Final Exam
CIS 341: Introduction to Logic and Automata — Spring 2002, day sections
Prof. Marvin K. Nakayama

Print Name: ____

Student Number: ____

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

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Signature and Date

- This exam has 12 pages in total, numbered 1 to 12. Make sure your exam has all the pages.
- This exam will be 2.5 hours in length.
- This is an open-book, open-note exam.
- For all problems, follow these instructions:
  1. Show your work and give reasons (except for question 1).
  2. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area to work out your answers before filling in the answer space.
  3. FA stands for finite automaton; TG stands for transition graph; CFG stands for context-free grammar; CFL stands for context-free language; PDA stands for push-down automaton; TM stands for Turing machine.
  4. For any proofs, be sure to provide a step-by-step argument, with justifications (e.g., cite a theorem or definition in the textbook) for every step. You may assume that the theorems in the textbook hold; i.e., you do not have to reprove the theorems in the textbook. When using a theorem or result from the textbook, make sure you refer to it by number (e.g., Theorem 3) or page number.

Problem	1	2	3	4	5	6	Total
Points							

1. **[20 points]** For each of the following, circle TRUE if the statement is always correct. Otherwise, circle FALSE

(a) TRUE FALSE — If a language L is accepted by a nondeterministic finite automaton, then there must be some PDA that also accepts L.

(b) TRUE FALSE — If L is generated by a context-free grammar that is *not* a regular grammar, then L must *not* be a regular language.

(c) TRUE FALSE — If L_1 is a context-free language and L_2 is not context-free, then L_1L_2 must be context-free.

(d) TRUE FALSE — If L_1 is a context-free language and L_2 is not context-free, then L_1L_2 must not be context-free.

(e) TRUE FALSE — Every context-free language is a nonregular language.

(f) TRUE FALSE — Every nondeterministic PDA can be transformed into an equivalent deterministic PDA.

(g) TRUE FALSE — We can construct a PDA for the language $L = \{a^{2n}b^n : n = 1, 2, 3, \ldots\}$ by first constructing an FA for L, and then converting the FA into a PDA for L.

(h) TRUE FALSE — If w is an input string and T is a transition graph, then T must either accept or reject w.

(i) TRUE FALSE — If w is an input string and T is a Turing machine, then T must either accept or reject w.

(j) TRUE FALSE — There is a Turing machine that can take as input any encoded Turing machine P and any input w for P and decide if P halts on input w.

2. **[20 points]** For each of the following multiple-choice questions, circle the letter of the correct answer.

   2.1. If a language L is finite, then

   (a) there must not exist a pushdown automaton for L.
   (b) L must have a transition graph.
   (c) L must be a non-regular language.

   2.2. If L_1 and L_2 are context-free languages, then $L_1 \cap L_2$

   (a) must be context-free.
   (b) must not be context-free.
   (c) may or may not be context-free.

   2.3. If L is a language accepted by a nondeterministic finite automaton, then L

   (a) must be context-free.
   (b) must not be context-free.
   (c) may or may not be context-free.

   2.4. If L is not a context-free language, then

   (a) L must be finite.
   (b) L must be a non-regular language.
   (c) L must be a regular language.

   2.5. If L is generated by a context-free language in Chomsky normal form, then

   (a) L must be regular.
   (b) L must be non-regular.
   (c) L can be either regular or non-regular.

   2.6. If L is accepted by a finite automaton, then

   (a) there must exist some Turing machine that accepts L.
   (b) there must not exist some Turing machine that accepts L.
   (c) there may or may not exist some Turing machine that accepts L.

   2.7. If L is a language accepted by some Turing machine M, then

   (a) $w \in L'$ must be rejected by M.
   (b) running M with $w \in L$ initially loaded on the TAPE will result in M ending in a HALT state.
   (c) L must be a nonregular language.

2.8. Let $\Sigma = \{a, b\}$, and let L be the language over Σ exactly consisting of all strings that do not contain the substring aaa. Consider the following regular expressions:

(i) $(a+b)^*(a+aa)^*(a+b)^*$

(ii) $(b+ab+aab)^*(\Lambda+a+aa)$

(iii) $(\Lambda+a+aa)(b+ba+baa)^*$

Which of the following is correct?

(a) Only regular expression (i) generates L.
(b) Only regular expression (ii) generates L.
(c) Only regular expression (iii) generates L.
(d) Only regular expressions (i) and (ii) generate L.
(e) Only regular expressions (i) and (iii) generate L.
(f) Only regular expressions (ii) and (iii) generate L.
(g) All 3 regular expressions generate L.
(h) None of the 3 regular expressions generates L.

2.9. Let $\Sigma = \{a, b\}$, and let $L = \{a^n w a^n : n \geq 1, w \in \Sigma^*\}$. Consider the following statements:

(i) L has regular expression $a^*(a+b)^*a^*$.

(ii) L is a non-regular language.

(iii) L has context-free grammar

$$S \rightarrow aSa \mid aS \mid bS \mid aa$$

where S, X are nonterminals.

(iv) L has context-free grammar

$$\begin{aligned} S &\rightarrow aSa \mid aXa \\ X &\rightarrow aX \mid bX \mid \Lambda \end{aligned}$$

where S, X are nonterminals.

Which of the following is true?

(a) Only statement (i) is correct.
(b) Only statement (ii) is correct.
(c) Only statement (iii) is correct.
(d) Only statement (iv) is correct.
(e) Only statements (i) and (iii) are correct.
(f) Only statements (i) and (iv) are correct.
(g) Only statements (ii) and (iii) are correct.
(h) Only statements (ii) and (iv) are correct.
(i) Only statements (ii), (iii) and (iv) are correct.
(j) All the 4 statements are correct.
(k) None of the 4 statements is correct.

2.10. Suppose L is a language generated by a regular grammar, and consider the following statements:

(i) L is a regular language.

(ii) L is a context-free language.

(iii) There is a Turing machine that accepts L.

Which of the following is correct?

(a) Only statement (i) is true.
(b) Only statement (ii) is true.
(c) Only statement (iii) is true.
(d) Only statements (i) and (ii) are true.
(e) Only statements (i) and (iii) are true.
(f) Only statements (ii) and (iii) are true.
(g) All 3 statements are true.
(h) None of the 3 statements is true.

3. **[20 points]** Let $\Sigma = \{a, b\}$. Each of the following languages L defined over Σ falls into one of the following categories:

(i) L is a regular language.

(ii) L is a context-free language, but not a regular language.

(iii) L is recursively enumerable, but not a context-free language.

For each of the following languages, specify which category it is in. If a language L is in category (i), give a regular expression for L. If a language L is in category (ii), give a context-free grammar for L. If a language L is in category (iii), you only need to specify that it is in this category.

(a) L exactly consists of all strings $w \in \Sigma^*$ such that $w = \text{reverse}(w)$ and the length of w is divisible by 4 (i.e., w has length $4n$ for some $n = 0, 1, 2, 3, \ldots$).

Circle one: Category (i) Category (ii) Category (iii)

(b) $L = \{b^n a^n b^n : n \geq 0\}$.

Circle one: Category (i) Category (ii) Category (iii)

(c) $L = \{a^{2n} : n \geq 0\}$.

Circle one: Category (i) Category (ii) Category (iii)

(d) $L = \{a^m b^m a^n b^n : m \geq 0, n \geq 0\}$.

Circle one: Category (i) Category (ii) Category (iii)

Scratch-work area for problem 3.

4. **[10 points]** Let $L = \{a^{3n}b^{2n} : n = 0, 1, 2, \ldots\}$.

(a) **[5 points]** Give a pushdown automaton that accepts L.

Draw PDA here:

(b) **[5 points]** Give a Turing machine that accepts L.

Draw Turing machine here:

Scratch-work area for problem 4.

5. **[10 points]** Let $L_1, L_2, L_3, \ldots$ be an infinite sequence of context-free languages, each of which is defined over a common alphabet Σ. Let L be the infinite union of $L_1, L_2, L_3, \ldots$; i.e., $L = L_1 + L_2 + L_3 + \cdots$. Is it always the case that L is a context-free language?

YES NO (Circle one)

If your answer is YES, give a proof. If your answer is NO, give a counterexample. Explain your answer.

6. **[20 points]** Let L_1, L_2, and L_3 be languages defined over the alphabet $\Sigma = \{a, b\}$, where

- L_1 consists of all possible words over Σ except the words $w_1, w_2, \ldots, w_{100}$; i.e., start with all possible words over the alphabet, take out 100 particular words, and the remaining words form the language L_1;
- L_2 can be generated by a context-free grammar that is a regular grammar; and
- L_3 is accepted by some push-down automaton.

Prove that $(L_1 \cap L_2)L_3$ is a context-free language.