Math 451H, Spring 2014: DLA simulations of Hele-Shaw flow for Newtonian and Non-Newtonian Fluids

Use Monte-Carlo method (random walkers) to simulate radial Hele-Shaw flow of an air bubble expanding in a Newtonian fluid, as explained in class. The procedure outlined in what follows is supposed to serve as an introductory guide only and will be updated during the semester.

Procedure:

- Background. Explain connection between random walkers and unstable two-phase flow in a Hele-Shaw cell. Read about Hausdorff dimension and fractal nature of diffusion limited aggregation (DLA).
- Computational part: (a) Newtonian fluid. Define your domain as interior of a circle of radius R with center at origin. Overlay the region $-R \leq x \leq R$, $-R \leq y \leq R$ by a rectangular lattice (for convenience, assume that lattice spacing = 1 unit). As initial configuration, assume that all lattice points inside the circle of radius $R_1 = 10$ have the value of 1 (air), and all the other ones are 0 (liquid). Start propagating random walkers from a random position at the distance R from the origin. If $d^2 = x^2 + y^2$ of these walkers at any point becomes larger than R^2, remove them (or bring them back inside of the circle). If they hit the air interface, let them stick there, according to the rule and suggestions given below. Stop your simulation when the distance to the origin of the most protruding part of the air interface is close to R. Use such a value of R that allows the simulations to be completed in a reasonable time.
- Computational part: (b) Non-Newtonian fluid. Consider a non-Newtonian fluid such that viscosity is a decreasing function of shear rate. In the context of DLA, one modeling approach could be to assume that there is an increased sticking probability at the parts of the interface that move faster; considering other implementations is encouraged. Exact viscosity model to be implemented will be discussed.
- Interpretation of the results. Discuss similarities and differences between the results of your simulations and experiments. Compare fractal dimensions between simulations and experiments. Discuss influence of shear-thinning. Compare fractal dimensions between Newtonian and shear-thinning fluids.

  Your report should contain: (1) description of the method; (2) computer code; (3) description of the results, including plots of the emerging shapes and the discussion of the influence of relevant parameters; (4) comparison of your results and the experimental ones, and your interpretation of similarities and differences between them.

  Sticking rule:
  Use one of two approaches: (1) The approach from Ref. (3): probability of sticking depends on local curvature; the relevant parameters, influence of which should be

checked, are A and B from Ref. (1)); (2) The approach from Refs. (1-2) below: the relevant parameter, influence of which should be checked, is relative frequency of the walks of type 1 and of type 2.

Literature:

1. Kadanoff, J. Stat. Phys. **39** 267 (1985);
2. Liang, Phys. Rev. A. **33** 2663 (1986);
3. Vicsek, Phys. Rev. Lett. **53** 2281 (1984);
4. Mandelbrot, Kol, and Aharony, Phys. Rev. Lett. **88** 055501-1 (2002);
5. Witten and Sander, Phys. Rev. B **27**, 5686 (1983);
6. Paterson, Phys. Rev. Lett. **52**, 1621 (1984);
7. Other papers/books of your choice.

Time frame

- March 24: Progress report that will include description of already completed research, and outline of what remains to be done.
- April 21: Preliminary version of final report; 1/2 hour presentation of your results.
- Final exam week: Public presentation, 1/2 hour. Final report due.

Suggestions:

- Initial position of a walker: choose randomly an angle in interval $0 \leq \theta < 2\pi$, and use as initial position the lattice site that is characterized by an angle θ_1 which is closest to θ, while its distance from the origin is closest to R.
- Efficiency: whenever possible, use integer arithmetic; if floating numbers/constants are needed, use single precision; avoid calculating square roots. You may want to use initially small R (about 20) and than increase it as the bubble grow.
- Experiment with the tricks given in the literature; in particular, allow to the sticking walkers to settle at the lattice site with largest number of close neighbors, etc.
- Ref. (2) has u number of useful practical suggestions, in particular regarding calculating local curvature.