WEEK 12

Balances on Transient Systems

Mass balances

- Transient (unsteady-state) processes: system variable changes with time
  - Ex: Batch and semibatch process

- The general balance equation is still valid:
  - Accumulation = input – output + generation – consumption

$$\frac{dM}{dt} = \dot{m}_{in} - \dot{m}_{out} + \dot{r}_{gen} - \dot{r}_{cons}$$

- We need an initial condition M(t=0)

- To do in class: page 545, ex. 11.1-1
  - Notice the difference between total mass balance and component balance.
  - Pay close attention to the units

Example: Mass balances

- To do in class: page 548, ex. 11.2-1
  - No reaction
  - Solution of ordinary differential equations
  - Concepts of practical ranges and domains

- To do in class: page 552, ex. 11.2-2

$$V\frac{dC_A}{dt} = F_{in}C_{Ain} - F_{out}C_{Aout} + \dot{r}_{gen} - V\dot{r}_{cons}$$

  - The steady-state concentration is obtained by setting the accumulation term to zero.
  - Integration tables may be necessary

Energy Balances

- Single phase nonreactive processes: accumulation = input - output

$$accumulation = \Delta E_{sys} = \Delta U_{sys} + \Delta E_{k,sys} + \Delta E_{p,sys}$$

$$input = \dot{m}_{in}\left(\hat{H}_{in} + \frac{u_{in}^2}{2} + gz_{in}\right)\Delta t + \dot{Q}\Delta t$$

$$output = \dot{m}_{out}\left(\hat{H}_{out} + \frac{u_{out}^2}{2} + gz_{out}\right)\Delta t + \dot{W}_s\Delta t$$

$$\frac{dU_{sys}}{dt} + \frac{dE_{k,sys}}{dt} + \frac{dE_{p,sys}}{dt} = \dot{m}_{in}\left(\hat{H}_{in} + \frac{u_{in}^2}{2} + gz_{in}\right) - \dot{m}_{out}\left(\hat{H}_{out} + \frac{u_{out}^2}{2} + gz_{out}\right) + \dot{Q} - \dot{W}_s$$

Energy Balances

- Simplifications
  - Single input stream, single output stream, same flow rate: $\dot{m}_{in} = \dot{m}_{out} = \dot{m}$
  - Kinetic and potential energy changes in the system and between inlet and outlet streams are negligible:

$$\frac{dE_{k,sys}}{dt} \approx \frac{dE_{p,sys}}{dt} \approx 0 \qquad \dot{m}\left(\frac{u_{in}^2}{2} - \frac{u_{out}^2}{2}\right) \approx 0 \qquad \dot{m}\left(gz_{in} - gz_{out}\right) \approx 0$$

As a result:

$$\frac{dU_{sys}}{dt} = \dot{m}\left(\hat{H}_{in} - \hat{H}_{out}\right) + \dot{Q} - \dot{W}_s$$

Energy Balances

– Perfectly mixed: $T_{out} = T_{sys} = T$

– No phase change or reactions:

$$U_{sys} = M\hat{U}_{sys} = M\left[\hat{U}(T_r) + C_v(T - T_r)\right]$$

$$\downarrow \; M, \hat{U}(T_r), C_v \text{ are constant}$$

$$\frac{dU_{sys}}{dt} = MC_v \frac{dT}{dt}$$

$$\hat{H}_{in} = C_p(T_{in} - T_r)$$

$$\hat{H}_{out} = C_p(T_{out} - T_r) \longrightarrow \hat{H}_{out} = C_p(T - T_r)$$

Open system: $$MC_v \frac{dT}{dt} = \dot{m}C_p(T_{in} - T) + \dot{Q} - \dot{W}_s$$

Closed system: $$MC_v \frac{dT}{dt} = \dot{Q} - \dot{W}$$

To do in class: Page 556, Ex. 11.3-1; Page 557, Ex. 11.3-2