Midterm Exam I
CIS 341: Introduction to Logic and Automata — Fall 2000
Prof. Marvin K. Nakayama

Print Name (last name first): ______________________________

Student Number: ______________________________

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

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Signature and Date

- This exam has 6 pages in total, numbered 1 to 6. Make sure your exam has all the pages.
- This exam will be 1 hour and 25 minutes in length.
- This is a closed-book, closed-note exam.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area to work out your answers before filling in the answer space.
  2. FA stands for finite automaton; TG stands for transition graph.
  3. For any proofs, be sure to provide a step-by-step argument, with justifications for every step.

Problem	1	2	3	4	5	Total
Points						

1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — A finite automaton accepts Λ if and only if the initial state is also a final state.

(b) TRUE FALSE — If S^* is infinite, then S must be infinite.

(c) TRUE FALSE — If S^* is infinite, then S must be finite.

(d) TRUE FALSE — If S is finite, then S^* must be infinite.

(e) TRUE FALSE — If S is infinite, then S^* must be finite.

(f) TRUE FALSE — The string *abaab* can be generated by the regular expression $(\mathbf{\Lambda} + \mathbf{b})(\mathbf{b}^*\mathbf{a}^*\mathbf{b}^*)^*\mathbf{a}^*$.

(g) TRUE FALSE — If a finite automaton has at least one final state, then the FA must accept at least one word.

(h) TRUE FALSE — If a transition graph has at least one final state, then the TG must accept at least one word.

(i) TRUE FALSE — A finite automaton may have more than one initial state.

(j) TRUE FALSE — A finite automaton may have no final states.

2. **[20 points]** For each of the following languages L over the alphabet $\Sigma = \{a, b\}$, give a regular expression for L.

   (a) L exactly consists of all non-empty strings whose first letter and last letter are the same.

   Regular Expression: ______________________

   (b) L exactly consists of all strings that do not contain the substring bb.

   Regular Expression: ______________________

Scratch-work area

3. **[20 points]** For each of the following languages L over the alphabet $\Sigma = \{a, b\}$, give a finite automaton that accepts exactly L.

   (a) L exactly consists of all strings that have length exactly 3.

   Draw finite automaton here:

   (b) L exactly consists of all strings that do not contain the substring aaa.

   Draw finite automaton here:

Scratch-work area

4. **[20 points]** Suppose that S and T are two sets of strings such that $S^{**} = T^*$. Is it necessarily the case that $S = T$?

YES NO (Circle one)

If your answer is YES, give a proof. If your answer is NO, give a counterexample. Explain your answer, and be sure to provide all details.

5. **[20 points]**

Let L be any language. Define the transpose of L, denoted by L^t, to be the language of exactly those strings that are words in L spelled backward; i.e., $L^t = \{\text{reverse}(w) : w \in L\}$. For example, if $L = \{a, abb, bbaab, bbbaa\}$, then $L^t = \{a, bba, baabb, aabbb\}$. Prove that $(L_1L_2)^t = L_2^tL_1^t$ for any languages L_1 and L_2.