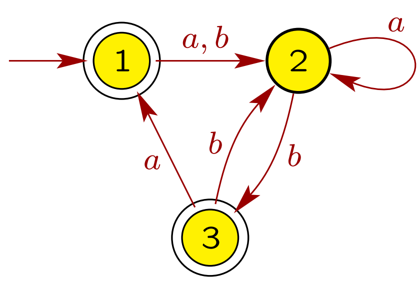
Homework 4

1. Use the procedure described in Lemma 1.55 to convert the regular expression $(((\texttt{00})^*(\texttt{11}))\cup\texttt{01})^*$ into an NFA.

2. Use the procedure described in Lemma 1.60 to convert the following DFA to a regular expression.



3. Each of the following languages is either regular or nonregular. If a language is regular, give a DFA and regular expression for it. If a language is nonregular, give a proof.

   (a) $A_1 = \{ www \mid w \in \{a,b\}^* \}$.

   (b) $A_2 = \{ w \in \{a,b\}^* \mid w = w^{\mathcal{R}} \}$.

   (c) $A_3 = \{ a^{2n}b^{3n}a^n \mid n \geq 0 \}$.

   (d) $A_4 = \{ w \in \{a,b\}^* \mid w$ has more a's than b's $\}$.

   (e) $A_5 = \{ w \in \{a,b\}^* \mid n_{ab}(w) = n_{ba}(w) \}$, where $n_s(w)$ is the number of occurrences of the substring $s \in \{a,b\}^*$ in w. For example, the string $w_1 = aaabbabbaa$ has $n_{ab}(w_1) = 2$ and $n_{ba}(w_1) = 2$, so $w_1 \in A_5$. Also, the string $w_2 = aaabbabbaab$ has $n_{ab}(w_2) = 3$ and $n_{ba}(w_2) = 2$, so $w_2 \notin A_5$.

4. Suppose that language A is accepted by an NFA N, and language B is the collection of strings *not* accepted by some DFA M. Prove that $A \circ B$ is a regular language.

5. (a) Prove that if we add a finite set of strings to a regular language, the result is a regular language.

   (b) Prove that if we remove a finite set of strings from a regular language, the result is a regular language.

   (c) Prove that if we add a finite set of strings to a nonregular language, the result is a nonregular language.

(d) Prove that if we remove a finite set of strings from a nonregular language, the result is a nonregular language.

6. Consider the following statement: "If A is a nonregular language and B is a language such that $B \subseteq A$, then B must be nonregular." If the statement is true, give a proof. If it is not true, give a counterexample showing that the statement doesn't always hold.