MST-Prim(G,w,r)

Q=V

for each u in Q

key[u]=∞

key[r]=0

while Q is not empty

u=Extract-Min(Q)

for each v ∈ Adj [u]

if v ∈ Q and w(u,v)<key[v] then key[v]=w(u,v)

Running time: $O(V+V \lg V+E \lg V)=O(E \lg V)$

using balanced binary search tree for Q

using fancier data structures can get $O(E+V \lg V)$
MST-Kruskal(G,w)

A=∅

for each vertex v
    Make-Set(v)

sort edges of E by nondecreasing weight w

for each edge (u,v) in order by nondecreasing weight
    if Find-Set(u) ≠ Find-Set(v) then
        add (u,v) to A
    Union(u,v)

return A

Running time: O(V+E lg E+Eα(E,V))=O(E lg E)

using disjoint-set union data structure
Theorem: Let $A$ be a subset of $E$ that is included in some MST for $G$. Let $(S, V-S)$ be any cut of $G$ that respects $A$ and let $(u,v)$ be a light edge crossing $(S, V-S)$. Then $(u,v)$ is safe for $A$.

Proof: Let $T$ be a MST that includes $A$. For contradiction, assume $T$ does not contain $(u,v)$. We want to construct another MST $T''$ that includes $A$ and $(u,v)$. Look at the path $p$ from $u$ to $v$ in $T$. $u$ and $v$ are on opposite sides of cut $(S, V-S)$. So, there must be some other edge of $p$ that crosses this cut, say $(x,y)$. Since $(S, V-S)$ respects $A$, we know that $(x,y) \notin A$. Let $T'' = T - \{(x,y)\} + \{(u,v)\}$ $T''$ is a spanning tree since taking out $(x,y)$ breaks $T$ into 2 components, and adding $(u,v)$ reconnects these 2 components. $T''$ is a minimum spanning tree since $w(u,v) \leq w(x,y)$.
Generic-MST(G,w)
A=∅
while A does not form a spanning tree
    find edge (u,v) that is safe for A and add edge (u,v) to A
return A

How to find safe edges?

A cut (S,V-S) is a partition of V

A cut respects the set A if no edge in A crosses the cut

An edge is a light edge crossing a cut if its weight is the minimum of any edge crossing a cut
TOPIC 18: Minimum Spanning Trees (MST)

given weighted graph $G=(V,E)$, we want an acyclic subset $T \subseteq E$ that spans $V$ and has minimum weight

$w(T)=$sum of weights of $T$’s edges

Growing a MST
algorithm manages set $A$ that is a subset of an MST
at each step, algorithm determines an edge $(u,v)$ that can be added to $A$ while maintaining the invariant that $A$ is a subset of an MST such edge $(u,v)$ is called a safe edge for $A$