

Homework 11

1. Answer each part TRUE or FALSE.
 - (a) $2n = O(n)$.
 - (b) $n^2 = O(n)$.
 - (c) $n^2 = O(n \log^2 n)$.
 - (d) $n \log n = O(n^2)$.
 - (e) $3^n = O(2^n)$.
 - (f) $3^n = 2^{O(n)}$.
 - (g) $2^{2^n} = O(2^{2^n})$.
2. Let $b > 1$ be a constant. Show that $O(t(n)) \times O(b^{t(n)}) = 2^{O(t(n))}$.
3.
 - (a) Show that P is closed under union.
 - (b) Show that P is closed under concatenation.
 - (c) Show that P is closed under complementation.
4. A **triangle** in an undirected graph is a 3-clique. Show that $TRIANGLE \in P$, where $TRIANGLE = \{ \langle G \rangle \mid G \text{ contains a triangle} \}$.
5. Using the polynomial-time algorithm for context-free language recognition (i.e., the CYK algorithm or dynamic programming), fill out the table for string $w = abba$ and CFG G :

$$\begin{aligned} S &\rightarrow RT \\ R &\rightarrow TR \mid a \\ T &\rightarrow TR \mid b \end{aligned}$$