Physics 106 Lecture 8
Equilibrium II
SJ 7th Ed.: Chap 12.1 to 3

- Recap – static equilibrium conditions
- Problem solving methods
- Static equilibrium example problems
- Mechanics overview

Rules for solving equilibrium problems

First Condition: \[ \sum F_i = 0 \]

Second Condition: \[ \sum \tau_i = 0 \]

Choose any convenient axis for torque calculation.
First condition implies that result will be the same (see proof) for any rotation axis
Calculate torques due to weight of rigid bodies by placing their weights at their center of gravity locations (see proof)
Use static friction force where needed: \[ f_s \leq \mu_s N \]

For plane statics (flat world)
- All forces lie in x-y plane. \( F_z \) always equals zero
  \[ F_{\text{net},x} = \sum F_{i,x} = 0, \quad F_{\text{net},y} = \sum F_{i,y} = 0 \]
- All torques lie along +/- z axis. \( \tau_x, \tau_y = 0 \)
  \[ \tau_{\text{net},z} = \sum \tau_{i,z} = 0 \]
Method for Solving Equilibrium Problems
ESSENTIALLY THE SAME AS FOR SOLVING SECOND LAW PROBLEMS

- Draw labeled sketch. What is in or out of the "system"?
- Draw free body diagrams. Include forces ON each body being analyzed, showing point of application to indicate torques.
- Choose x, y, z axes.
- Choose a rotation axis/point to use in calculating torques.
- All choices yield the same net torque, so long as the First equilibrium condition applies…but...
- Some choices simplify the solution. Look for ways to give zero moment arm (zero torque) to irrelevant or troublesome forces.
- Apply \( \sum F_x = 0, \sum F_y = 0, \sum \tau_z = 0 \) (2 dimensions)
- Write the actual forces and torques on the left side of the equations.
- Count the unknowns; make sure there are N equations when there are N unknowns. Constraint equations are often needed.
- Solve the set of "simultaneous equations" algebraically as far as is reasonable before substituting numbers.
- Try to interpret resulting equations intuitively. Check that numerical answers make sense, have reasonable magnitudes, physical units, etc.

Find the forces on the pivot

8.1. The sketches show four overhead views of uniform disks that can slide or rotate on a frictionless floor. Three forces act on each disk, either at the rim, at the center, or halfway from the center to the rim. The force vectors rotate along with the disks, so the force arrangements remain the same. Which disks are in equilibrium?

\[
\begin{align*}
\sum F &= 0 \\
\sum \tau &= 0
\end{align*}
\]

1) 1, 2, 3, 4
2) 1, 3, 4
3) 3
4) 1, 4
5) 3, 4

\( F_{\text{net}} = 0 \quad \tau_{\text{net}} = 0 \)
Example 1: Massless beam supporting a weight

The 2.4 m long weightless beam shown in the figure is supported on the right by a cable that makes an angle of 50° with the horizontal beam. A 32 kg mass hangs from the beam 1.5 m from the pivot point on the left.

a) Calculate the torque caused by the hanging mass.

\[ \tau_{\text{mass}} = -mgx = 32 \times 9.8 \times 1.5 = 470 \text{ N.m} \]

b) Determine the cable tension needed to produce equilibrium

Choose: rotation axis at O

Draw FBD of beam

Part a)

\[ \theta = 50^\circ, \ x = 1.5 \text{ m, } L = 2.4 \text{ m} \]

Part b)

\[ \sum \tau_o = 0 \quad \text{for equilibrium} \]

\[ 0 = -mgx + T_y L \]

\[ T_y = \text{component of } T \perp \text{ to beam} = T \sin(\theta) \]

\[ T = \frac{mgx}{L \sin(\theta)} = 256 \text{ N.} \]

Example 1, continued: Find the forces on the pivot

8.2 For the preceding problem, find the x component of the force at the pivot point. What equations can you use?

A) \( F_x = 0 \) because the system is in equilibrium
B) \( F_x = 165 \text{ N.} \)
C) \( F_x = 117 \text{ N.} \)
D) \( F_x = 196 \text{ N.} \)
E) Result can only be determined by measuring the forces

8.3. For the preceding problem, find the y component of the force at the pivot point. What equations can you use?

A) \( F_y = 0 \) because the system is in equilibrium
B) \( F_y = 314 \text{ N.} \)
C) \( F_y = 117 \text{ N.} \)
D) \( F_y = 510 \text{ N.} \)
E) Result can only be determined by measuring the forces
Example 2: As previous but beam has mass now

The 2.4 m long beam shown in the figure is supported on the right by a cable that makes an angle of 50° with the horizontal beam. A 32 kg mass hangs from the beam 1.5 m from the pivot point on the left.

\( M_b = \text{mass of beam} = 30 \text{ kg acts as point mass at CG of beam.} \)

Choose: rotation axis at O

a) Determine the cable tension needed to produce equilibrium

b) Find the forces at point “O”

\( \theta = 50^\circ, \ x = 1.5 \text{ m, } L = 2.4 \text{ m} \)

Part a) \[ \sum \tau_O = 0 \] for equilibrium

\[ 0 = -mgx - M_bg \frac{L}{2} + T \sin(\theta) \]

\[ T = \frac{mgx + M_bg \frac{L}{2}}{L \sin(\theta)} = 448 \text{ N.} \]

Part b) \[ \sum F_x = 0 = F_x - T \cos(\theta) \]

\[ F_x = T \cos(\theta) = 288 \text{ N.} \]

\[ \sum F_y = 0 = F_y + T \sin(\theta) - mg - M_bg \]

\[ F_y = mg + M_bg - T \sin(\theta) = 264 \text{ N.} \]

Example 3:

A uniform beam, of length \( L \) and mass \( m = 1.8 \text{ kg} \), is at rest with its ends on two scales (see figure). A uniform block, with mass \( M = 2.7 \text{ kg} \), is at rest on the beam, with its center a distance \( L/4 \) from the beam’s left end.

What do the scales read?

Solve as equilibrium system

Axis at “o” is convenient
Example 4:

A safe whose mass is $M = 430$ kg is hanging by a rope from a boom with dimensions $a = 1.9$ m and $b = 2.5$ m. The boom consists of a hinged beam and a horizontal cable that connects the beam to a wall. The uniform beam has a mass $m$ of 85 kg; the mass of the cable and rope are negligible.

(a) What is the tension $T_c$ in the cable; i.e., what is the magnitude of the force $T_c$ on the beam from the horizontal cable?

(b) What is the force at the hinge?
Example 4 Solution

(a) The beam is the system. The weight of the beam acts at its CG.
\[ \sum F_x = F_h - T_c = 0 \quad F_h = T_c \]
\[ \sum F_y = F_v - mg - Mg = 0 \quad T_c = Mg \]
\[ \sum T = 0 = T_c \theta - F_h \frac{L}{3} - mg \frac{L}{3} \]
\[ T_c \theta = (m + M)g \frac{L}{3} \quad \theta = \cos^{-1} \left( \frac{m + M}{M} \right) \frac{L}{3} \]
\[ T_c = 60.93 N \]

(b) \[ F_h = T_c = 60.93 N \]
\[ F_v = (m + M)g \left( \frac{L}{3} \right) \frac{L}{2} \]
\[ F_v = 50.47 N \]
\[ F_\theta = (F_h + F_v) \frac{L}{3} = 79.12 N \]

Example 5: Climbing a ladder

A ladder of length \( L = 12 \) m and mass \( m = 45 \) kg leans against a slick (frictionless) wall. Its upper end is at height \( h = 9.3 \) m above the pavement on which the lower end rests (the pavement is not frictionless). The ladder's center of mass is \( L/3 \) from the lower end.

A firefighter of mass \( M = 72 \) kg climbs the ladder until her center of mass is \( s = L/2 \) from the lower end.

What are the magnitudes of the forces on the ladder from the wall and the pavement?

How far up the ladder can the firefighter climb if the static friction coefficient between ladder and pavement is 0.2?

\[ a = L \cos(\theta) \]
\[ h = L \sin(\theta) \]
Example 5 solution – Climbing a Ladder

Firefighter, a distance \( S \) from base

\[ S = S \cos \theta \]

\( F_N = \text{Normal Force} \)

\( F_{fr} = \text{Static Friction Force} \leq \mu \text{F}_N \)

\( M \) for fireman

\( m \) for ladder, CG at \( L/2 \)

\[ \sum F_x = 0 = F_W - F_x \]

\[ F_W = \frac{F_x}{3} \]

\[ \sum F_y = 0 = F_{fr} - Mg + mg \]

\[ F_{fr} = Mg \left( \frac{45 + 72}{108} \right) \approx 71.4 N \]

Choose \( \theta \) for moments at \( O \) - \( \text{Example} \)

\[ \sum \tau = 0 = Mg \frac{L}{3} \cos \theta + Mg \frac{L}{2} \cos \theta - \frac{F_{fr} L}{3} \sin \theta - 0 \times F_p + 0 \times F_{py} \]

Ladder Example Solution, continued

Solve:

\[ F_W = \frac{(mg \frac{L}{2} + Mg) \cos \theta}{\cos \theta} \]

\[ F_W = \frac{m + Ms}{L} g \cot \theta \]

If \( S = L/2 \) (halfway up):

\[ F_W = \frac{Mg + mg}{L} \cot \theta \]

\[ F_W = \frac{40Fg + 40g N = F_N} \]

How far up before slipping:

\[ F_{fr} = Ms \left( Mg + mg \right) \]

\[ Mg \cos \theta = F_W L \sin \theta - mg \cos \theta \]

\[ s = \frac{Ms (Mg + mg) L \tan \theta - Mg L}{Mg} \]

\[ = 0.2 \left( 1 + \frac{45}{72} \right) \times 1.223 \times 12 \]

\[ = 4.035 \times 2 \]

\[ s = 8.19 \text{ m} \approx 2.3 \text{ m} \]

Note: As \( s \) grows, so does \( F_W = F_{fr} \) (friction) needed to stay in equilibrium. As \( \theta \to 0 \), frictional force becomes huge, ladder slips

impending motion

solve torque equation for \( s \)

\[ \mu_s = 0.2 \]

\[ \tan \theta = 1.227 \]
Example 6: Does the wheel roll over the curb?

**Problem:** In the figure, force $F$ is applied horizontally at the axle of the wheel. What is the minimum magnitude of $F$ needed to raise the wheel over a curb of height $h$? The wheel’s radius is $r$ and its mass is $m$.

**Why such large wheels?**

In the figure, what magnitude of force $F$ applied horizontally at the axle of the wheel is necessary to raise the wheel over curb of height $h$? The wheel's radius is $r$ and its mass is $m$.

**Example 6: Does the wheel roll over the curb?**

In the figure, what magnitude of force $F$ applied horizontally at the axle of the wheel is necessary to raise the wheel over a curb of height $h$? The wheel’s radius is $r$ and its mass is $m$.

**Equations for equilibrium:**

1. $\sum F_y = N + P_y - mg = 0$
2. $\sum F_x = F - P_x = 0$
3. $\sum \tau = mgs - Ns - F(r - h) = 0$

As wheel is about to lift off and roll, $N \to 0$

\[ F = P_x \]

\[ F(r - h) = mgs = mg\sqrt{2rh - h^2} \]

For $r < h$, $F$ grows as $r \to h$. $F$ becomes infinite if $r = h$.

For $h > r$ vehicle is stopped. $F$ cannot pull it over curb.

When does $F = mg$? Ans: for $r = 2h$
Example 7: Rock Climber

In the figure, a rock climber with mass $m = 55$ kg rests during a “chimney climb,” pressing only with her shoulders and feet against the walls of a fissure of width $w = 1.0$ m. Her center of mass is a horizontal distance $d = 0.20$ m from the wall against which her shoulders are pressed. The coefficient of static friction between her shoes and the wall is $\mu_1 = 1.1$, and between her shoulders and the wall it is $\mu_2 = 0.70$. To rest, the climber wants to minimize her horizontal push on the walls. The minimum occurs when her feet and her shoulders are both on the verge of sliding.

(a) What is the minimum horizontal push on the walls? (impending motion)

(b) What should the vertical distance $h$ be between the shoulders and feet, in order for her to remain stationary?

Solution of Example 7: Rock Climber

Choose axis at point "O"

\[
\sum F_0 = 0 = f_2 x 0 + N x 0 + mgd + hN - f_1 w
\]

\[
\therefore h = \frac{\mu_1 N w - mgd}{N} = \mu_1 (w - d) - \mu_2 d = 0.74 \text{ m}
\]

This choice of $h$ produces impending motion.
Structure of rotational and translational mechanics are parallel

NEWTONIAN MECHANICS
forces & torques cause changes in the motion

TRANSLATIONAL DYNAMICS
movement from one place to another

IMPULSE-MOMENTUM
impulse changes the linear momentum

WORK-ENERGY
external work changes the total energy

NEWTON'S SECOND LAW
acceleration is proportional to net force

STATIC EQUILIBRIUM
linear and rotational accelerations are zero

ROTATIONAL DYNAMICS
rotation from one orientation to another

IMPULSE-MOMENTUM
angular impulse changes the angular momentum

ROTATIONAL WORK-ENERGY
external work changes the total energy, including rotational KE

CONSERVATION LAWS
some quantities remain constant for an isolated system

LINEAR MOMENTUM IS CONSERVED
for isolated system of translating bodies

TOTAL ENERGY IS CONSERVED
for isolated system of particles

ANGULAR MOMENTUM IS CONSERVED
for isolated system of rotating bodies

Physics B Course Overview

- PHYSICS A
  COVERED:
  - motion of point bodies
  - kinematics - translation
  - dynamics \[ \sum F_{ext} = ma \]
  - conservation laws: energy & momentum

- PHYSICS B
  COVERS:
  - motion of "Rigid Bodies" (extended, finite size)
  - rotation + translation, more complex motions possible
  - rigid bodies: fixed size & shape, orientation matters
  - kinematics of rotation
  - dynamics
    \[ \sum F_{ext} = ma_{lin} \quad \text{and} \quad \sum T_{ext} = I\alpha \]
  - rotational terms for energy conservation
  - conservation laws: energy & angular momentum

- TOPICS:
  - 8 weeks - rotation:
    - angular versions of kinematics & second law
    - angular momentum
    - equilibrium
  - 2 weeks - gravitation
  - 2 weeks - oscillations
  - 2 weeks - Phys 105/106 review