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Onset of flow in a horizontally vibrated granular bed: Convection by horizontal shearing

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Abstract. – We present experimental observations of the onset of flow for horizontally vibrated 3D granular systems. For accelerations Γ above Γ^*, the top layers of granular material flow, leading to convection with motion both parallel and transverse to the shaking; the lower part of the layer moves with the shaker in solid body motion. With increasing Γ, more of the layer becomes liquefied. The initial bifurcation is backward, but a small amount of fluidization by gas flow lifts the hysteresis. A new convective mechanism, which we explore both experimentally and computationally, associated with horizontal shearing at the walls, is identified as the mechanism driving the transverse convective flow.

Although granular materials are common in nature and industrial applications, the complete understanding of their dynamical behavior is still an open problem. Consequently, the dynamics of granular materials have attracted considerable interest in recent years (for comprehensive reviews see [1] and citations therein). Granular materials can exhibit both fluid-like and solid-like properties depending on the circumstances: they resist shearing up to a point, but flow freely under strong enough shear or at low enough density. These materials also display a number of different dynamical states including liquefaction, heap formation and convection under vibration, the spontaneous formation of stable arches, segregation, a variety of density waves, stick-slip motion during avalanches, etc.

Much recent attention has been focused on the dynamics of vertically vibrated granular materials. Although there have been some studies [2-7], much less is known about the corresponding dynamics of granular materials subject to horizontal vibration, and of the existing work much is very recent. Very recently, several authors [5, 6] have investigated 2D systems; the present study focuses on the dynamics of horizontally vibrated 3D systems, particularly the transition to flow —sometimes referred to as liquefaction. A better understanding of this second case is of interest scientifically because it gives insight into phenomena associated with shearing and into the competition between dilation and friction —*i.e.* a dense granular layer resists shearing both because of friction, and because it must dilate to deform. Horizontally driven flow is also of interest because both horizontal and vertical vibration are commonly used in industries as an aid to mixing, segregating and transporting granular materials. Soil liquefaction during earthquakes is a common and destructive phenomenon associated with horizontal shaking. Finally, the present experiments show a novel shear-induced flow which is the motor for convection transverse to the shaking direction; this mechanism is likely to be present in other shear flows as well. This transverse flow is explored carefully, both experimentally and by molecular dynamics (MD) simulations.

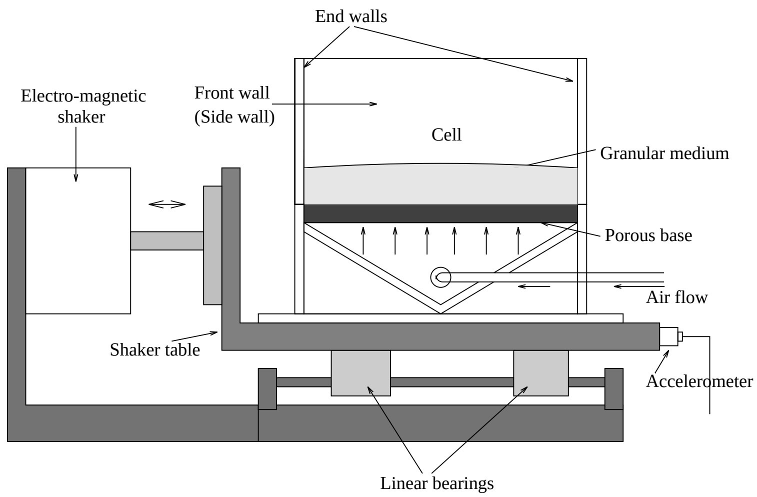

Fig. 1. – Schematic of the apparatus. The cell is made of Plexiglas, and is mounted on a Plexiglas base of the same cross-sectional dimensions which is attached to a table. An electro-mechanical actuator, driven by a sinusoidal AC signal, provides the horizontal driving.

Our experimental setup is shown in fig. 1. To facilitate the discussion below, we will refer to the direction of shaking as x, the horizontal direction perpendicular to x as z, and the vertical direction as y. The heart of the experiment is a rectangular cell with dimensions $L_z = 1.93\,\mathrm{cm}$ by $L_x = 12.1\,\mathrm{cm}$. The height, L_y, of the cell is much taller than the fill height. The base is made of a porous medium (average pore size $50\,\mu\mathrm{m}$) through which gas can flow in order to fluidize the granular medium. This provides an independent control over the dilation of the material. The cell is mounted on a base, which is in turn mounted on a movable table; the base acts as the gas distributor to the system. The pressure gradient across the distributor is more than 50% of the total pressure gradient; hence, there is reasonably even air distribution across the granular bed. An electro-mechanical actuator provides a sinusoidal drive of the form $x = A \sin \omega t$ at frequencies $f(= \omega/2\pi)$ of 3–15 Hz and at amplitudes A of 0–15 mm. A calibrated PCB accelerometer yields the acceleration of the system. This device indicates less than 1% extraneous noise.

In a typical run, we observed the evolution of the system as A was increased from zero while keeping ω fixed. Once flow was well initiated, A was decreased. We used several types of approximately monodisperse granular materials, including spherical glass beads, smooth Ottawa sand, and seived rough sand, with diameters of $\sim 0.6\,\mathrm{mm}$ (also other sizes, $0.2 \leq d \leq 1.0\,\mathrm{mm}$ —with similar effects). The material was poured into the cell and partially compacted by flattening the surface —a process which was enhanced by shaking at relatively low Γ before proceeding to higher values. Typically, the layer had a height $h/d \simeq 40$; as long as $h/d > 20$, we found no dependence on h. The relevant control parameter is the dimensionless acceleration $\Gamma = A\omega^2/g$, where g is the acceleration due to gravity. Other parameters, such as $E = (A\omega)^2/gd$, which are important in describing higher-order phenomena in vertically shaken materials (*e.g.*, traveling waves [8], coarsening [9]) are not necessary to describe the onset of flow in these experiments. However, frictional properties are important, and recent experiments [3] show that the ratio A/d may be important because highly monodisperse grains can lock into a crystalline state, depending on that ratio. However, the materials used here were not highly monodisperse, and no dependence on A/d was observed.

We focus on the flow at the initial instability which consists of a sloshing of the grains in the direction of the shaking, plus slower convective flow, including flow in the direction transverse

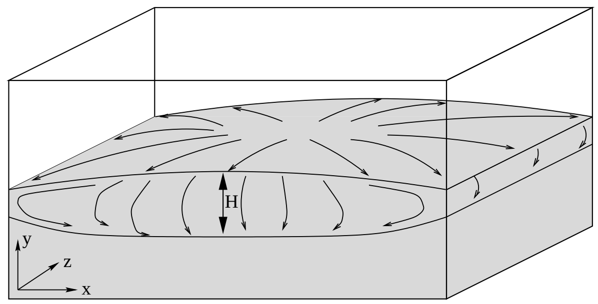

Fig. 2. – Sketch of time-averaged convection flow lines in the liquified layer. Magnitude of the flow in y and z directions has been greatly exaggerated.

to the shaking. Figure 2 shows a sketch of convective flow lines for Γ somewhat above onset in a cell that is 30 grains wide and 50 grains deep. Convective lines were observed by coloring some of the particles and by then following them over time. (In this regard, considerable care must be taken with the coloring, since it can affect the surface friction, leading in turn to strong segregation.) Both lab-mounted and shaker-mounted cameras were used to observe the system. The dominant flow is in the direction (x) of shaking; further, grains rise up in the middle of the cell and flow transversely along the surface towards the side walls and then sink at the wall boundaries. The top surface of the liquefied layer has a dome shape which is concave down, and the bottom surface of this layer is concave upwards near the side walls. Thus, the thickness of the fluidized layer is largest in the middle of the cell and smallest at the end walls, as seen previously [2]. In the case of thin cells, where the width of the cells is less than six grains wide, the convective flow transverse to the shaking direction is suppressed and the curvature of the top surface in the z-direction vanishes. In our experiments (maximum $\Gamma = 2$), we do not observe upward convective flow next to the end walls or 4-roll convective state, as reported in recent computational study [5]. Some of the reasons for this discrepancy might be different properties of the particles in experiment and simulations, modifications introduced by the 3D aspect of the experiment, or the details of the simulations, which lead to increased deformability of simulational particles, as pointed out in [5].

A useful measure of the strength of the flow is the thickness, H, of the liquefied material in the middle ($L_x/2$) of the cell (fig. 2). H was measured by following the motion of the grains next to the front wall using close-up images from a shaker mounted camera. Grain displacements near $x = L_x/2$ decrease linearly with the distance y from the top surface, reaching zero smoothly at the liquefied layer boundary. We note that H is time-independent, so that the effect of recurrent swelling [7], reported for the system consisting of very small particles driven at high frequencies, is not observed in our system.

Figure 3(a) shows typical hysteretic behavior for H as a function of Γ. With increasing Γ, there is a well-defined transition to finite amplitude (including finite H) flow at Γ^*. If we then decrease Γ below Γ^* once flow has begun, H decreases but remains nonzero until Γ reaches a critical value $\Gamma_c < \Gamma^*$, where the relative motion completely stops. As Γ is decreased from above towards Γ_c, grains near the side walls (x-y planes) stop moving first, while grains in the middle keep moving.

It is perhaps not surprising that the initial transition to flow is hysteretic, since the onset of flow must occur by the breaking of static friction, whereas once flow has begun, dynamical friction is involved, assuming that grains remain in motion throughout the cycle. In addition, the grains must dilate in order to flow [10], but once dilated, the flow is more easily sustained. Some hysteresis was reported in 2D studies [6]; higher dimensionality or perhaps roughness of the grains could explain much larger hysteresis in our system.

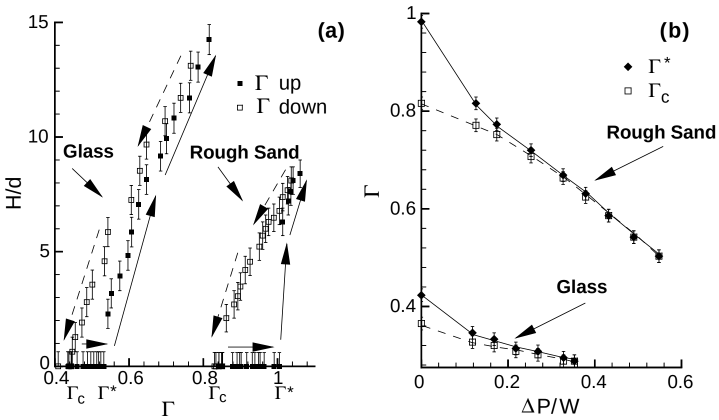

Fig. 3. – (a) Thickness of the liquefied layer next to the side walls (x-y plane) *vs.* Γ for glass beads, $d = 0.6$ mm and rough sand $d_{\mathrm{ave}} = 0.6$ mm. Γ^* and Γ_{c} are the bifurcation points when Γ is increased and decreased quasi-statically. (b) Bifurcation points Γ^* and Γ_{c} *vs.* dimensionless lifting pressure.

We note that different ω's or A's yield the same critical Γ_{c} and Γ^* for a given material and fill height for the full range of A's and ω's which we explored. Thus, Γ is the relevant control parameter, as opposed to, say, E or some other dynamical measure.

These Γ's depend on the physical properties of the material (see fig. 3(a)). For instance, Γ_{c} and Γ^* increase as the roughness of the granular materials increases. The same is true for the difference $(\Gamma^* - \Gamma_{\mathrm{c}})$, which is larger for rougher granular beds than for smooth ones. This can be attributed to two factors: first, the ability of rough grains to move is reduced because of the stronger interlocking of grains; and second, the effective macroscopic frictional forces between grains and between grains and walls may also be higher for rough grains. These effects are distinct, although in an experiment it may be difficult to distinguish between them.

To obtain additional insight into the relative importance of interlocking and friction at grain contacts, we have fluidized the granular bed by passing air through it, using the flow-controlled air supply (see fig. 1). Figure 3(b), which presents Γ^* and Γ_{c} as a function of the measured pressure difference ΔP across the granular medium, nondimensionalized by the weight W of the granular material per unit area, indicates a very strong dependence of these quantities on the air flow. In particular, a modest air flow reduces the critical Γ's and ultimately effectively removes the hysteresis in the initial transition. A key point is that the measured dilation of the bed due to the air flow is very small; the maximum dilation corresponds to less than one granular layer for a bed of 45 layers, *i.e.* less than 2% dilation. As a rough comparison, all interlocking is removed for a cubic sphere packing (solids fraction $\gamma = 0.52$), whereas a typical dense sphere packing has $\gamma = 0.64$; these differ by 18%. Since the dilation in our experiment is much smaller, we conjecture that the most important role played by the gas levitation is a reduction in the static friction at contacts (perhaps by introducing a thin air gap between the grains), rather than a significant reduction of the interlocking effect. It appears that this reduction in static friction is also responsible for removing hysteresis at the transition.

An important issue concerns the sources or "motors" which drive both the convective flow parallel to the shaker plane (shown in fig. 1) and the convective flow perpendicular to it. It is easy to understand why the layer begins to slosh back and forth above a critical acceleration, but it is much less clear why there should be a transverse flow and the formation of a dome-shaped heap. The convective flow in the x-direction is due to the avalanching of grains at the end walls (y-z planes) during the sloshing motion of the liquefied layer. Each half of the shaker cycle, grains pile up on one end wall and open a gap near the opposite one.

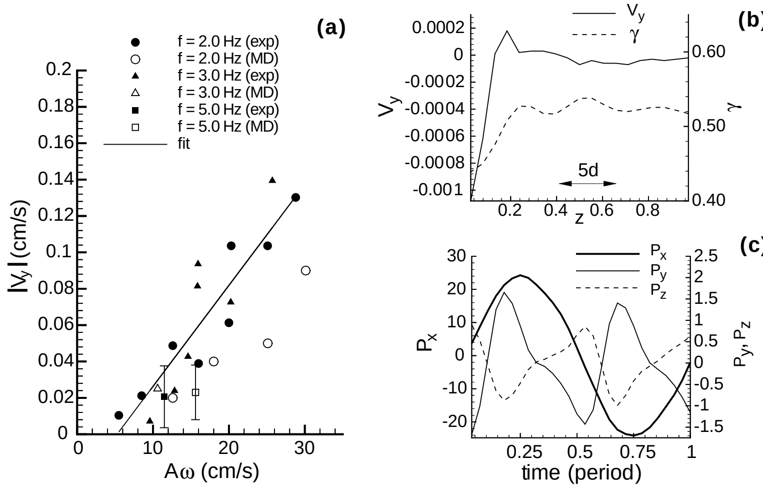

Fig. 4. – (a) The average speed at which grains fall along the shearing wall *vs.* shearing speed $A\omega$ of the wall; the line is the least square fit to experimental data. The error bars show two typical standard deviations. (b) MD results for v_y (in units of $A\omega$), and for solids fraction γ ($A = 1\,\text{cm}$, $f = 2\,\text{Hz}$). The diameter of the particles, $d = 2\,\text{mm}$, is also shown. (c) Total momentum (in arbitrary units) during one period (MD).

When the opening at the end wall is sufficiently large, grains near the opening avalanche down to fill part of the gap. This process repeats at each endwall every cycle.

The convective flow in the z-direction, *i.e.* transverse to the shaking, is due to a completely different and novel mechanism. To investigate this cross flow, we have carried out additional experiments as well as MD simulations in which we only oscillate one of the long side walls, keeping the cell fixed. Experimentally, we find a convective flow where grains fall at the sidewall, and are then pushed inward and upward within a shearing layer.

Better insight is obtained by MD simulations, which are performed on a 3D system of 7000 monodisperse spherical particles. Periodic boundary conditions are assumed in the shearing (x) direction, the top surface is free, while impermeable walls bound the system in the remaining directions. The system is mapped to a $(0, 1)^3$ computational domain. The particles are initially distributed on a regular lattice, given random velocities, and left to relax under gravity. The results given below are obtained by averaging over 30 periods of the wall motion and are found not to be influenced by diameter or by monodispersity of the particles. The MD method is outlined in [11], with particle-particle and particle-wall interactions modeled as in [12], allowing for both normal and tangential interactions. Details are to be given elsewhere; here we just present the main results.

Figure 4(a) shows both MD and experimental results for the average speed v_y at which the grains fall along the shearing wall. We find that v_y varies approximately linearly with shearing speed $A\omega$. The experimental and MD results follow the same trend and are consistent up to the experimental and computational accuracy. Figure 4(b) gives MD results for the solids fraction and for v_y ($-$ sign indicates downward flow). The dilation close to the shearing wall ($z = 0$) allows the particles to flow predominantly down close to the wall. These particles have to move inward and they develop upward flow in the interior of the shearing layer which is 2–4 grain diameters wide. Figure 4(c) shows the total momentum P of all particles in the system, during a period. While the motion of the particles in the x-direction closely follows the harmonic wall motion, we observe non-trivial structure of P_y and P_z. Downward flow ($P_y < 0$), as well as flow away from the wall ($P_z > 0$), develop during the parts of the cycle when the wall velocity changes sign. We consider that in particular this observation provides better understanding of this novel shear-induced convection, which has not, to our knowledge, been previously characterized, but it is likely to occur generally when a 3D material is sheared in the presence of a gravitational field [13].

When the whole cell is now shaken horizontally, the following scenario describes the onset of the instability and the mechanism for the cross-convective flow. The flow begins when the grains overcome frictional and dilatancy effects. At this point there is a shear flow of surface grains relative to the side walls (*i.e.* in the x-direction) which is a maximum at $L_x/2$. Near the side walls (x-y planes), there is a strong dilatancy of the moving grains, due to shearing. The highly dilated grains near the side walls are mobile, and grains reaching these walls at the top of the layer percolate downward. However, grains falling along these walls may participate in a stress chain which pushes grains inwards. Such an effect can also push grains upward towards the free surface, but not into the compacted non-mobile layer at the bottom of the sample. This leads to the heaping and the resultant flow back towards the side walls.

To conclude, we have characterized the transition to flow, or liquefaction, for horizontal shaking of granular materials in a 3D geometry. In the absence of additional fluidization by vertical gas flow, this transition is hysteretic. However, by applying a very modest vertical gas flow which creates a dilation of $\sim 2\%$, the hysteresis is lifted, and the onset to flow occurs at lower Γ. We have observed, both experimentally and computationally, convective flow patterns perpendicular to the horizontal shaker motion. The mechanism that drives this convective flow is particularly interesting since it likely occurs in other 3D systems with shearing and gravity.

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