# PHEN 612 

## SPRING 2008 <br> WEEK 3 <br> LAURENT SIMON

## Chapter 6*

Types of reactions: series, parallel, complex, and independent

1) Parallel (or competing) reactions: $A$
2) Series (or consecutive reactions): $A \xrightarrow{k_{1}} B \xrightarrow{k_{2}} C$
3) Complex reactions: parallel + series reactions
4) Independent reactions: $A \rightarrow B+C$

$$
D \rightarrow E+F
$$

*Elements of Chemical Reaction Engineering - Fourth Edition by H. Scott Fogler, Prentice Hall, 2006

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5) Desired or undesired reactions
$A \xrightarrow{A \xrightarrow{k_{D}} D} U$
D is the desired product; U is undesired.
Also $A \xrightarrow{k_{D}} D \xrightarrow{k_{U}} U$
Instantaneous selectivity of $D$ with respect to $U$
$S_{D / U}=r_{D} / r_{U}=($ rate of formation of $D) /($ rate of formation of $U$ )
Overall selectivity $=F_{D} / F_{U}=$ (exit molar flow rate of desired product)/(exit molar flow rate of undesired product)
For batch systems: Overall selectivity $=N_{D} / N_{U}$

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6) Instantaneous reaction yield: $Y_{D}=r_{D} /-r_{A}$ Overall reaction yield (batch) $=\mathrm{N}_{\mathrm{D}} /\left(\mathrm{N}_{\mathrm{A} 0}-\mathrm{N}_{\mathrm{A}}\right)$
Overall reaction yield (flow) $=\mathrm{F}_{\mathrm{D}} /\left(\mathrm{F}_{\mathrm{A} 0}-\mathrm{F}_{\mathrm{A}}\right)$
6.2 Parallel Reactions

Undesired products ( U ) can be minimized through selection of the reactor types and conditions.


The rate laws: $\mathrm{r}_{\mathrm{D}}=\mathrm{k}_{\mathrm{D}} \mathrm{C}_{\mathrm{A}}{ }^{\alpha 1} ; \mathrm{r}_{\mathrm{U}}=\mathrm{k}_{\mathrm{U}} \mathrm{C}_{\mathrm{A}}{ }^{\alpha 2} \quad-r_{A}=r_{D}+r_{U}=k_{D} C_{A}^{\alpha 1}+k_{U} C_{A}^{\alpha 2}$ $\alpha 1$ and $\alpha 2$ are positive reaction orders

Goal: we want $r_{D}$ to be high with respect to $r_{u}$

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- Selectivity parameter: $S_{D / U}=r_{D} / r_{U}=\left(k_{D} / k_{U}\right) C_{A}^{\alpha 1-\alpha 2}$


### 6.2.1 Maximizing the desired product for one reactant

Maximize $\mathrm{S}_{\mathrm{D} / \mathrm{U}}$
Case 1: $\alpha 1>\alpha 2 S_{D / U}=\left(k_{D} / k_{U}\right) C_{A}^{a}$
Let $\mathrm{a}=\alpha 1-\alpha 2$,
For gas phase: high pressure; for liquid phase: minimum diluent
-- A batch or plug flow reactor should be used
--- In a CSTR, the concentration of the reactant is at its lowest value
Case 2: $\alpha 2>\alpha 1 \quad S_{D / U}=\left(k_{D} / k_{U} C_{A}^{b}\right)$
Let $\mathrm{b}=\alpha 2-\alpha 1$,
$\mathrm{S}_{\mathrm{D} / \mathrm{U}}$ is large if A is kept as low as possible
--- A CSTR should be used; dilution of feed
Note: Temperature also has an effect on selectivity

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### 6.2.2 Reactor Selection and Operating Conditions

$$
\begin{array}{r}
A+B \xrightarrow[k_{1}]{A+B \xrightarrow[k_{2}]{\longrightarrow} U}
\end{array} \quad r_{D}=k_{1} C_{A}^{\alpha 1} C_{B}^{\beta 1} ; \quad r_{U}=k_{2} C_{A}^{\alpha 2} C_{B}^{\beta 2}
$$

Rate selectivity parameter:

- Different reactors and schemes are shown in Fig. 6.3
- Tubular or batch if we need to keep conc. high. CSTR if we need to keep conc. Low.


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6.3 Maximizing the desired product in series reactions

To optimize $B$, one need to pay attention to how fast both $B$ and C are being formed.

$$
A \xrightarrow{k_{1}} B \xrightarrow{k_{2}} C
$$

6.4 Algorithm for solution of complex reactions

Mathematical software should be used to solve most of these problems
6.4.1 Mole balances: Mole balances for multiple reactions are summarized in Table 6-1, page 327.

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### 6.4.2 Net Rates of Reaction

We need to write the net rate of formation for each species. We have q reactions:

$$
\begin{aligned}
& A+B \xrightarrow{k_{1 n}} 3 C+D \\
& A+2 C \xrightarrow{k_{2 A}} 3 E \\
& 2 B+3 E \xrightarrow{k_{34}} 4 F \\
& E+2 F \xrightarrow{k_{\text {en }}} G \\
& r_{A}=r_{1 A}+r_{2 A}+r_{3 A}+\ldots+r_{q A}=\sum_{i=1}^{q} r_{\text {iA }} \\
& \mathrm{r}_{\mathrm{B}}=\mathrm{r}_{1 \mathrm{~B}}+\mathrm{r}_{2 \mathrm{~B}}+\mathrm{r}_{3 \mathrm{~B}}+\ldots+\mathrm{r}_{\mathrm{qB}}=\sum_{i=1}^{q} r_{\mathrm{BB}} \quad r_{j}=\sum_{i=1}^{q} r_{i j} \\
& \text { 6.4.2-B Rate laws } r_{i j}=k_{i j} f_{i}\left(C_{A}, C_{B,}, \ldots, C_{j}, \ldots, C_{n}\right)
\end{aligned}
$$

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6.4.2-C Stoichiometry: Relative Rates of Reaction
$\mathrm{aA}+\mathrm{bB} \rightarrow \mathrm{cC}+\mathrm{dD}$
$\left(-r_{A} / a\right)=\left(-r_{B} / b\right)=\left(r_{C} / c\right)=\left(r_{D} / d\right)$

- Reaction (1): $A+B \xrightarrow{k_{A}} 3 C+D \quad r_{1 A}=-k_{1 A} C_{A} C_{B}$
$\left(-r_{1 A} / 1\right)=\left(-r_{1 B} / 1\right)=\left(r_{1 \mathrm{C}} / 3\right)=\left(\mathrm{r}_{1 \mathrm{D}} / 1\right)$
As a result: $r_{1 B}=-k_{1 A} C_{A} C_{B}$
The same calculations can be done for reactions 2 and 3.


## Chapter 6

6.4.2-D Combine individual rate laws to find the net rate

$$
\begin{align*}
& A+B \xrightarrow{k_{1 A}} 3 C+D  \tag{1}\\
& A+2 C \xrightarrow{k_{2 A}} 3 E \tag{2}
\end{align*}
$$

$$
\begin{aligned}
& r_{A}=r_{1 A}+r_{2 A}=-k_{1 A} C_{A} C_{B}-k_{2 A} C_{A} C_{C}^{2} \\
& r_{B}=r_{1 B}=-k_{1 A} C_{A} C_{B} \\
& r_{C}=r_{1 C}+r_{2 C}=3 k_{1 A} C_{A} C_{B}-2 k_{2 A} C_{A} C_{C}^{2} \\
& r_{D}=r_{1 D}=k_{1 A} C_{A} C_{B} \\
& r_{E}=r_{2 E}=3 k_{2 A} C_{A} C_{C}^{2}
\end{aligned}
$$

6.4.3 For gas phase, we have to take changes in volumetric flow rate (v) into account and use appropriate equations.

## Chapter 7*

Reaction mechanisms, pathways, bioreactions, and bioreactors

- An active intermediate ( $\mathrm{A}^{*}$ ) is a highenergy molecule that reacts virtually as fast as it is formed: $A+M \rightarrow A^{*}+M$
7.1.1. Pseudo-steady state hypothesis: $r_{A^{*}}=0$ : Net rate of formation is zero
*Elements of Chemical Reaction Engineering - Fourth Edition
by H. Scott Fogler, Prentice Hall, 2006


## Chapter 7

## Ex:

Activation: (1) $A+M \xrightarrow{k_{1}} A^{*}+M$
Deactivation: (2) $A^{*}+M \xrightarrow{k_{2}} A+M$
Decomposition: (3) $A^{*} \xrightarrow{k_{3}} P$
Rate of product formation: $\quad r_{p}=k_{3} C_{A^{*}}$

$$
\begin{aligned}
& r_{1 A^{*}}=k_{1} C_{A} C_{M} ; \quad r_{2 A^{*}}=-k_{2} C_{A^{*}} C_{M} ; \quad r_{3 A^{*}}=-k_{3} C_{A^{*}} \\
& r_{A^{*}}=r_{1 A^{*}}+r_{2 A^{*}}+r_{3 A^{*}} ; \\
& r_{A^{*}}=k_{1} C_{A} C_{M}-k_{2} C_{A^{*}} C_{M}-k_{3} C_{A^{*}}=0 ;
\end{aligned}
$$

$$
C_{A^{*}}=\frac{k_{1} C_{A} C_{M}}{k_{2} C_{M}+k_{3}} ;
$$

$$
r_{p}=\frac{k_{1} k_{3} C_{A} C_{M}}{k_{2} C_{M}+k_{3}}=-r_{A}
$$

Since $C_{M}$ is constant (i.e., inert)

$$
\begin{aligned}
& r_{p}=k C_{A} \\
& k=\frac{k_{1} k_{3} C_{M}}{k_{2} C_{M}+k_{3}}
\end{aligned}
$$

## Chapter 7

Chain reactions

1) Initiation: formation of an active intermediate
2) Propagation or chain transfer: interaction of an active intermediate with the reactant or product to produce another active intermediate
3) Termination: deactivation of the active intermediate to form products

## Chapter 7

Enzyme: high-molecular-weight protein that accelerates a biological reaction:

$$
S+E \Leftrightarrow E . S \rightarrow E+P
$$

7.2.1 Enzyme-substrate complex

Two models for enzyme-substrate interactions:

1) Lock and key model: topographical, structural compatibility between enzyme and substrate
2) Induced model: enzyme and substrate molecules are distorted

### 7.2.2 Mechanisms

1) Complex formation
2) Decomposition of complex
3) Product formation

## Chapter 7

## Ex:

$$
\begin{aligned}
& S+E \xrightarrow{k_{1}} E . S \\
& E . S \xrightarrow{k_{2}} E+S \\
& E . S+W \xrightarrow{k_{3}} P+E
\end{aligned}
$$

$$
\begin{aligned}
& r_{1 S}=-k_{1}(E)(S) ; \quad r_{2 S}=-k_{2}(E . S) ; \quad r_{3 P}=k_{3}(E . S)(W) ; \\
& -r_{s}=k_{1}(E)(S)-k_{2}(E . S) ; \\
& r_{E . S}=k_{1}(E)(S)-k_{2}(E . S)-k_{3}(E . S)(W) ; \\
& P S S H: r_{E . S}=0
\end{aligned}
$$

$-r_{S}=\frac{k_{1} k_{3}(E)(S)(W)}{k_{2}+k_{3} W} \quad$ but $\quad E_{t}=E+E . S$
As a result:

$$
-r_{S}=\frac{k_{1} k_{3}(W)\left(E_{t}\right)(S)}{k_{1}(S)+k_{2}+k_{3} W}
$$

## ?nanter?

### 7.2.3 Michaelis-Menten Equation

$$
k_{c a t}=k_{3}(W)
$$

Since water is in excess and $K_{M}=\frac{k_{\text {cow }}+k_{2}}{k_{1}}$

$$
-r_{S}=\frac{k_{\text {cat }}\left(E_{t}\right)(S)}{(S)+k_{M}}
$$

called Michaelis-Menten equation
$\mathrm{K}_{\text {cat }}$ (1/time) is the Turnover number = number of substrate molecules converted to product in a given time on a single-enzyme molecule when the enzyme is saturated with the substrate
$\mathrm{K}_{\mathrm{M}}$ (mole/volume): Michaelis constant = affinity constant = measure of the attraction of the enzyme for the substrate.

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- $\mathrm{V}_{\text {max }}=\mathrm{k}_{\text {cat }}\left[\mathrm{E}_{\mathrm{t}}\right]$

Maximum rate of reaction for a given total enzyme concentration $\quad-r_{s}=\frac{V_{m u n}(s)}{k_{w}+S}$

- Evaluation of Michaelis-Menten parameters: $\mathrm{V}_{\max }$ and $\mathrm{K}_{\mathrm{M}}$ : : Lineweaver - Burk plot : $\frac{1}{-r_{s}}=\frac{1}{V_{\max }}+\frac{K_{M}}{V_{\max }}\left(\frac{1}{(S)}\right)$;

Hanes-Woolf plot: $\frac{(S)}{-r_{s}}=\frac{K_{M}}{V_{\max }}+\frac{1}{V_{\max }}(S)$;
Eadie-Hofstee plot: $-r_{s}=V_{\max }-K_{M}\left(\frac{-r_{s}}{(S)}\right)$

## Chapter 7

- The product-enzyme complex:

$$
E+S \Leftrightarrow E . S \Leftrightarrow P+E
$$

- Briggs-Haldane rate law: $-r_{s}=\frac{V_{m \times x}+\left(C_{s}-\frac{C_{p}}{K_{E}}\right)}{C_{s}+K_{\text {Kem }}+K_{s} C_{P}}$
- 7.2.4 Batch reactor calculations for enzyme reactions $-\frac{d N_{S}}{d t}=-r_{S} V ;-\frac{d C_{S}}{d t}=-r_{S} ;-r_{S}=\frac{V_{\max } C_{S}}{k_{M}+C_{S}}$

$$
\begin{aligned}
& t=\int_{C_{S} 0}^{C_{S 0}} \frac{d C_{S}}{-r_{S}} ; t=\frac{K_{M}}{V_{\max }} \ln \left(\frac{C_{S 0}}{C_{S}}\right)+\frac{C_{S 0}-C_{S}}{V_{\max }} ; \\
& t=\frac{K_{M}}{V_{\max }} \ln \left(\frac{1}{1-X}\right)+\frac{C_{S 0} X}{V_{\max }} ; \\
& \frac{1}{t} \ln \left(\frac{1}{1-X}\right)=\frac{V_{\max }}{K_{M}}-\frac{C_{S 0} X}{K_{M} t}
\end{aligned}
$$

- Effect of temperature is the result of arrhenius equation and enzyme de-activation

