As a student at NJIT, I (sign) _______________ , will conduct myself in a professional manner and will comply with the provisions of the NJIT Academic Honor Code. I also understand that I must subscribe to the following pledge on major work submitted for credit as described in the NJIT Academic Honor code: On my honor, I pledge that I have not violated the provisions of the NJIT Academic Honor Code.

Multiple Choice
Identify the choice that best completes the statement or answers the question.

1. A uniform rod of mass $M = 1.2$ kg and length $L = 0.80$ m, lying on a frictionless horizontal plane, is free to pivot about a vertical axis through one end, as shown. The moment of inertia of the rod about this axis is given by $ML^2/3$ if a force $(F = 5.0$ N, $\theta = 40^\circ)$ acts as shown, what is the resulting angular acceleration about the pivot point?

   \[ \tau = I \alpha \]

   \[ \tau = L F \sin \theta \]

   \[ I = \frac{1}{3} ML^2 \]

   \[ \therefore \alpha = \frac{\tau}{I} = \frac{L F \sin \theta}{\frac{1}{3} ML^2} = \frac{3 F \sin \theta}{ML} \]

   a. $12 \text{ rad/s}^2$
   b. $33 \text{ rad/s}^2$
   c. $14 \text{ rad/s}^2$
   d. $16 \text{ rad/s}^2$
   e. $10 \text{ rad/s}^2$

   \[ \therefore \alpha = \frac{3 \times 5 \times \sin 40^\circ}{1.2 \times 0.8} = 0.04 \text{ rad/s}^2 \]

2. A wheel starts from rest and rotates with a constant angular acceleration about a fixed axis. It completes the first revolution $6.0$ s after it started. How long after it started will the wheel complete the second revolution?

   \[ \frac{\Delta \theta}{\Delta t} = \omega_0 \Delta t + \frac{1}{2} \alpha \Delta t^2 \]

   \[ 2\pi = \frac{1}{2} \alpha \cdot 6^2 \quad \therefore \alpha = \frac{2 \times \frac{2\pi}{36}}{36} = \frac{\pi}{9} \]

   a. $7.8 \text{ s}$
   b. $9.9 \text{ s}$
   c. $9.2 \text{ s}$
   d. $8.5 \text{ s}$
   e. $6.4 \text{ s}$

   \[ 2\pi = \frac{1}{2} \alpha \Delta t^2 \]

   \[ \Delta t_2 = \frac{8\pi}{\alpha} \quad \therefore \Delta t_2 = \frac{8\pi}{\pi/9} = 9.8 \sqrt{9}=9.2 \sqrt{9} \]

3. A thin uniform rod (length = $1.2$ m, mass = $2.0$ kg) is pivoted about a horizontal, frictionless pin through one end of the rod. (The moment of inertia of the rod about this axis is $ML^2/3$.) The rod is released when it makes an angle of $37^\circ$ with the horizontal. What is the angular acceleration of the rod at the instant it is released?

   \[ \tau = I \alpha \]

   \[ \tau = \frac{1}{2} (Mg) \sin (90^\circ - \theta) \]

   \[ I = \frac{1}{3} ML^2 \]

   \[ \therefore \alpha = \frac{\tau}{I} = \frac{\frac{1}{2} (Mg) \sin (90 - \theta)}{\frac{1}{3} ML^2} = \frac{3 \frac{9.8 \cos \theta}{L}}{\frac{1}{2} L} = \frac{3 \frac{9.8 \cos \theta}{2}}{1.2} = 9.8 \]
4. The turntable of a record player has an angular velocity of 8.0 rad/s when it is turned off. The turntable comes to rest 2.5 s after being turned off. Through how many radians does the turntable rotate after being turned off? Assume constant angular acceleration.

\[ \omega = \omega_0 + \alpha t \]
\[ 0 = 8.0 + \alpha \cdot 2.5 \quad \Rightarrow \quad \alpha = -\frac{8}{2.5} \]
\[ \Delta \theta = \frac{\omega^2 - \omega_0^2}{2\alpha} = -\frac{8^2}{2 \cdot (-\frac{8}{2.5})} = \frac{8 \times 2.5}{2} = 10 \text{ rad} \]

5. The rigid body shown is rotated about an axis perpendicular to the paper and through the point P. If \( M = 0.40 \) kg, \( a = 30 \text{ cm} \), and \( b = 50 \text{ cm} \), how much work is required to take the body from rest to an angular speed of 5.0 rad/s? Neglect the mass of the connecting rods and treat the masses as particles.

\[ \Delta W = K_f - K_i = K_f \]
\[ K_f = \frac{1}{2} I \omega^2 \]
\[ I = 3M \cdot a^2 + M \cdot b^2 = 3 \times 0.4 \times 0.3^2 + 0.4 \times 0.5^2 = 0.208 \text{ kg} \cdot \text{m}^2 \]
\[ \Delta W = K_f = \frac{1}{2} \times 0.208 \times 5^2 = 2.6 \text{ J} \]

6. A uniform meter stick is pivoted to rotate about a horizontal axis through the 25-cm mark on the stick. The stick is released from rest in a horizontal position. The moment of inertia of a uniform rod about an axis perpendicular to the rod and through the center of mass of the rod is given by \((1/12)ML^2\). Determine the magnitude of the initial angular acceleration of the stick. (Hint: Gravitational force is acting at the center of mass.)

\[ \tau = I \alpha \]
\[ \ddot{\alpha} = \frac{\tau}{I} \quad \Rightarrow \quad \tau = r Mg \]
\[ I = I_{cm} + \frac{1}{12} ML^2 + Mr^2 \]
\[ \ddot{\alpha} = \frac{r M g}{\frac{1}{12} ML^2 + Mr^2} = \frac{0.25 \times 9.8}{\frac{1}{12} \cdot 0.25^2 + 0.25} = 16.8 \text{ rad/s}^2 \]
7. A uniform rod (mass = 1.5 kg) is 2.0 m long. The rod is pivoted about a horizontal, frictionless pin through one end. The rod is released from rest in a horizontal position. What is the angular speed of the rod when the rod makes an angle of 30° with the horizontal? (The moment of inertia of the rod about the pin is 2.0 kg m²).
   a. 2.2 rad/s
   b. 3.1 rad/s
   c. 2.7 rad/s
   d. 3.6 rad/s
   e. 1.8 rad/s

   \[ \omega = \sqrt{\frac{2Mgh}{I}} = \sqrt{\frac{2 \times 1.5 \times 9.8 \times 0.5}{2}} = \sqrt{7.35} \approx 2.7 \text{ rad/s} \]

8. Four identical particles (mass of each = 0.40 kg) are placed at the vertices of a rectangle (2.0 m x 3.0 m) and held in those positions by four light rods which form the sides of the rectangle. What is the moment of inertia of this rigid body about an axis that passes through the mid-points of the longer sides and is parallel to the shorter sides?
   a. 2.7 kg m²
   b. 3.1 kg m²
   c. 4.1 kg m²
   d. 1.6 kg m²
   e. 3.6 kg m²

   \[ I = \sum m_i \cdot r_i^2 = m \times 1.5^2 \times 2 = 0.4 \times 1.5^2 \times 4 = 3.6 \text{ kg m}^2 \]

9. A wheel rotating about a fixed axis with a constant angular acceleration of 2.0 rad/s² starts from rest at \( t = 0 \). The wheel has a diameter of 20 cm. What is the magnitude of the total linear acceleration of a point on the outer edge of the wheel at \( t = 0.60 \text{ s} \)?
   a. 0.20 m/s²
   b. 0.14 m/s²
   c. 0.34 m/s²
   d. 0.25 m/s²
   e. 0.50 m/s²

   \[ \alpha = 2 \text{ rad/s}^2 \]

   \[ \omega_t = 0.1 \times 2 = 0.2 \text{ rad/s} \]

   \[ a_t = \frac{v_t^2}{r} = \frac{0.25}{0.1} = 2.5 \text{ m/s}^2 \]

   \[ v_t = \omega_t \cdot r = 0.1 \times 0.1 \times 0.6 = 0.006 \text{ m/s} \]

   \[ \omega = \alpha \cdot t = 2 \times 0.6 = 1.2 \text{ rad/s} \]
10. A mass \( m = 4.0 \text{ kg} \) is connected, as shown, by a light cord to a mass \( M = 6.0 \text{ kg} \), which slides on a smooth horizontal surface. The pulley rotates about a frictionless axle and has a radius \( R = 0.12 \text{ m} \) and a moment of inertia \( I = 0.090 \text{ kg} \cdot \text{m}^2 \). The cord does not slip on the pulley. What is the magnitude of the acceleration of \( m \)?

\[
\begin{align*}
T_1 - mg &= ma_1 \quad \text{(1)} \\
-T_2 &= Ma_2 \quad \text{(2)} \\
-T_1 R + T_2 R &= I \alpha \quad \text{(3)} \\
a_1 &= R \alpha, \quad a_2 = R \alpha \quad \text{(4)}
\end{align*}
\]

- \( a = 3.2 \text{ m/s}^2 \)
- \( b = 1.7 \text{ m/s}^2 \)
- \( c = 4.2 \text{ m/s}^2 \)
- \( d = 2.4 \text{ m/s}^2 \)
- \( e = 2.8 \text{ m/s}^2 \)

11. When the center of a bicycle wheel has linear velocity \( \vec{v}_{CM} \) relative to the ground, the velocity relative to the ground of point \( P' \) at the top of the wheel is

\[
\begin{align*}
\vec{v}_{CM} \\
\alpha &= -\frac{mgR}{I + MR^2 + mR^2} \\
a' &= R\alpha = -\frac{mgR}{I + MR^2 + mR^2}
\end{align*}
\]

\[
\begin{align*}
\alpha &= \frac{mg}{I/10^2 + m + M} \\
&= -\frac{\frac{mg}{0.5 \text{ kg}^2 + 6 + 4}}{4 \times 9.8} \\
&= -2.4
\end{align*}
\]
12. At $t = 0$, a wheel rotating about a fixed axis at a constant angular acceleration has an angular velocity of $2.0$ rad/s. Two seconds later it has turned through $5.0$ complete revolutions. What is the angular acceleration of this wheel?
   a. $20 \text{ rad/s}^2$
   b. $14 \text{ rad/s}^2$
   c. $13 \text{ rad/s}^2$
   d. $17 \text{ rad/s}^2$
   e. $23 \text{ rad/s}^2$

   $\Delta \theta = \omega_0 t + \frac{1}{2} \alpha t^2$
   $\Delta \theta - \omega_0 t = \frac{1}{2} \alpha t^2$
   $\alpha = \frac{2 (\Delta \theta - \omega_0 t)}{t^2} = \frac{2 \times (5\pi \text{ rad} - 2\pi \text{ rad})}{2^2} = \frac{5\pi - 2}{2} \text{ rad/s}^2$
   $= \frac{13 \pi}{2} \approx 13.7$

13. A wheel (radius $= 0.20 \text{ m}$) is mounted on a frictionless, horizontal axis. A light cord wrapped around the wheel supports a $0.50$-kg object, as shown in the figure. When released from rest the object falls with a downward acceleration of $5.0 \text{ m/s}^2$. What is the moment of inertia of the wheel?

   ![Diagram of a pulley system with a wheel and a hanging mass]

   $a = -5 \text{ m/s}^2$
   $R = 0.2 \text{ m}$
   $a = R \alpha$, $\alpha = \frac{a}{R}$
   $T = m g + ma = m (g + a)$
   $T = m g + m a = m (g + \frac{a}{R})$
   $\tau = m (g + a) R = I \alpha$
   $- m (g + a) R = I \frac{a}{R}$
   $I = \frac{a}{R} m R^2$

   a. $0.023 \text{ kg} \cdot \text{m}^2$
   b. $0.019 \text{ kg} \cdot \text{m}^2$
   c. $0.032 \text{ kg} \cdot \text{m}^2$
   d. $0.027 \text{ kg} \cdot \text{m}^2$
   e. $0.016 \text{ kg} \cdot \text{m}^2$

14. You throw a Frisbee of mass $m$ and radius $r$ so that it is spinning about a horizontal axis perpendicular to the plane of the Frisbee. Ignoring air resistance, the torque exerted about its center of mass by gravity is
   a. $2m g$
   b. small at first, then increasing as the Frisbee loses the torque given it by your hand.
   c. a function of the angular velocity.
   d. $m g r$. 

   [Diagram of a spinning Frisbee]
15. A uniform cylinder of radius $R$, mass $M$, and length $L$ rotates freely about a horizontal axis parallel and tangent to the cylinder, as shown below. The moment of inertia of the cylinder about this axis is

\[ I = \frac{1}{2} MR^2 \]

Options:
- \[ \frac{3}{2} MR^2 \]
- \[ \frac{7}{5} MR^2 \]
- \[ \frac{1}{2} MR^2 \]
- $MR^2$
- \[ \frac{2}{3} MR^2 \]

\[ I_p = I_{cm} + Mh^2 \]
\[ I_{cm} = \frac{1}{2} MR^2 \]
\[ I_p = \frac{1}{2} MR^2 + Mh^2 \]
\[ = \frac{3}{2} MR^2 \]