Homework 3

1. Give NFAs with the specified number of states recognizing each of the following languages. In all cases, the alphabet is $\Sigma = \{\texttt{0}, \texttt{1}\}$.

   (a) The language $\{ w \in \Sigma^* \mid w \text{ ends with } \texttt{00} \}$ with three states.

   (b) The language $\{ w \in \Sigma^* \mid w \text{ contains the substring } \texttt{0101}, \text{ i.e., } w = x\texttt{0101}y \text{ for some } x, y \in \Sigma^* \}$ with five states.

   (c) The language $\{ w \in \Sigma^* \mid w \text{ contains at least two } \texttt{0}\text{s, or exactly two } \texttt{1}\text{s} \}$ with six states.

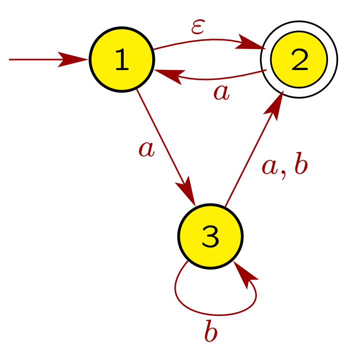
   (d) The language $\{\varepsilon\}$ with one state.

   (e) The language $\texttt{0}^*\texttt{1}^*\texttt{0}^*\texttt{0}$ with three states.

2. (a) Show by giving an example that, if M is an NFA that recognizes language C, swapping the accept and non-accept states in M doesn't necessarily yield a new NFA that recognizes $\overline{C}$.

   (b) Is the class of languages recognized by NFAs closed under complement? Explain your answer.

3. Use the construction given in Theorem 1.39 to convert the following NFA N into an equivalent DFA.



4. Give regular expressions that generate each of the following languages. In all cases, the alphabet is $\Sigma = \{a, b\}$.

   (a) The language $\{ w \in \Sigma^* \mid |w| \text{ is odd} \}$.

   (b) The language $\{ w \in \Sigma^* \mid w \text{ has an odd number of } a\text{'s} \}$.

(c) The language $\{ w \mid w$ contains at least two a's, or exactly two b's $\}$.

(d) The language $\{ w \in \Sigma^* \mid w$ ends in a double letter $\}$. (A string contains a *double letter* if it contains aa or bb as a substring.)

(e) The language $\{ w \in \Sigma^* \mid w$ does not end in a double letter $\}$.

(f) The language $\{ w \in \Sigma^* \mid w$ contains exactly one double letter $\}$. For example, $baaba$ has exactly one double letter, but $baaaba$ has two double letters.

5. Suppose we define a restricted version of the Java programming language in which variable names must satisfy all of the following conditions:

- A variable name can only use Roman letters (i.e., `a`, `b`, ..., `z`, `A`, `B`, ..., `Z`) or Arabic numerals (i.e., `0`, `1`, `2`, ..., `9`); i.e., underscore and dollar sign are not allowed.
- A variable name must start with a Roman letter: `a`, `b`, ..., `z`, `A`, `B`, ..., `Z`
- The length of a variable name must be no greater than 8.
- A variable name cannot be a keyword (e.g., if). The set of keywords is finite.

Let L be the set of all valid variable names in our restricted version of Java.

(a) Let L_0 be the set of strings satisfying the first 3 conditions above; i.e., we do not require the last condition. Give a regular expression for L_0.

(b) Prove that L has a regular expression, where L is the set of strings satisfying all four conditions.

(c) Give a DFA for the language L_0 in part (a), where the alphabet Σ is the set of all printable characters on a computer keyboard (no control characters), except for parentheses to avoid confusion.

6. Define L to be the set of strings that represent numbers in a modified version of Java. The goal in this problem is to define a regular expression and an NFA for L. To precisely define L, let the set of *digits* be $\Sigma_1 = \{ \texttt{0}, \texttt{1}, \texttt{2}, \ldots, \texttt{9} \}$, and define the set of *signs* to be $\Sigma_2 = \{ \texttt{+}, \texttt{-} \}$. Then $L = L_1 \cup L_2 \cup L_3$, where

- L_1 is the set of all strings that are decimal integer numbers. Specifically, L_1 consists of strings that start with an optional sign, followed by one or more digits. Examples of strings in L_1 are "`02`", "`+9`", and "`-241`".
- L_2 is the set of all strings that are floating-point numbers that are not in exponential notation. Specifically, L_2 consists of strings that start with an optional sign, followed by zero or more digits, followed by a decimal point, and end with zero or more digits, where there must be at least one digit in the string. Examples of strings in L_2 are "`13.231`", "`-28.`" and "`.124`". All strings in L_2 have exactly one decimal point.

- L_3 is the set of all strings that are floating-point numbers in exponential notation. Specifically, L_3 consists of strings that start with a string from L_1 or L_2, followed by "`E`" or "`e`", and end with a string from L_1. Examples of strings in L_3 are "`-80.1E-083`", "`+8.E5`" and "`1e+31`".

Assume that there is no limit on the number of digits in a string in L. Also, we do not allow for the suffixes `L`, `l`, `F`, `f`, `D`, `d`, at the end of numbers to denote types (long integers, floats, and doubles). Define Σ as the alphabet of all printable characters on a computer keyboard (no control characters), except for parentheses to avoid confusion.

(a) Give a regular expression for L_1. Also, give an NFA and a DFA for L_1 over the alphabet Σ.

(b) Give a regular expression for L_2. Also, give an NFA for L_2 over the alphabet Σ.

(c) Give a regular expression for L_3. Also, give an NFA for L_3 over the alphabet Σ.

(d) Give a regular expression for the language L. Also, give an NFA for L over the alphabet Σ.