Common Exam 2

During class (1 - 2:55 pm) on 6/14, Tue
Room: 412 FMH (classroom)

Bring scientific calculators
Exam covers lectures after common exam 1.

Review session: Monday during class

Last class

In 1D:
  Kinetic Energy
  Work
  Net Work
  Work-Energy Theorem

Today

   Kinetic Energy, Work, Net Work, W-E Theorem in 2D

  Conservative vs. Non-conservative forces

  Gravitational Potential Energy
For 2D motion...

Kinetic energy

\[ K.E. = \frac{1}{2} m v^2 = \frac{1}{2} m \left( v_x^2 + v_y^2 \right) \]

What if we have force and displacement with an angle?

⇒ Work in 2D
Work in 2D
What if force and displacement are perpendicular?

Example: Uniform circular motion

No change in “magnitude” of velocity
→ No kinetic energy change → No work, sorry!
(Velocity does change, because the “direction” changes.)
Work done by a constant force in 2D

\[ W = |\vec{F}| |\vec{d}| \cos \theta_{F,d} \equiv \vec{F} \cdot \vec{d} \]

Scalar (dot) product

**Magnitude**
- Force
- Displacement

Note:
- \( 0 \leq \theta < 90 \) positive work
- \( \theta = 90 \) zero work
- \( 90 < \theta \leq 180 \) negative work

Example

Angle = 20 degree, \( F = 5 \) N, \( d = 2 \) m.
Find the work done by force \( F \).
Math Review: Scalar (dot) product using components

\[ \vec{F} = (F_x, F_y) \quad \vec{d} = (d_x, d_y) \]

\[ W = |\vec{F}| |\vec{d}| \cos \theta_{\vec{F},\vec{d}} = \vec{F} \cdot \vec{d} \]
\[ = F_x d_x + F_y d_y \]

Commutative property \[ \vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A} \]

Distributive property \[ (\vec{A} + \vec{B}) \cdot \vec{C} = \vec{A} \cdot \vec{C} + \vec{B} \cdot \vec{C} \]

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iClicker Quiz

Force \[ \vec{F} = (3N, -2N) \] is acting on an object, which makes a displacement \[ \vec{d} = (2m, 4m) \]

Find the work done by this force.

A) \( (6J, -8J) \]
B) \( \sqrt{3^2 + 2^2} \times \sqrt{2^2 + 4^2} \)
C) \( 6 + 12 -4 = 6 \ J \)
D) -2 J
E) Not enough information. I need to know theta.
If multiple forces are applied on an object,

\[ W_{net} = \vec{F}_{net} \cdot \vec{d} = (\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots) \cdot \vec{d} \]

\[ = \vec{F}_1 \cdot \vec{d} + \vec{F}_2 \cdot \vec{d} + \vec{F}_3 \cdot \vec{d} + \ldots = W_1 + W_2 + W_3 + \ldots \]

\[ = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \]

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**Work-Energy Theorem in 2D**

If multiple forces are applied on an object,

\[ W_{net} = W_1 + W_2 + \ldots = K_f - K_i \]
1. Find the forces on the box.
2. Find the work done by each force.
3. Find the net work
4. If initial velocity is 3 m/s, what is final kinetic energy?

Next topic
Conservative vs. Non-conservative forces
Gravitational Potential Energy
Motivation
Why do we learn about potential energy and conservation of mechanical energy?

### iClicker Quiz

**Work done by a gravity force**

- \( h_1 = 4m \)
- \( h_3 = 3m \)
- \( h_2 = 2m \)
- \( h_4 = 1m \)

**Mass = 1 kg**

\[ \vec{F}_g = mg \]

(Work along \( h_1 \to h_2 \to h_3 \to h_4 \)) vs. (Work along \( h_1 \to h_4 \))

Which work is greater?

- A) Work along \( h_1 \to h_2 \to h_3 \to h_4 \)
- B) They are equal
- C) Work along \( h_1 \to h_4 \)
- D) Not enough information
iClicker Quiz

Work done by a gravity force

\[ h_1 = 3 \text{m} \]
\[ x_1 = 0 \text{m} \]
\[ x_2 = 4 \text{m} \]
\[ \text{Mass } = 1 \text{ kg} \]
\[ F_g = mg \]

\[ h_4 = 0 \text{m} \]

Which work is the greatest?
A) Work along green arrows
B) Work along blue arrow
C) Work along red arrows
D) Work along red and green are equal and greater than work along blue arrow
E) They are all equal

Conservative force

If the work done by a force depends only on initial and final positions, the force is called a conservative force.

Gravity force is "a" conservative force.
iClicker Q: Work done by friction force

Position

1m 2m 3m 4m

vs.

\[ f_i = 1N \] : opposite to displacement

In which case, the work by friction is more negative?

A) Top  
B) Bottom  
C) They are equal  
D) N.E.I.

Is friction force a conservative force?

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Non-Conservative force

If the work done by a force depends not only on initial and final positions, but also on the path between them, the force is called a non-conservative force.

Example: Friction force, tension, normal force, and force applied by a person.
Conservative forces in Phys 102:
Gravity, Spring force (to be studied in future)

Non-Conservative forces in Phys 102:
Friction, Normal force, Tension, Other applied forces

Can you find the work done by the gravity force along the path?
A) Yes, of course.
B) Please... No.
C) Not enough information

Mass = 1 kg
height = 10 m

height = 2 m
Gravitational Potential Energy

\[ W_g = mg(h_i - h_f) \]

\[ F_g = mg \]

\[ = mgh_i - mgh_f \]

\[ = U_{g,i} - U_{g,f} \]

\[ h_f \]

\[ h_i \]

\[ U_g = \text{Gravitational "Potential Energy", which depends on position only.} \]

Energy related to position, which potentially converted to kinetic energy.

\[ W_g = U_{g,i} - U_{g,f} = -(U_{g,f} - U_{g,i}) = -\Delta U_g \]

\[ \rightarrow \text{Work done by gravity is negative of gravity Potential Energy change} \]

Gravitational Potential Energy:

\[ U_g = mgh \]

(Note: Height, \( h \), should be measured along vertical direction.)

The higher and the heavier an object is, the greater the gravitational potential energy is.
Example: Bungee Jump and Gravitational potential energy

(a) Is that, by any chance, ...uh... Prof. Ahn?

(b) Suppose that the person's mass is 70 kg and the height of the bridge from the water surface is 120 m. What is the person's gravitational potential energy when the person starts to jump? The height and the potential energy is measured from the water surface.

New Zealand, 2004

iClicker Quiz
The change in gravitational potential energy is _________
(a) $3 \times 9.8 \times 10\ J$
(b) $3 \times 9.8 \times (-10)\ J$
(c) something else
Example:
The change in gravitational potential energy is \( \underline{\text{_______ J}} \)

Example:
The work done by gravity is \( \underline{\text{_______}} \)
Work done by gravity force is negative of gravitational potential energy change:

\[ W_g = U_{g,i} - U_{g,f} = -(U_{g,f} - U_{g,i}) = -\Delta U_g \]

If gravitational force is the only force that does work, \( W_{net} = W_g \), then, from Work-Energy theorem, \( W_{net} = \Delta K \), and above.

\[ \Delta K = -\Delta U_g \]
\[ \Delta K + \Delta U_g = 0 \]
\[ K + U_g \] does not change.

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**Conservation of Mechanical Energy with gravity force**

Define Mechanical energy:

\[ E_{mech} \equiv K + U_g = \frac{1}{2}mv^2 + mgh \]

If \( W_{net} = W_g \)

or, if gravity force is the only forces that does work,

\[ E_{mech,f} = E_{mech,i} : \text{Mechanical energy does not change.} \]

Mechanical energy is conserved.

In general, if conservative forces are the only forces that do work, mechanical energy is conserved: “Conservation of mechanical energy”
Conservation of Mechanical Energy \[ K_2 + U_2 = K_1 + U_1 \]

If conservative forces only cause energy changes, the kinetic and potential energy can change, but their sum, the mechanical energy \( E_{mec} \) of the system, cannot change.

Example: Falling ball

- Height \( h_1 \) with velocity \( v_1 \), less kinetic energy, but more potential energy.
- Height \( h_2 \) with velocity \( v_2 \), more kinetic energy, but less potential energy.

Mechanical energy

\[
E_{mec} = \frac{1}{2}mv^2 + mgh
\]

Conservation of mechanical energy

\[
\frac{1}{2}mv_1^2 + mgh_1 = \frac{1}{2}mv_2^2 + mgh_2
\]
iClicker Quiz

A person throws a ball 30 degree from horizontal from the top of a 20 m high building. Neglect the air resistance.

True or false?

If he throws the ball at a different angle but with the same speed, the ball would hit the ground at a different speed.

(a) True     (b) False

Example: Pendulum

2 m rope

30 degree

What is the speed at the bottom?
Example. A car with its engine off costs along a straight highway, which goes uphill. How far along the highway will the car go before it stops, if its initial speed was 80 km/h, and the slope is 15°? The tires roll, not skid.