points
A simple Atwood's machine uses two masses $m_1$ and $m_2$. Starting from rest, the speed of
the two masses is 7.6 m/s at the end of 2.7 s.
At that time, the kinetic energy of the system
is 51 J and each mass has moved a distance of
10.26 m.
The acceleration of gravity is 9.81 m/s².

Find the value of heavier mass. Answer in
units of kg.

003 (part 2 of 2) 10 points
Find the value of lighter mass. Answer in
units of kg.

External force:
Gravity, Normal force from pulley

Internal force:
Tensions between rope and objects

Extra problem: Find the work done
on $m_2$ by Tension
Work done by a constant force

\[ W = Fd \cos \theta \]

The horizontal force and the friction force are doing work on the object.

\[ W_{\text{total}} = W_1 + W_2 = K_f - K_i \]

HW#9

004 (part 1 of 2) 10 points
A horizontal force of 156 N is used to push a 49.0 kg packing crate a distance of 6.00 m on a rough horizontal surface. The acceleration of gravity is 9.81 m/s². If the crate moves with constant velocity, calculate:

a) the work done by the force. Answer in units of J.

See Lecture Note 9

b) the coefficient of kinetic friction.

HW#8

001 (part 1 of 1) 10 points
A 16.3 kg block is dragged over a rough, horizontal surface by a constant force of 78.7 N acting at an angle of 26.8° above the horizontal. The block is displaced 40.9 m, and the coefficient of kinetic friction is 0.146. The acceleration of gravity is 9.8 m/s².

First, find the normal force by balancing the forces along the vertical direction.

Then, find the friction force.

You are ready to find the work done by friction force.

Find the work done by the force of friction. Answer in units of J.
**HW#7**

**006** (part 1 of 2) 10 points

The coefficient of static friction between the person and the wall is 0.69. The radius of the cylinder is 5.34 m.

The acceleration of gravity is 9.8 m/s².

An amusement park ride consists of a large vertical cylinder that spins about its axis fast enough that any person inside is held up against the wall when the floor drops away.

What is the minimum angular velocity \( \omega_{\text{min}} \) needed to keep the person from slipping downward? Answer in units of rad/s.

**See Sample Prob 6-8**

**007** (part 2 of 2) 10 points

Suppose the person, whose mass is \( m \), is being held up against the wall with an angular velocity of \( \omega' = 2\omega_{\text{min}} \).

The magnitude of the frictional force between the person and the wall is

\[ T = \frac{2\pi r}{v} \]

**Angular velocity** \[ \omega = \frac{2\pi}{T} \]

**Velocity** \[ v = \frac{2\pi r}{T} = \omega \cdot r \]