Homework 7

1. Give an implementation-level description of a Turing machine that decides the language $B = \{\, 0^n 1^n 2^n \mid n \geq 0 \,\}$.

2. (a) Show that the class of decidable languages is closed under union.

   (b) Show that the class of Turing-recognizable languages is closed under union.

3. In Theorem 3.21 we showed that a language is Turing-recognizable iff some enumerator enumerates it. Why didn't we use the following simpler TM for the forward direction of the proof? As before, $s_1, s_2, \ldots$ is a list of all strings in Σ^*.

   E = "Ignore the input.
   1. Repeat the following for $i = 1, 2, 3, \ldots$
   2. Run M on s_i.
   3. If it accepts, print out s_i."

4. A Turing machine with doubly infinite tape is similar to an ordinary Turing machine, but its tape is infinite to the left as well as to the right. The tape is initially filled with blanks except for the portion that contains the input. Computation is defined as usual except that the head never encounters an end to the tape as it moves leftward. Show that this type of Turing machine recognizes the class of Turing-recognizable languages.