Final Exam
CIS 341: Introduction to Logic and Automata — Spring 2001
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Instructions:

- Write all of your answers in a Microsoft Word document, which you are to e-mail as an attachment to cis341DL@cis.njit.edu when you are done. Be sure to include your full name and student ID at the top of the document.
- **The name of the Word file must be your last name followed by a dash and followed by your first name.** For example, if your name is Joe Smith, then name your file Smith-Joe.doc .
- Right below your name and student ID, type in the following: "I have read and understand all of the instructions, and I will obey the Academic Honor Code. I will not discuss the exam with anyone other than possibly the course instructor." **If you do not do this, then you will get a 0 on the exam.**
- This exam has 4 pages in total, numbered 1 to 4. Make sure your exam has all the pages.
- This exam will be 2 hour and 30 minutes in length. You must e-mail as an attachment your Microsoft Word document containing your answers to cis341DL@cis.njit.edu by the end of the exam period. If you do not e-mail your solutions by the end of the exam period, you will receive a 0 on the exam.
- **After you e-mail in your solutions, you must wait for an acknowledgment from me saying that I got your file and was able to read it. You should receive an acknowledgment within 10 minutes.**
- Send any questions you have during the exam to cis341DL@cis.njit.edu .
- This is an open-book, open-note exam.
- For all problems, follow these instructions:
    1. FA stands for finite automaton; TG stands for transition graph; CFG stands for context-free grammar; CFL stands for context-free language; PDA stands for pushdown automaton.
    2. For any proofs, be sure to provide a step-by-step argument, with justifications for every step. You may assume that the theorems in the textbook hold; i.e., you do not have to reprove the theorems in the textbook. When using a theorem from the textbook, make sure you refer to it by number (e.g., Theorem 3).

1. **[20 points]** For each of the following, specify TRUE if the statement is always correct. Otherwise, specify FALSE

    a. TRUE or FALSE: If L_1 and L_2 are regular languages, then there is some Turing machine that accepts L_1 L_2.

    b. TRUE or FALSE: If L is generated by a context-free grammar that is not a regular grammar, then L is not a regular language.

    c. TRUE or FALSE: If L is generated by a context-free grammar that is a regular grammar, then L is a regular language.

    d. TRUE or FALSE: If L is accepted by a pushdown automaton, then L* is a context-free language.

    e. TRUE or FALSE: All context-free languages are non-regular languages.

    f. TRUE or FALSE: If L is a language accepted by a Turing machine, then L is a context-free language.

    g. TRUE or FALSE: A pushdown automaton may accept infinitely many different strings.

    h. TRUE or FALSE: If L is a context-free language, then there is a deterministic pushdown automaton that accepts exactly L.

    i. TRUE or FALSE: There is a Turing machine that accepts the language EVEN-EVEN.

    j. TRUE or FALSE: A Turing machine may crash when processing a string.

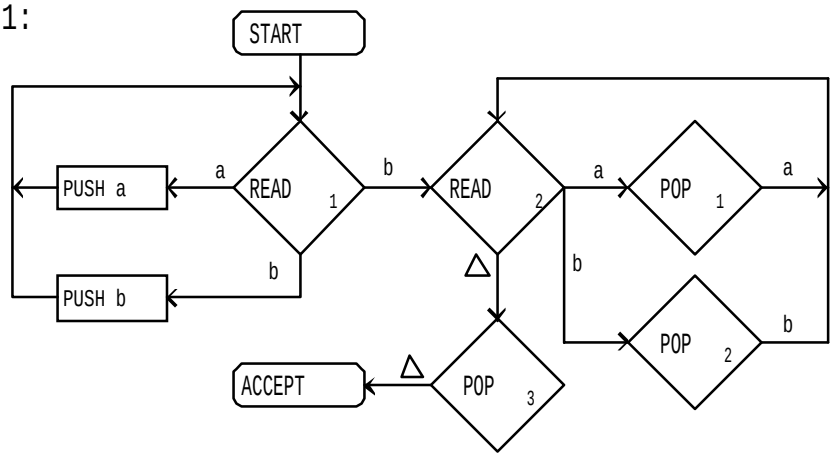
2. **[30 points]** Let L be the language generated by the context-free grammar:

$$S \rightarrow aSa \mid bSb \mid a$$

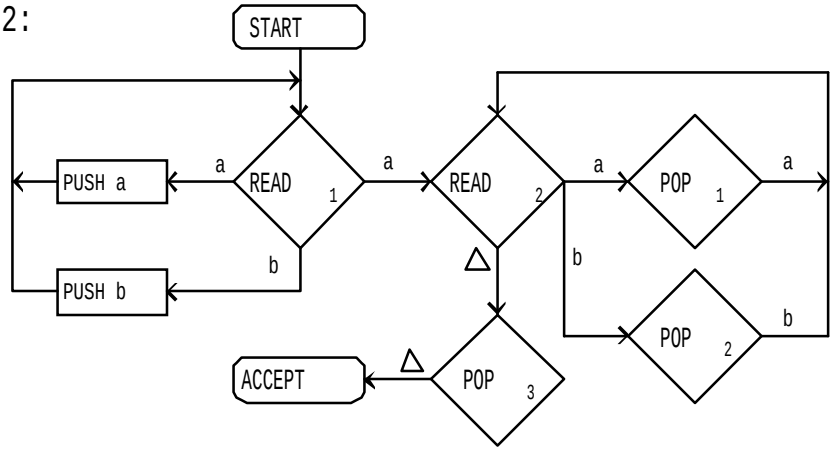
with alphabet $\Sigma = \{ a, b \}$. Consider the 3 PDAs, labeled M1, M2, M3, on the next page. Observe that the differences in the 3 PDAs are the labels of the arc from $READ_1$ to $READ_2$ and what is pushed onto the stack. For each machine, indicate if it accepts exactly L or not. For each machine that does not accept exactly L, do the following:

- Show that it does not accept exactly L by giving either an example of a string w not in L that is accepted by the machine, or an example of a string w in L that is not accepted by the machine. Explain your example.
- Give a context-free grammar for the language exactly accepted by the machine.

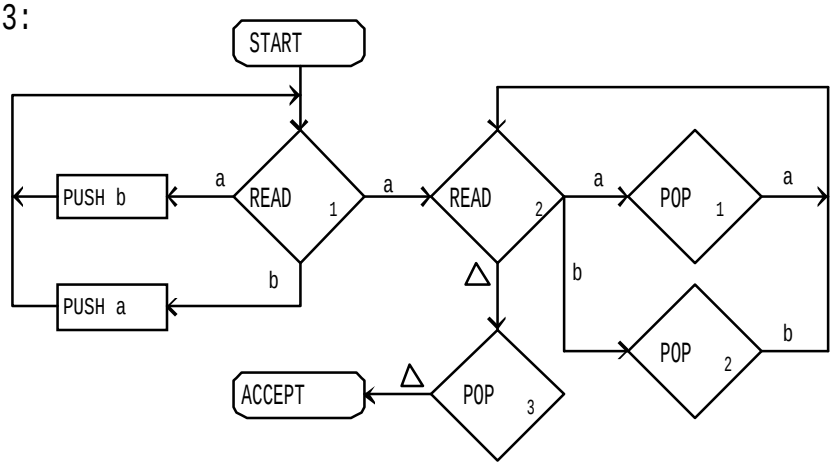
M1:


START
READ 1
PUSH a
PUSH b
READ 2
POP 1
POP 2
POP 3
ACCEPT
a
b

M2:


START
READ 1
PUSH a
PUSH b
READ 2
POP 1
POP 2
POP 3
ACCEPT
a
b

M3:


START
READ 1
PUSH b
PUSH a
READ 2
POP 1
POP 2
POP 3
ACCEPT
a
b

3. **[20 points]** For each of the following languages L over the alphabet $\Sigma = \{a, b\}$, give a context-free grammar for L.
    a. L exactly consists of all strings of odd length.
    b. $L = \{ b^{3n} a^{n+1} : n = 0, 1, 2, \ldots \}$.

4. **[15 points]** For each of the languages in question 3, do the following:
    - State whether it is a regular language or a non-regular language.
    - If the language is regular, give a regular expression.
    - If the language is non-regular, prove it.

5. **[15 points]** Let L_1, L_2, L_3 be languages, each having the same finite alphabet Σ. Suppose that
    - L_1 consists of all strings whose length is at least 20.
    - L_2 is accepted by some nondeterministic finite automaton.
    - L_3 is accepted by some nondeterministic pushdown automaton.

    Prove that $L_1 (L_2' + L_3)$ is a context-free language.