

Homework 13

1. The *Set Partition Problem* takes as input a set S of numbers. The question is whether the numbers can be partitioned into two sets A and $\bar{A} = S - A$ such that

$$\sum_{x \in A} x = \sum_{x \in \bar{A}} x.$$

Show that *SET-PARTITION* is NP-Complete. (Hint: Reduce *SUBSET-SUM*.)

2. Let

$DOUBLE-SAT = \{ \langle \phi \rangle \mid \phi \text{ is a Boolean formula with two satisfying assignments} \}$.

Show that *DOUBLE-SAT* is NP-Complete. (Hint: Reduce *3SAT*.)

3. Let G represent an undirected graph. Also let

$SPATH = \{ \langle G, a, b, k \rangle \mid G \text{ contains a simple path of length at most } k \text{ from } a \text{ to } b \}$

and

$LPATH = \{ \langle G, a, b, k \rangle \mid G \text{ contains a simple path of length at least } k \text{ from } a \text{ to } b \}$.

- (a) Show that $SPATH \in P$.
- (b) Show that $LPATH$ is NP-Complete. You may assume the NP-completeness of *UHAMPATH*, the Hamiltonian path problem for undirected graphs.