CS 341: Foundations of Computer Science II
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Homework 1

1. Answer the questions below for the sets of strings:

$$\begin{aligned} C &= \{\varepsilon, aab, baa\}, \\ D &= \{bb, aab\}, \\ E &= \{\varepsilon\}, \\ F &= \emptyset. \end{aligned}$$

Except for parts (c) and (h), write out any sets with the elements in *string order*, i.e., shorter strings appear before longer strings, and strings of the same length are in alphabetical order. For any infinite sets, list only the first 8 elements in string order, but be sure to also indicate that the set is infinite by including 3 dots after listing the first 8 elements. For any set S of strings, we define

$$\begin{aligned} S^* &= \{\, x_1 x_2 \cdots x_k \mid k \geq 0 \text{ and each } x_i \in S \,\}, \\ S^+ &= \{\, x_1 x_2 \cdots x_k \mid k \geq 1 \text{ and each } x_i \in S \,\}, \end{aligned}$$

where the concatenation of $k = 0$ strings is the empty string ε.

(a) What is $D \cup C$?

(b) What is $C \cup F$?

(c) What is $C \times D$?

(d) What is $C \cap D$?

(e) What is $D \circ C$?

(f) What is $C \circ E$?

(g) What is $D \circ D \circ D$?

(h) What is $\mathcal{P}(C)$, the power set of C ?

(i) What is $D - C$?

(j) What is C^+ ?

(k) What is F^* ?

(l) Is $E \subseteq C$?

(m) Is $D \subseteq C$?

(n) For this part, we first need some definitions.

- The *reverse* of a string w, written $w^{\mathcal{R}}$, is the same string with the symbols written in reverse order. For example, $(cat)^{\mathcal{R}} = tac$.
- A collection of objects is *closed* under some operation if applying that operation to members of the collection returns an object still in the collection. For example, the set $\mathcal{N} = \{0, 1, 2, 3, \ldots\}$ is closed under the function $f(x) = x^2$ since the square of any nonnegative integer always results in a nonnegative integer. However, $\mathcal{N}$ is not closed under square root since, for example, $3 \in \mathcal{N}$ but $\sqrt{3} \notin \mathcal{N}$.

Which of C, D, and E are closed under reversal? Explain your answers.

2. Let w be a string of symbols and let the language T be defined as adding w to the language S; i.e., $T = S \cup \{w\}$. Suppose further that $T^* = S^*$.

   (a) Is it necessarily true that $w \in S$? If this is necessarily true, give a proof. If this is not necessarily true, give a counterexample.

   (b) Is it necessarily true that $w \in S^*$? If this is necessarily true, give a proof. If this is not necessarily true, give a counterexample.

3. For each of the following parts, give an example satisfying the given conditions. Give a brief explanation for each of your examples.

   (a) Give an example of a set S of strings such that $S^* = S^+$.

   (b) Give an example of a set S of strings such that $S^* \neq S^+$.

   (c) Give an example of a set S of strings such that $S = S^*$.

   (d) Give an example of a set S of strings such that $S \neq S^*$.

   (e) Give an example of a set S of strings such that S^* is finite.

4. Suppose we define a restricted version of the Java programming language in which variable names must satisfy all of the following conditions:

   - A variable name can only use Roman letters (i.e., a, b, ..., z, A, B, ..., Z) or Arabic numerals (i.e., 0, 1, 2, ..., 9); i.e., underscore is not allowed.
   - A variable name must start with a Roman letter: a, b, ..., z, A, B, ..., Z
   - The length of a variable name must be no greater than 8.
   - A variable name cannot be a keyword (e.g., if). The set of keywords is finite.

   Let L be the set of all valid variable names in our restricted version of Java.

   (a) Let L_0 be the set of strings satisfying the first 3 conditions above; i.e., we do not require the last condition. How many strings are in L_0? You can just give a general formula for your answer; you do not need to give a single number.

   (b) Prove that L is finite, where L is the set of strings satisfying all four conditions.

5. Let S be any set of strings. Prove that $S^* = S^+$ if and only if $\varepsilon \in S$.