Homework 8

1. Consider the decision problem of testing whether a DFA and a regular expression are equivalent. Express this problem as a language and show that it is decidable.

2. Consider the decision problem of testing whether a CFG generates the empty string. Express this problem as a language and show that it is decidable.

3. Let $\Sigma = \{\texttt{0}, \texttt{1}\}$, and consider the decision problem of testing whether a regular expression with alphabet Σ generates at least one string w that has `111` as a substring. Express this problem as a language and show that it is decidable.

4. Consider the emptiness problem for Turing machines:

$$E_{\text{TM}} = \{ \langle M \rangle \mid M \text{ is a Turing machine with } L(M) = \emptyset \}.$$

Show that E_{TM} is co-Turing-recognizable. (A language L is co-Turing-recognizable if its complement $\overline{L}$ is Turing-recognizable.) Note that the complement of E_{TM} is

$$\overline{E_{\text{TM}}} = \{ \langle M \rangle \mid M \text{ is a Turing machine with } L(M) \neq \emptyset \}.$$

(Actually, $\overline{E_{\text{TM}}}$ also contains all $\langle M \rangle$ such that $\langle M \rangle$ is not a valid Turing-machine encoding, but we will ignore this technicality.)

5. Recall that the equivalence problem for DFAs has language

$$\begin{aligned} EQ_{\text{DFA}} &= \{ \langle A, B \rangle \mid A \text{ and } B \text{ are DFAs and } L(A) = L(B) \} \\ &\subseteq \{ \langle A, B \rangle \mid A \text{ and } B \text{ are DFAs} \} \equiv \Omega. \end{aligned}$$

In other words, for any $\langle A, B \rangle \in \Omega$, the pair $\langle A, B \rangle \in EQ_{\text{DFA}}$ if and only if A and B *agree* on every possible input string $s \in \Sigma^*$, where Σ is the input alphabet for both A and B. Specifically, we say that A and B *agree* on a string s if A and B both accept s or both reject s. Similarly, A and B *disagree* on s if one of the DFAs accepts and the other rejects. Theorem 4.5 establishes that EQ_{DFA} is decidable. The proof builds a DFA C for the symmetric difference of $L(A)$ and $L(B)$, which is possible because the class of regular languages is closed under complementation, intersection, and union. Then it checks if the symmetric difference is empty using the decider for E_{DFA}.

Instead of the proof that was given for Theorem 4.5, suppose we instead try to prove that EQ_{DFA} is decidable using the following TM M' that checks if the two inputted

DFAs A and B always agree on every possible input string. Let $s_1, s_2, \ldots$ be an enumeration of Σ^*, and define TM M' as follows:

M' = "On input $\langle A, B \rangle \in \Omega$, where A and B are DFAs:

0. Check if $\langle A, B \rangle$ is a proper encoding of 2 DFAs. If not, *reject.*

1. Repeat the following for $i = 1, 2, 3, \ldots$

2. Run A and B on s_i.

3. If A and B disagree on s_i, then *reject*.

4. If A and B always agree on each s_i, then *accept.* "

What is the problem with this approach?