Math 451H, Spring 2014: Saffman-Taylor instability

Use experimental, theoretical and numerical methods to analyze two-fluid flow in a Hele-Shaw cell. Concentrate on the "radial" case where inviscid fluid (air) spreads into viscous incompressible fluid.

- Theoretical part (a): Newtonian flow.
  1. Derive Darcy's law ($\mathbf{u} = -b^2/(12\mu)\nabla P$) for the flow in a Hele-Shaw cell.
  2. In the case of two fluid flow, explain the Laplace-Young boundary conditions satisfied by the pressure in the driven fluid (jump in pressure across the interface between two fluids is equal to $\gamma\kappa$, where γ is the surface tension, and κ is the curvature);
  3. Explain the basics of Saffman-Taylor instability;
  4. Put the relevant equations in nondimensional form, and define capillary number appropriately (see literature below).
  5. Derive exact solution for the simple case of circular boundaries.
  6. Perform linear stability analysis of the flow. Explain the role of surface tension and make predictions regarding the length-scales that are expected to emerge, as a function of plate separation, the flow rate, and the viscosity of the driven fluid. (See Ref. (3)).
- Theoretical part (b): Non-Newtonian flow. Consider shear-thinning fluid as in Refs. (5,6) below, and, by going through the items in the part (a) analyze the influence of shear-thinning on the instability development.
- Experimental part. Work with the Lab Assistant on carrying out experiments with the Newtonian and non-Newtonian fluids. Compare the experimental results to the theoretical ones.

Literature:

1. Homsy, Annual Review of Fluid Mechanics **19**, 271 (1987).
2. Walker, Scientific American **257**, 114 (1987).
3. Paterson, Journal of Fluid Mechanics **113**, 513 (1981).
4. Chen, Journal of Fluid Mechanics **201**, 223 (1989).
5. Kondic etal, Phys. Rev. E **54**, 4536 (1996).
6. Kondic etal, Phys. Rev. Lett. **80**, 1433 (1998).
7. Other papers/books of your choice.

Time frame

- March 24: Progress report that will include description of already completed research, and outline of what remains to be done.
- April 21: Preliminary version of final report; 1/2 hour presentation of your results.
- Final exam week: Public presentation, 1/2 hour. Final report due.