CS 341: Foundations of Computer Science II
Prof. Marvin Nakayama

Homework 6

1. Give pushdown automata that recognize the following languages. Give both a drawing and 6-tuple specification for each PDA.

   (a) $A = \{ w \in \{0,1\}^* \mid w \text{ contains at least three } 1\text{s} \}$

   (b) $B = \{ w \in \{0,1\}^* \mid w = w^{\mathcal{R}} \text{ and the length of } w \text{ is odd} \}$

   (c) $C = \{ w \in \{0,1\}^* \mid w = w^{\mathcal{R}} \}$

   (d) $D = \{ a^i b^j c^k \mid i,j,k \geq 0, \text{ and } i = j \text{ or } j = k \}$

   (e) $E = \{ a^i b^j c^k \mid i,j,k \geq 0 \text{ and } i + j = k \}$

   (f) $F = \{ a^{2n} b^{3n} \mid n \geq 0 \}$

   (g) $H = \{ a^i b^j c^k \mid i,j,k \geq 0 \text{ and } i + k = j \}$

   (h) $L = \{ ab^n acab^n a \mid n \geq 0 \}$.

   (i) $\emptyset$, with $\Sigma = \{0,1\}$

   (j) The language J of strings of properly balanced left and right brackets: every left bracket can be paired with a unique subsequent right bracket, and every right bracket can be paired with a unique preceding left bracket. Moreover, the string between any such pair has the same property. For example, [ ] [[[ ] [ ]] [ ]] $\in J$.

2. (a) Use the languages

$$\begin{aligned} A &= \{ a^m b^n c^n \mid m,n \geq 0 \} \text{ and} \\ B &= \{ a^n b^n c^m \mid m,n \geq 0 \} \end{aligned}$$

   together with Example 2.36 of the textbook to show that the class of context-free languages is not closed under intersection.

   (b) Use part (a) and DeMorgan's law (Theorem 0.20 of the textbook) to show that the class of context-free languages is not closed under complementation.

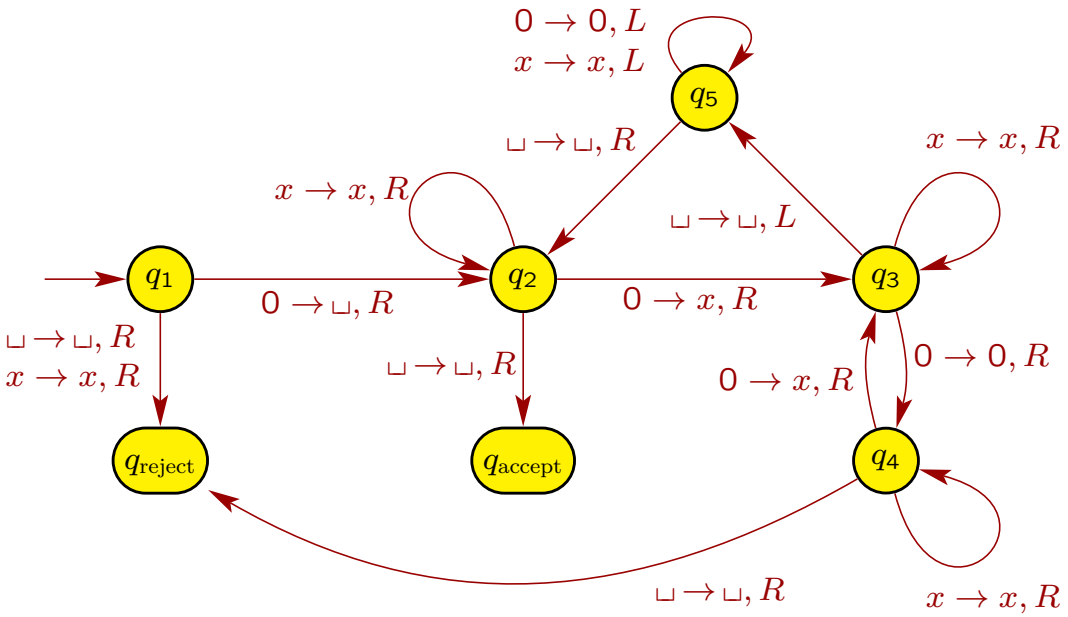
3. (Optional) Consider the following CFG $G = (V, \Sigma, R, S)$, where $V = \{S, T, X\}$, $\Sigma = \{a, b\}$, the start variable is S, and the rules R are

$$\begin{aligned} S &\rightarrow aTXb \\ T &\rightarrow XTS \mid \varepsilon \\ X &\rightarrow a \mid b \end{aligned}$$

   Convert G to an equivalent PDA using the procedure given in Lemma 2.21.

4. Use the pumping lemma to prove that the language $A = \{ 0^{2n} 1^{3n} 0^n \mid n \geq 0 \}$ is not context free.

5. The Turing machine M below recognizes the language $A = \{ 0^{2^n} \mid n \geq 0 \}$.



In each of the parts below, give the sequence of configurations that M enters when started on the indicated input string.

(a) 00

(b) 000000