24.2-4 Suppose that all edge weights in a graph are integers in the range from 1 to $|V|$. How fast can you make Kruskal’s algorithm run? What if the edge weights are integers in the range from 1 to $W$ for some constant $W$?

25.1-2 Give an example of a weighted, directed graph $G = (V, E)$ with weight function $w : E \to \mathbb{R}$ and source $s$ such that $G$ satisfies the following property: For every edge $(u, v) \in E$, there is a shortest paths tree rooted at $s$ that contains $(u, v)$ and another shortest-paths tree rooted at $s$ that does not contain $(u, v)$.

25.2-3 Suppose we change line 4 of Dijkstra’s algorithm to the following.

4 while $|Q| > 1$

This change causes the while loop to execute $|V| - 1$ times instead of $|V|$ times. Is this proposed algorithm correct?

25.3 Arbitrage is the use of discrepancies in currency exchange rates to transform one unit of a currency into more than one unit of the same currency. For example, suppose that 1 U.S. dollar buys 0.7 British pound, 1 British pound buys 9.5 French francs, and 1 French franc buys 0.16 U.S. dollar. Then, by converting currencies, a trader can start with 1 U.S. dollar and buy $0.7 \times 9.5 \times 0.16 = 1.064$ U.S. dollars, thus turning a profit of 6.4 percent.

Suppose that we are given $n$ currencies $c_1, c_2, \ldots, c_n$ and an $n \times n$ table $R$ of exchange rates, such that one unit of currency $c_i$ buys $R[i,j]$ units of currency $c_j$.

a. Give an efficient algorithm to determine whether or not there exists a sequence of currencies $(c_{i_1}, c_{i_2}, \ldots, c_{i_k})$ such that

$$R[i_1, i_2] \cdot R[i_2, i_3] \cdots R[i_{k-1}, i_k] \cdot R[i_k, i_1] > 1$$

Analyze the running time of your algorithm.