Quantitative comparison of two-point correlation functions from real and mock galaxy surveys

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ABSTRACT

We present a novel bootstrap bandwidth selection method to obtain the optimal bin-size for estimation of the two-point correlation function (2PCF). As an example for demonstrating its effectiveness, we apply it to the real 2dF galaxy data as well as to 20 mock catalogs with varying cosmologies. More specifically, we use 20 mock catalogs generated from \( N \)-body simulations where the two parameters \( \Omega_m \) and \( \Omega_\lambda \) are varied. We quantify the differences between the real 2dF 2PCF estimates and those of the mock catalogs by means of a discrepancy measure \( X^2 \). This allows us to identify cosmologies that are close to the observed data. Since the number of mock catalogs is small, we are unable to create a confidence region for the two cosmological parameters. Instead, we assess uncertainty by creating a confidence set of cosmological models based on \( X^2 \). We do this using two methods, first, by assuming a \( \chi^2 \) distribution of the deviations of \( X^2 \) from the minimum, and second, by means of a bootstrap method. Both methods yield very similar confidence sets.

Subject headings: cosmology: cosmological parameters - cosmology: large-scale structure of universe - methods: statistical
1. Introduction

The two-point correlation function (2PCF), usually denoted by $\xi$, is an important quantity in studies in astronomy and cosmology. Intuitively $\xi(r)$ is related to the probability of finding two objects separated by distance $r$. More precisely, following Diggle (2003),

$$\lambda^2 \{ 1 + \xi(r) \} = \lim_{|\delta S| \to 0} \left\{ \frac{N(\delta S)N(\delta S + r)}{|\delta S|^2} \right\}, \quad (1)$$

where $\lambda$ is the number density, $\delta S$ is an infinitesimal region, $|\delta S|$ its volume, $\delta S + r$ the region shifted by distance $r$ and $N(\delta S)$ is the number of objects in $\delta S$.

The standard cosmological model describes the universe as beginning with an almost homogeneous mass of matter, with structures forming as the inhomogeneity became more pronounced through the effects of gravitation. Hence studying the correlations present in astronomical data, and specifically the 2PCF, is immensely helpful in understanding the evolution of the universe (Peebles 1980; Springel et al. 2006).

In planning for a survey of the sky, astronomers commonly generate mock catalogs that follow the constraints of the proposed survey, using a variety of cosmological models/parameters. The resulting mock catalogs reflect in some way the kind of observations the planned survey is expected to yield given the particular combinations of cosmological model and parameters used. The expectation is that, if the underlying workings of the cosmological model match the processes through which our universe evolved and the initial parameters are close to the values at the beginning of our universe, then the resulting simulated realization will “look” like our own.

It is, however, not straightforward to take two spatial data sets of points and decide whether they are similar or not. One approach that is commonly taken is to compare the correlation structures of the two data catalogs. In particular, since the 2PCF is often used to describe the large-scale structure of the universe, it is natural to use this quantity for
Estimating the two-point correlation function involves a choice of the bin-size or the bandwidth. This is in contrast to another measure of second-order correlation, the Ripley’s $K$ function, related to $\xi$ by $K(r) = \int_0^r [1 + \xi(u)]4\pi u^2 \, du$ in $\mathbb{R}^3$, where, because it is a cumulative measure, does not require a choice of bandwidth. See Illian et al. (2008) for more a more in-depth discussion of the use of these quantities in the analysis of spatial point patterns as well as applications to astronomy. Statisticians generally prefer the $K$ function because it is unbinned and, being an integrated function, may be simpler to estimate. However, direct links between the 2PCF and various astronomical processes in the early universe have been derived (e.g. Eisenstein et al. 2005), and astronomers tend to prefer the 2PCF and its Fourier transforms.

This choice of bin-size for estimating the 2PCF is important as it affects the efficiency of the resulting estimates. A large bin-size would over-smooth the data, reducing the variability of the estimate but increasing its bias. Conversely, a bin-size that is too small will result in an estimate that, although has small bias, has large variability. An optimal choice of the bin-size will balance between the bias and variance and can significantly improve estimates (Hastie et al. 2009). In many instances, however, estimates of $\xi$ are obtained using bandwidths that are selected heuristically or simply set to be the intervals between the values of $r$ at which $\xi$ is to be estimated. For example, Sanchez et al. (2012) used the Sloan Digital Sky Survey data to obtain 2PCF estimates using the Landy-Szalay estimator (Landy and Szalay 1993). The bin sizes appear to have been selected heuristically.

Loh and Jang (2010) introduced a method to select optimal bandwidths by means of a semiparametric bootstrap approach. Briefly, it involves selecting a bandwidth that minimizes an expected mean squared error that is estimated using a variance term and a bias term. The variance term is obtained by spatial bootstrap of the data set under
study while the bias term is obtained from the difference between two 2PCF estimates, one
obtained nonparametrically (e.g. the Landy-Szalay or Hamilton estimate) and the other
obtained parametrically (e.g. based on a power law model). This is described in more detail
in Section 3.

To illustrate our method, we estimate the 2PCF of the 2dF galaxy survey catalog as
well as 20 mock catalogs generated from variations of the standard cosmological model,
using the semiparametric bootstrap method to select optimal bandwidths. With estimates
of the two-point correlation function of each mock catalog and of the 2dF survey, a measure
$X^2$ is defined that quantifies the discrepancy between the estimates. The mock catalog with
the smallest $X^2$ then matches most closely to the 2dF survey in terms of the two-point
correlation function.

Ideally, to incorporate uncertainty in our results, we would like a confidence region
around the two cosmological parameters. However, this is not possible with only 20 mock
catalogs. Instead, we construct a confidence set containing of the set of mock catalogs
that are close enough to the 2dF catalog. This is done in two ways, first, by assuming
a $\chi^2$ distribution for $X^2 - X^2_{\text{min}}$, the difference between $X^2$ and the minimum value of
$X^2$ over the 20 mock catalogs, and second, by using a bootstrap approach applied to the
residuals. Details are provided in Section 3. This method of creating confidence sets of
simulations/models is useful in other scenarios where computationally intensive simulations
prohibit a large enough number of models to be run, so that the sets of parameters
considered are sparse.

We briefly describe the 2dF galaxy redshift survey and the mock catalogs in Section 2.
The discrepancy measure $X^2$ is defined in Section 3, which also contains a description of
the bootstrap bandwidth approach. Section 5 summarizes the results and also details our
methods for obtaining the confidence sets. A discussion in Section 6 concludes the paper.
2. The 2dF galaxy redshift survey and the mock catalogs

The 2dF Galaxy Redshift Survey (2dFGRS) is a spectroscopic survey that ran from 1998 to 2003 and measured redshifts of about 220,000 galaxies using the multi-fibre spectrograph on the Anglo-Australian Telescope (AAT) (Cole et al. 1998; Colless et al. 2001)). The observation region consists mostly of two strips of the sky, one each in the southern and northern Galactic hemispheres, referred to as the SGP and NGP strips respectively. The final data release was in June 2003. Detailed information about the 2dFGRS can be found in Colless et al. (2001). The data can be accessed at http://www.magnum.anu.edu.au/~TDFgg.

Cole et al. (1998) created mock 2dF galaxy redshift catalogs that follow the constraints of the actual 2dF survey. Briefly, these catalogs were obtained from $N$-body simulations using various cosmological models, such as bias models and different values of $\Omega_m$ and $\Omega_\Lambda$, respectively representing the present-day dimensionless matter density and cosmological energy density constant. These catalogs can be downloaded from http://star-www.dur.ac.uk/~cole/mocks/main.html. We chose all the 20 mock catalogs under the heading “Standard Models” on the website, consisting of two main subsets of models, referred to as “COBE normalized” and “structure normalized” by Cole et al. (1998). The COBE satellite measurements of the cosmic microwave background temperature fluctuations were used to set the amplitude density fluctuations for the COBE normalized models, while the structure normalized models (labeled STRC) were set to reproduce roughly the same observed abundance of rich galaxy clusters. In both subsets of models, two cosmological parameters $\Omega_m$ and $\Omega_\Lambda$ are varied, allowing for both open and flat models. Table 1 provides information about the values of $\Omega_m$ and $\Omega_\Lambda$ used. In addition there is a mock catalog each based on the standard CDM model (labeled SCDM) and a tilted Einstein-de Sitter model. Full details are available in Cole et al. (1998).
3. Obtaining optimal bandwidths for 2PCF estimation

The methods used in this paper consists of two parts. The first is a semiparametric bootstrap procedure to select the optimal bandwidths for 2PCF estimation and the second is the comparison of estimated 2PCF’s, specifically between estimates from the real galaxy survey data and each of the mock catalogs. The first method aims to obtain the best estimates of the 2PCF given the available data while the second identifies mock catalogs that match the actual 2dFGRS well.

There are many estimators for the 2PCF $\xi$. The angular 2PCF is also widely used as it does not require redshift information (see http://lbc.oa-roma.inaf.it/goods/art_drgclust/node7.html). In this work, we focus on the distance 2PCF and in particular use the Hamilton estimate (Hamilton 1993), specifically,

$$\hat{\xi}_{\text{Ham}}(r) = \frac{DD(r) \times RR(r)}{DR(r)^2} - 1,$$

where $DD(r) = \sum_i \sum_j 1\{|x_i - x_j| \in (r - \delta r/2, r + \delta r/2)\}$ is the number of data point pairs with distance separation close to $r$. Similar definitions hold for $RR(r)$ and $DR(r)$ with, respectively, both points from the random set and one point each from the data set and the random set. The Hamilton estimate is known for its reduced variability and bias. We set the random set $R$ to be the same size as the data set, with the RA and declination values randomized within the constraints of the SGP and NGP strips. Another common estimator is the Landy-Szalay estimator (Landy and Szalay 1993), given by

$$\hat{\xi}_{\text{LS}}(r) = \frac{DD(r) - 2DR(r) + RR(r)}{RR(r)}.$$

These two estimators are related to the two estimators recommended by Stein (1991) for their reduced variability. Our semiparametric bootstrap bandwidth selection method applies to either of these two and other estimators of $\xi$ and for the purposes of this paper, we will only consider the Hamilton estimate.
Computation of the values \( RR, DD \) and \( DR \) require the choice of a bin size \( \delta r \). We use a bootstrap bandwidth selection method to select the optimal bin size (Loh and Jang 2010). The optimal bin size is chosen to reduce the estimated mean squared error of the resulting estimate. The estimated mean squared error has two components, a bias and a variance component:

\[
\text{MSE}(r) = \text{E}[(\hat{\xi}(r) - \xi(r))^2]
\]

\[
= [\text{E}(\hat{\xi}(r)) - \xi(r)]^2 + \text{Var}[\hat{\xi}(r)],
\]

for each distance \( r \). The two quantities on the second line of (5) are not known and have to be estimated. We estimate the variance term by spatial bootstrap, which involves resampling from the data to create new data sets from which \( \xi \) is estimated. The variance of these bootstrapped estimates then provide an estimate of \( \text{Var}[\hat{\xi}(r)] \). The specific spatial bootstrap method we use is described in Loh (2008a,b).

The bias term is the difference between the non-parametric estimate of the 2PCF from the data and the true \( \xi \). Since \( \xi \) is not known some other estimate of the 2PCF, which we denote by \( \xi_p \), has to be chosen. This quantity \( \xi_p \) serves as a sort of anchor to prevent overfitting with a small bandwidth and the non-parametric estimate from deviating too far away, which would happen if the bandwidth were too large. Thus the bias and variance terms balance each other and the optimal bandwidth finds the best tradeoff between the variance and the bias.

The choice of \( \xi_p \) is quite flexible, e.g. \( \xi_p \) can be estimates based on some theoretical model separate from any data. Alternatively, \( \xi_p \) can be parametric or non-parametric estimates obtained from data sets other than the one being studied. As an illustration, in this paper, we use the power-law model for \( \xi_p \), specifically,

\[
\xi_p(r) = (r/r_0)\gamma,
\]
with parameters \( r_0 \) and \( \gamma \). This functional model for the 2PCF is well-known to fit observational data quite well and estimates of the 2PCF based on this model are often computed from galaxy and other surveys (e.g. Sawangwit et al. 2011; Ivashchenko et al. 2012). Rather than fit this model to our data, we use estimates obtained by Basilakos and Plionis (2004) for a subset of the SDSS data, specifically, \( \gamma = 1.6 \) and \( r_0 = 20.7 \). Basilakos and Plionis (2004) noted that their estimates were similar to those obtained by Peacock and West (1992) using the Abell catalog.

Let \( \hat{\xi}_b^* \) denote the \( b \)-th bootstrap estimate of \( \xi \), \( b = 1, \ldots B = 1000 \). We then set the estimate of the mean squared error at \( r_i \) corresponding to the \( i \)-th bin to be

\[
\text{MSE}_{\delta r}(r_i) = [\xi_p(r_i) - E_*(\hat{\xi}_b^*(r_i))]^2 + \text{Var}_*[\hat{\xi}_b^*(r_i)],
\]

(6)

where \( E_* \) and \( \text{Var}_* \) denote the bootstrap mean and variance of \( \hat{\xi}_b \) respectively. We do this for \( i = 1, \ldots 11 \) corresponding to \( r_1 = 5, r_2 = 10, r_3 = 20, \ldots r_{11} = 100 \). The integrated mean squared error is the sum of the mean squared errors over all the values of \( r_i \):

\[
\text{IMSE}_{\delta r} = \sum_{i=1}^{R} \text{MSE}_{\delta r}(r_i)
\]

(7)

where \( R \), the number of bins, is 11 in our case.

Both the MSE and IMSE are functions of the bin size \( \delta r \) used in the estimation of \( \xi \). The overall optimal bin size is the value of \( \delta r \) that minimizes IMSE. This overall optimal bin size balances between bias and variance for all the distances together. However, it is reasonable to expect that the optimal amount of smoothing (i.e. optimal bin size) varies with \( r \), so that, for example, a smaller bin-size is optimal at small \( r \) compared with larger \( r \). Hence we find adaptive optimal bin sizes instead, finding an optimal bin size \( \delta r_i \) for each value of \( r_i \), by minimizing MSE, i.e. (6), separately for each \( r_i \) instead of minimizing (7). Note that we chose optimal bin sizes \( \delta r \) for the 2dF catalog and each mock catalog separately, so that the estimate of \( \xi \) for each catalog is the “best” in terms of its mean.
squared error.

4. Measuring discrepancy and identifying confidence sets of cosmological models

With the adaptive optimal bandwidths obtained for the 2dFGRS and each of the 20 mock catalogs using the methods described in Section 3, estimates of the 2PCF can be obtained. In each case, redshifts were converted to radial distances using a cold dark matter model depending on the parameters $\Omega_m$ and $\Omega_\Lambda$ of the cosmological model that generated the mock catalog.

To compare the 2PCF estimates of the mock catalogs to that of the 2dFGRS, we compute the discrepancy measure

$$X_j^2 = \sum_i \left( \frac{\hat{\xi}_{\text{real}}(r_i) - \hat{\xi}_{\text{mock}}^j(r_i)}{\hat{\sigma}_i} \right)^2,$$

(8)

where $\hat{\xi}_{\text{real}}(r_i)$ and $\hat{\xi}_{\text{mock}}^j(r_i)$ are respectively the 2PCF estimates of the 2dFGRS and the $j$-th mock catalog at $r_i$. The quantity $\hat{\sigma}_i$ is an estimate of the uncertainty of $\hat{\xi}_{\text{real}}(r_i)$ obtained by means of bootstrap. Specifically, we bootstrap the real 2dF data to obtain data resamples, from which estimates of the two-point correlation function are computed. The value of $\hat{\sigma}_i$ is then set to be the standard error of the resulting bootstrap two-point correlation function estimates at $r_i$. The cosmological models used to generate the mock catalogs can then be ranked by their values of $X^2$, with models having smaller $X^2$ considered to be better fits to the real 2dF data.

The quantity $X^2$ can be related to the $\chi^2$ distribution. Specifically, if $X_{\text{min}}^2$ is the minimum of the $X_j^2$’s, with the assumption of normality for $\hat{\xi}$, $X^2 - X_{\text{min}}^2$ has a $\chi^2$ distribution with 2 degrees of freedom (Berkson 1980).
5. Results

We computed the discrepancy measure $X^2$ for each of the 20 mock catalogs, using estimates of the two-point correlation function at $r = 10, 20, \ldots, 100$. Table 1 shows their ranking from smallest $X^2$ (i.e. best fit to the 2dFGRS) to largest $X^2$. Note that the best model, the standard COBE model with $\Omega_m = 0.3, \Omega_\lambda = 0.7$, agrees with current observational constraints from e.g. WMAP (Spergel et al. 2003; Larson et al. 2011).

However, with only 20 mock catalogs within a two-dimensional parameter space, we are unable to provide a confidence region for $\Omega_m$ and $\Omega_\lambda$. Figure 1 shows a plot of the $X^2$ values for the 20 mock catalogs within the two-dimensional parameter space. In the figure, the $x$ and $y$ axes represent $\Omega_m$ and $\Omega_\lambda$ respectively, with the size of the circles related to the size of $X^2$. Thus, the locations of the smallest circles show the models that are most compatible with the observed data. We find that “COBE” models (represented by black circles in Figure 1) tend to be a better fit than the “Cluster” models: the black circles tend to be smaller than the red ones.

So, instead of providing a confidence region for $(\Omega_m, \Omega_\lambda)$, we obtain a confidence set of mock catalogs. While a confidence region contains all the sets of values $(\Omega_m, \Omega_\lambda)$ that satisfy, say, a 95% confidence level, the confidence set does this for models instead. Models that fall outside this 95% confidence set can be considered to failed the hypothesis test of matching the true 2dFGRS in terms of the 2PCF, at the 5% significance level.

We construct this confidence set of models in two ways. In the first method, we use the $\chi^2$ distribution with 2 degrees of freedom for $X^2 - X^2_{\text{min}}$. Since $P(\chi^2_2 \leq 5.99) = 0.95$, we identify the confidence set as the set of mock catalogs whose $X^2$ values differ from $X^2_{\text{min}}$ by less than 5.99. We find that the 10 best fitting models, ranked 1 to 10 in column 4 of Table 1, fall within this set.
However, this requires a normality assumption for $\hat{\xi}$, which Loh and Jang (2010) suggested might not hold. Hence we consider a second method, we construct a similar confidence set using non-parametric bootstrap. In particular, we compute, for each mock catalog, the residuals $\hat{\xi}(r_i) - \xi^j_{\text{mock}}(r_i)$, where $\xi^j_{\text{mock}}$ is the two-point correlation function for model $j$. The residuals are resampled and added to $\xi^j_{\text{mock}}(r_i)$ to produce resampled $\xi$ values. The resampled $\xi$ values are compared with those of the real 2dFGRS, yielding resampled $X^2$ values. The cosmological model with the smallest resampled $X^2$ is noted. This is repeated 1000 times. The cosmological models are then ordered according to the number of times they were chosen out of the 1000 resamples. Table 1 shows the results in column 5, where we have indicated the top 8 models found using this bootstrap method. These eight models together account for more than 88% of the 1000 bootstrap samples, i.e. one of these 8 models was the best fit to the 2dFGRS two-point correlation function for more than 880 times out of the 1000 bootstrap samples.

We find that there is general agreement between the confidence sets found by the two methods: the 8 models found by the bootstrap method are in the confidence set found by the $\chi^2$ method. Thus, while the standard COBE model with $\Omega_m = 0.3, \Omega_\lambda = 0.7$ is identified as the model that best fits the real 2dFGRS data, the uncertainty in the model as assessed by our $\chi^2$ and bootstrap methods suggest that some of the other models may not be significantly different, in terms of the discrepancy between their resulting two-point correlation functions and that of the real 2dF data.

6. Discussion

Estimating the 2PCF requires a choice of bandwidth(s) in order to compute the values $DD(r), DR(r)$ and $RR(r)$ at all the values of $r$ considered. Bandwidths that are too large or small can result in estimates that are respectively too smooth (low variance but high
bias) or too jagged (low bias but high variance). Selecting bandwidths that optimize the tradeoff between bias and variance allows investigators to extract as much information as possible from the data. We described a procedure to obtain adaptive optimal bandwidths, i.e. optimal at each distance \( r \), that uses spatial bootstrap to estimate the variability and a pilot estimator to estimate the bias. This pilot estimator can be chosen to be a 2PCF estimator obtained from a similar but separate data set, for example.

We illustrated the application of this to the comparison of 2PCF estimates of the 2dFGRS and those of 2dF mock catalogs, with the aim of identifying mock catalogs, and the cosmological models that generated them, that are close to the 2dFGRS. Specifically, we compared all 20 mock 2dF catalogs generated from combinations of values of \((\Omega_m, \Omega_\lambda)\) under the standard cosmological models with the real 2dF catalog, using 2PCF estimates \( \hat{\xi} \) where the bin-sizes have been optimally selected using our approach. The model identified as being closest to the real 2dFGRS in terms of the 2PCF is the COBE model with \( \Omega_m = 0.3, \Omega_\lambda = 0.7 \), in agreement with the current understanding.

To take into account the uncertainty, we obtain a confidence set of models that are statistically close to the real catalog, instead of constructing a confidence region of parameter values. This method of identifying confidence sets of models is useful since the number of mock catalogs is too small and not enough to cover the parameter space. Our two methods of constructing the confidence sets, by using a \( \chi^2 \) distribution for \( X^2 - X^2_{\text{min}} \) and by using resampling of the residuals, yielded similar results.

While the 2dF survey may now be rather dated, the concepts introduced here are more broadly applicable. When comparing theoretical models with observed data, standard statistics texts recommend estimates such as maximum likelihood estimates and confidence intervals (regions) computed from e.g. the Fisher information. However, in cases where the models are too complicated or the data has serious selection effects, comparison can only
be done with data generated from these models. We performed our comparisons using the 2PCF which in turn led to questions about optimally estimating it. While using the best estimators (generally considered to be the Landy-Szalay and Hamilton estimators, for the 2PCF) are crucial, better estimates can be achieved easily by using optimal bandwidths with these estimators.

If the simulations are computationally expensive, as they often are, it is not possible to densely cover the parameter space. Constructing confidence sets of the simulations (or models) allows us to make some statistical statements about the simulations that were performed.

An interesting direction of research is a sampling design of the parameter space, identifying the set of parameter values with which to run the models so that the resulting simulations provide the greatest amount of information about the parameters. The bootstrap bandwidth selection method can be applied to bin-size selection for higher-order correlation functions, and these in turn can be used to compare catalogs. It is however not clear how one can effectively combine discrepancy measures based on different correlation functions or other statistics into an overall discrepancy measure. Work done on meta-analysis may be applicable here. If the discrepancy measures are $\chi^2$ distributed, Fisher’s $p$ value method may be used to combine the $p$ values.

Our work can also be applied to surveys of other astronomical objects. Here, again, combining the results from separate data sources can be a challenge. Finally, similar analyses can be done with the more modern simulations, such as the SDSS data and mock catalogs or the Marenosrum Institut de Ciències de l’Espai (MICE) simulations$^{1}$.

\footnote{http://maia.ice.cat/mice/}
Acknowledgments

We thank Russell Johnston for converting the 2dF mock catalogs downloaded from the mock catalog website into a form which we could use. We also thank him and Martin Hendry for advice and helpful suggestions.
Table 1: List of cosmological models with their rank based on the discrepancy measure $D$ (smaller rank means better fit). The best fitting models, falling within a confidence set of models, based on a $\chi^2$ criterion and based on bootstrap are indicated by an asterisk * and + respectively (see text).

<table>
<thead>
<tr>
<th>Model</th>
<th>$\Omega_m$</th>
<th>$\Omega_\Lambda$</th>
<th>rank of $D$</th>
<th>confidence set</th>
</tr>
</thead>
<tbody>
<tr>
<td>COBE</td>
<td>0.1</td>
<td>0.9</td>
<td>5</td>
<td>* +</td>
</tr>
<tr>
<td>COBE</td>
<td>0.2</td>
<td>0.8</td>
<td>8</td>
<td>* +</td>
</tr>
<tr>
<td>COBE</td>
<td>0.3</td>
<td>0.7</td>
<td>1</td>
<td>* +</td>
</tr>
<tr>
<td>COBE</td>
<td>0.4</td>
<td>0.6</td>
<td>2</td>
<td>* +</td>
</tr>
<tr>
<td>COBE</td>
<td>0.5</td>
<td>0.5</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>COBE</td>
<td>0.3</td>
<td>0</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>COBE</td>
<td>0.4</td>
<td>0</td>
<td>10</td>
<td>*</td>
</tr>
<tr>
<td>COBE</td>
<td>0.5</td>
<td>0</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>COBE</td>
<td>1</td>
<td>0</td>
<td>19</td>
<td></td>
</tr>
<tr>
<td>STRC</td>
<td>0.2</td>
<td>0.8</td>
<td>17</td>
<td></td>
</tr>
<tr>
<td>STRC</td>
<td>0.3</td>
<td>0.7</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>STRC</td>
<td>0.4</td>
<td>0.6</td>
<td>9</td>
<td>* +</td>
</tr>
<tr>
<td>STRC</td>
<td>0.5</td>
<td>0.5</td>
<td>4</td>
<td>* +</td>
</tr>
<tr>
<td>STRC</td>
<td>0.2</td>
<td>0</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>STRC</td>
<td>0.3</td>
<td>0</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>STRC</td>
<td>0.4</td>
<td>0</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>STRC</td>
<td>0.5</td>
<td>0</td>
<td>7</td>
<td>*</td>
</tr>
<tr>
<td>STRC</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>* +</td>
</tr>
<tr>
<td>SCDM</td>
<td>1</td>
<td>0</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>TILT</td>
<td>1</td>
<td>0</td>
<td>6</td>
<td>* +</td>
</tr>
</tbody>
</table>
Fig. 1.— Plot of discrepancy measure $D$, represented by the size of the circles, on the two dimensional parameter space of $\Omega_m$ and $\Omega_\lambda$. 
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