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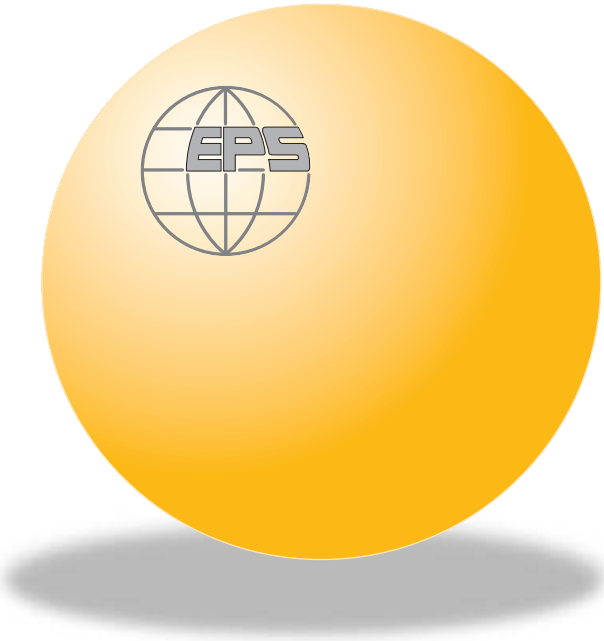

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Segregation by friction

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PACS. 45.70.Mg – Granular flow: mixing, segregation and stratification.
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Abstract. – Granular materials are known to separate by size under a variety of circumstances. Experiments presented here and elucidated by modeling and MD simulation document a new segregation mechanism, namely segregation by friction. The experiments are carried out by placing steel spheres on a horizontal plane enclosed by rectangular sidewalls, and subjecting them to horizontal shaking. Half the spheres are highly smooth; the remainder are identical to the first half, except that their surfaces have been roughened by chemical etching, giving them higher coefficients of friction. Segregation due to this difference in friction occurs, particularly when the grains have a relatively long mean free path. In the presence of an appropriately chosen small "hill" in the middle of the container, the grains can be made to completely segregate by friction type.

Granular materials have been the subject of much recent as well as past attention [1–3]. A steady input of energy in the presence of gravity, for instance by shaking, causes materials of different sizes to segregate, with the largest grains tending toward the top. This phenomenon is called the "Brazil Nut Effect". Explanations [4–11] involve various geometric arguments as well as flows due to wall-driven convection. While other sources of segregation and/or clustering have been studied [12, 13], a much less explored issue is the role played by friction in segregation. Here, we investigate the issue of friction-related segregation in a regime where a two-dimensional granular material is relatively fluid-like [14–16], and in an apparatus where wall effects are generally not important. Although the material may be roughly analogous to a fluid in these experiments, we observe a kind of segregation that does not occur in a true thermodynamic system.

The geometry for these experiments is sketched in fig. 1. We use an electromagnetically driven shaker [17] to provide a sinusoidal horizontal displacement, $x = A\sin(\omega t)$. Here, A and ω provide useful measures for length and time, and the dimensionless acceleration is $\Gamma \equiv A\omega^2/g$, where g is the acceleration of gravity. Two configurations have been explored. In the first, C1, the surface on which the spheres reside is flat, horizontal, and oscillates

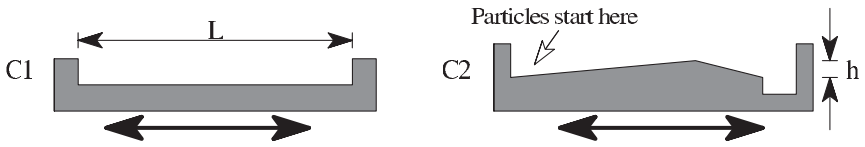

Fig. 1 – Sketch of the flat shaken surface (C1) and the hill configuration (C2). Here, $L = 28.73\,\text{cm}$, $h = 0.22\,\text{cm}$, and the dimension of the system in the transverse direction (not shown) is $11.23\,\text{cm}$.

horizontally. In the second, C2, the surface is bent so that there is a small hill, and all the spheres are initially at one side of the plane. If the particles cross over the hill, they are not allowed to return, and we indicate this schematically in fig. 1 by showing a well on the side of the hill opposite the starting point. The main difference between this setup and the one by Kudrolli *et al.* [18] is that here the energy is supplied to the granular particles by the substrate and boundaries system, while in [18] the energy is supplied only by a moving boundary.

The experiments use a 50-50 mixture of "rough" and "smooth" steel spheres of $d \approx 4.05\,\text{mm}$. The spheres are identical except for their static, kinematic and rolling friction coefficients, μ_s, μ_k, and μ_r, respectively. The rough spheres (d) have been modified by chemical etching and are darker than the lighter colored unetched spheres (l), and we use this as a simple way to distinguish the two types in the experiments. For the smooth particles, $\mu_\text{s} = 0.33 \pm 0.04$, $\mu_\text{k} = 0.10 \pm 0.03$, and $\mu_\text{r} = (1.9 \pm 0.3) \times 10^{-3}$, while for rough particles, $\mu_\text{s} = 0.41 \pm 0.04$, $\mu_\text{k} = 0.12 \pm 0.02$, and $\mu_\text{r} = (4.2 \pm 0.6) \times 10^{-3}$. The frictional properties were measured using the fast camera techniques given in [19]. Specifically, to determine μ_s and μ_k, we glued three particles together and placed them on an inclined plane. The tangent of the angle at which the particles just began to slide defined μ_s; the trajectory of the sliding particles then yielded μ_k. Finally, to determine μ_r, we placed a single particle on a flat substrate, and tracked its motion from an initially rolling state. The deceleration then yielded μ_r. We emphasize that the measurement techniques have relatively large error bars. But, by design, the friction coefficients are higher for the etched particles. It is also interesting to note that segregation occurs for even modest differences in friction, *i.e.* a large difference is not needed to obtain the effect.

Figure 2 shows the results of the experiments performed under the C1 protocol. Here we use a mixture of 1000 smooth and 1000 rough spheres that fill roughly 80% of the available surface area of the shaker. Initially, the particles are completely segregated, with all the l particles to the right, and all the d particles to the left. Figure 2 shows the distribution of the particles after $t = 1, 6$, and 16 minutes. If the spheres were identical except for color, then we would anticipate that they would be randomly distributed in space. A visual inspection may suggest that for late times the l spheres form chain-like structures, and that d spheres form separated structures that are interleaved with the light particles. To check this visual impression, we explore quantitatively the color distribution of the particles. If the particles are placed randomly without regard to color, then the probability of finding n nearest-neighbor grains of any given friction type (*i.e.* color) in a cluster of N (typically six) particles around a particle of a given type is a simple binomial distribution: $P(n) = (1/2)^N N!/(n!(N-n)!)$. Careful examination of this probability, using the data given in fig. 2, shows that the visual impression of structure is an illusion: the spheres are completely mixed for late times, and they closely follow a binomial distribution. Alternatively, if an experimental image of the spheres is processed so that the colors are reassigned randomly, it is not possible to distinguish an arbitrary original image from an arbitrary processed image. Thus, in the dense region of the sample, in the absence of a gravitational bias, mixing remains complete. We note, however, that a large number of shakes (hundreds) is needed to completely mix the system (compare

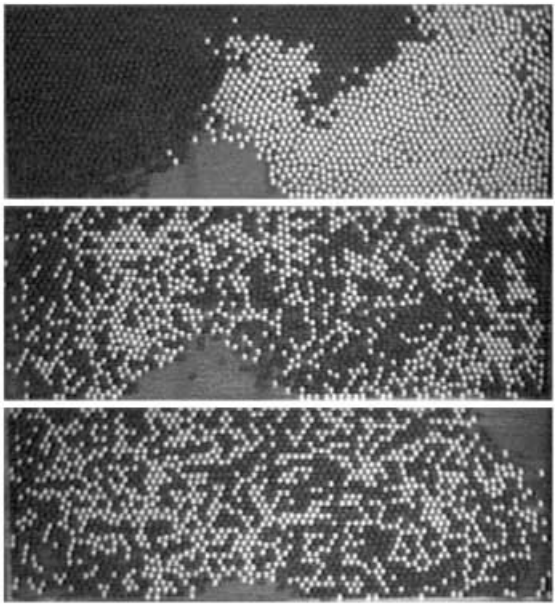

Fig. 2 – Typical pattern of spheres after shaking ($\Gamma = 0.130 \pm 0.005$, $A = 1.80\,\mathrm{cm}$, and $f = 1.339\,\mathrm{Hz}$) for a horizontal surface (C1 protocol) after 1, 6 and 16 minutes (top to bottom). Dark grains have higher rolling (and other) friction coefficients than light grains. The distribution by friction type evolves from almost completely segregated for $t = 1\,\mathrm{min}$ to randomized for $t = 16\,\mathrm{min}$.

fig. 2 for $t = 6$ and $t = 16$). Similar results have been observed using different Γ's.

We continue exploring the possibility of segregation by friction using the C2 protocol. In this experiment, the spheres (a 50/50 mixture of smooth and rough) are all initially placed in a uniformly mixed state on the left side of the hill. We can envision spheres crossing the crest of the hill as being analogous to a thermally activated process. If we ignore friction, the probability of a particle getting over the hill has the form $P = \exp[-E_\mathrm{b}/E_\mathrm{T}]$. Here $E_\mathrm{b} = mgh$ is a measure of the gravitational energy barrier. E_T represents a random or "thermal" energy, the so-called granular temperature. Assuming that the grains are rolling (we will see later that this is not always the case), they experience an energy loss per distance traveled of $\mathrm{d}E_\mathrm{T}/\mathrm{d}x = -\mu_\mathrm{r} mg$ that is higher for rough spheres. Then, if the particles start at the bottom with a nearly identical random energy, $E_{\mathrm{T}0}$, the spheres reaching the barrier have a reduced E_T of $E_\mathrm{T} = E_{\mathrm{T}0} - \mu_\mathrm{r} mgx$, where x is the typical distance traveled to reach the crest of the hill. In the case of the rougher spheres, it appears to be possible to adjust the energy input so that $E_\mathrm{T} = 0$ by or before the point that a typical sphere has reached the crest of the hill. In that event, the cross-over probability falls to zero, and segregation is complete. In practice, we choose Γ so that all l particles cross to the other side of the hill, whereas very few of d particles cross.

Figure 3 shows the results for the number of each type of sphere, N_i with $i = l, d$, remaining on the starting side of the hill as a function of time. This figure shows the mean and standard deviation of four different experiments. For simplicity, the Γ used here is also the one in fig. 2, namely, $\Gamma = 0.130$. The final approach to the steady state (*i.e.* no l particles on the original side) appears to be exponential, implying that an escape rate for the l spheres is given by $-\mathrm{d}N_l/\mathrm{d}t \propto N_l$. However, from fig. 3, it is apparent that there is some initial curvature to the N_l *vs.* t data, *i.e.*, departure from exponential decay, which requires further explanation.

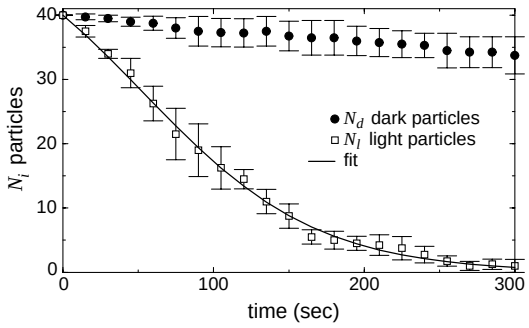

Fig. 3 – The populations of higher- and lower-μ spheres, N_d and N_l, *vs.* time for the "hill" (C2) configuration. The line is the least-squares fit based on the simple model discussed in the text, giving $b = 0.01737$ and $c = 0.0003117$.

This slow-down of the segregation for early times can be understood by noting that there are two processes involved. One (slow) part consists of collision-dominated diffusion of the l particles to the free surface of the layer close to the bottom of the hill. This layer acts like a liquid phase, preventing the l particles from reaching the less dense phase further up the slope. The second (fast) part consists of their "evaporation" *i.e.* ejection away from the bulk. Once free of the bulk, the dominant energy dissipation comes from friction with the substrate. In the dense collision-dominated regime, the particles are essentially indistinguishable; but once they are evaporated into the gas phase, the particles experience different energy loss rates because of the difference in the friction. A simple model that adequately describes the data and captures at least some of the relevant physics assumes that

$$\mathrm{d}N_l/\mathrm{d}t = -a(N_l)N_l, \tag{1}$$

where $a(N_l)$ is now a coefficient that depends on N_l as $a = b - cN_l$, where b and c are positive constants. Roughly, the idea is that when the number of low friction particles is low, the escape rate is at its greatest, namely, b. However, when N_l is relatively high, there is a collective process that reduces the rate of escape. That is, it is harder for particles captured inside a dense cluster to escape. The smaller the cluster, the easier it is for a particle to escape the cluster and then possibly make it over the hill. The solution of this equation

$$N_l(t) = \frac{b}{c + \frac{b-cN_0}{N_0}e^{bt}}\,, \tag{2}$$

is shown in fig. 3 with b, c fitted using the method of least squares.

As an additional test of eq. (1), we have performed experiments where only light spheres are present, but their initial number is varied. In fig. 4 we immediately observe an increased curvature of the profiles for larger N. This is as expected, since more particles lead to an increased dissipation of energy, therefore slowing down the escape process. Based on these results, we conjecture that, while the detailed values of the parameters depend (weakly) on a particular experimental configuration, the model specified by eq. (1) captures the most relevant physics that governs the escaping probability.

So far, we have concentrated only on the rolling friction as the parameter that governs the segregation process. However, a back-of-the-envelope energy estimate shows that there is a problem with this simple picture. For example, consider a single (rough) particle moving under

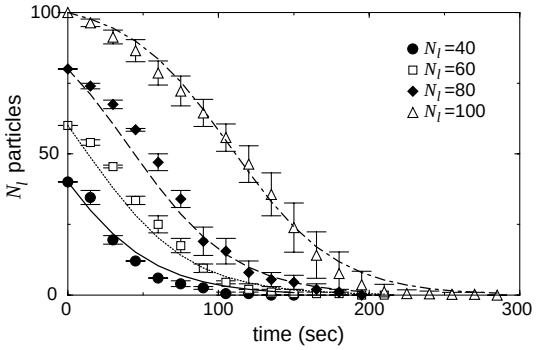

Fig. 4 – The populations of lower-μ spheres in the C2 experiments in which higher-μ spheres are absent. The lines are the solutions of eq. (1) using $b = 0.0288$ and $c = 0.0002758$. The values of the constants b and c are different from those in fig. 3 since there are only l particles in this system.

gravity, and experiencing collisions with the left wall shown in the C2 protocol in fig. 1. It is easy to see that this particle would be provided with enough energy to escape to the other side of the hill. In the case of many particles there is an additional dissipation due to interparticle collisions, but still one would expect that during a few hundred cycles, all the particles (rough and smooth) would escape. To gain better insight into this issue and into the segregation mechanism in general, we have performed discrete element simulations (DES) of this system.

These simulations follow rather closely the model described in [20]. For completeness, a brief overview will be given here; for details, and for the list of relevant earlier works, the reader is referred to that paper. During a collision, a sphere experiences normal and tangential collisional forces. The normal force is given by

$$\boldsymbol{F}_{\rm N}^{\rm c} = \left[k(d - r_{i,j}) - \gamma_{\rm N}\bar{m}(\boldsymbol{v}_{i,j}\cdot\hat{\boldsymbol{n}})\right], \tag{3}$$

where k is a force constant, $r_{i,j} = |\boldsymbol{r}_{i,j}|$, $\boldsymbol{r}_{i,j} = \boldsymbol{r}_i - \boldsymbol{r}_j$, $\hat{\boldsymbol{n}} = \boldsymbol{r}_{i,j}/r_{i,j}$, $\boldsymbol{v}_{i,j} = \boldsymbol{v}_i - \boldsymbol{v}_j$, $\bar{m}$ is the reduced mass, and d is the particle diameter. The damping constant is chosen so that 10% of the energy is lost in a typical collision, as appropriate for steel particles; see [20] for other parameters. We note that Hertzian (nonlinear) interactions would be appropriate to model the collision between spheres; however, we have not seen any relevant difference in the results if the linear model specified above is used, and the parameters are chosen appropriately [20]. The tangential force in the plane of the substrate is given by

$$\boldsymbol{F}_{\rm S}^{\rm c} = \mathrm{sign}(-v_{\rm rel}^{\rm t})\min\left(\gamma_{\rm S}\bar{m}|v_{\rm rel}^{\rm t}|, \nu_k|\boldsymbol{F}_{\rm N}^{\rm c}|\right)\hat{\boldsymbol{s}}\,, \tag{4}$$

where $v_{\rm rel}^{\rm t}$ is the relative velocity in the tangential direction $\hat{\boldsymbol{s}}$, $\gamma_{\rm S} = \gamma_{\rm N}/2$ and ν_k is the coefficient of friction between the particles. In principle, there is also a tangential collisional force in the direction normal to the surface, but this is not included here. Next, the particles interact with the substrate via rolling friction, as explained earlier. More importantly, as shown theoretically [20], and measured experimentally [19,21], the particles *slide* on the substrate after a typical collision, due to large momentum exchange that occurs between the colliding particles. The basic picture is that during the collision process, the rotational motion of the particles cannot immediately follow their translational motion so as to maintain a no-slip condition. This leads to a large relative velocity at the contact point between a colliding particle and the substrate, and hence to sliding. Sliding with a coefficient of friction, $\mu_{\rm k}$, continues until friction brings the linear and angular velocities back to a state of rolling without slipping. The

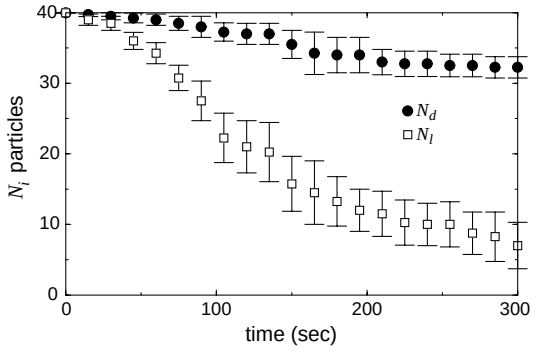

Fig. 5 – The populations of higher- and lower-μ particles, N_d and N_l, *vs.* time for the "hill" configuration (C2), but in DES.

resulting loss of energy is large, since $\mu_{\rm k}$ is so much larger than $\mu_{\rm r}$. While the exact loss of energy due to sliding depends on the details of the collision, and on the time period until the next collision, this loss could reach 80% of the original energy of a particle [19–21]. Therefore, much more energy can be lost due to sliding, than due to inelasticity of the collisions. Sliding also occurs in collisions with the bottom wall. In principle, sliding could also occur due to acceleration of the substrate. However, this is expected only if the acceleration of the substrates, $a_{\rm S}$, satisfies $a_{\rm S} \geq (1 + mR^2/I)\mu_{\rm s}g$, where I is the moment of inertia of a particle of radius R and mass m [20]. This condition is not satisfied for the weakly driven case considered here.

Some additional details of the DES are relevant here. The geometry of the DE domain is similar (but not exactly identical) to the C2 setup. In particular, the simulations have been performed by using both periodic boundary conditions, and rigid (and inelastic) side walls in the direction transverse to the shaking, without any noticeable differences. Initially the particles are placed on a lattice, given random initial velocities, and left to equilibrate without driving and without friction. The particles are then randomly assigned either low- or high-friction coefficients, and the simulations are started using this initial configuration. We note that for convenience we increase the inclination angle on the right-hand side of the domain in the DES, so that particles that cross the hill do not return. In the experiments, a similar outcome was reached by using a trough and/or highly absorbent right wall.

Figure 5 shows results for the number of remaining particles *vs.* time for each type, as obtained from the DES. These data were obtained by averaging over four different runs which differ only in the seed for the random number generator used to assign the initial particle velocities, as well as their frictional properties. We show the mean and standard deviation resulting from these four runs.

The escape rates of the DE particles are similar to the ones observed in experiments, showing that the simulation, while relatively simple, is complete enough to explain the main features of the experiment. The only noticeable differences between the simulations and the experiment are 1) a larger spread of the results of DES compared to experiments, and 2) somewhat fewer light particles escaping in the DES. We note that without inclusion of sliding friction, all the particles escape to the right side of the domain within the shown time period. However, rolling friction is also important in the segregation process. For the parameters we explored in the simulations, we could not achieve a high degree of segregation based on sliding friction alone (simulations allow for simple removal of rolling friction by specifying $\mu_{\rm r} = 0$). Therefore, both sliding and rolling frictional properties are relevant to the segregation process.

It is also worth mentioning that for all the simulations and experiments carried out here, in the presence of a barrier, less frictional particles escape across a potential barrier more easily than those of higher friction. This observation appears to cover a broad driving regime, *i.e.* the particular values given to the amplitude or frequency of shaking, or of the inclination angle. This is in contrast to segregation by size/density, where the outcome may depend on a shaking regime (see, *e.g.*, [9, 11]).

To conclude, we have shown that granular systems undergo segregation due to frictional differences between grain types. This segregation process is similar to the case of size differences, although there are also important differences. Related frictional segregation may well occur in other situations when grains are free to flow along a surface, as in avalanching flows, or possibly in shear layers near boundaries.

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