Midterm Exam II
CIS 341: Introduction to Logic and Automata — Fall 2000
Prof. Marvin K. Nakayama

Print Name (last name first): ______________________________

Student Number: ______________________________

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

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Signature and Date

- This exam has 7 pages in total, numbered 1 to 7. Make sure your exam has all the pages.
- This exam will be 1 hour and 25 minutes in length.
- This is an closed-book, closed-note exam.
- For all problems, follow these instructions:
  1. Show your work and give reasons (except for question 1).
  2. Give only your answers in the spaces provided. Only what you put in the answer space will be graded, and points will be deducted for any scratch work in the answer space. Use the scratch-work area to work out your answers before filling in the answer space.
  3. FA stands for finite automaton; TG stands for transition graph.
  4. For any proofs, be sure to provide a detailed, step-by-step argument, with justifications for each step. Unless a problem specifically asks you to prove a theorem from the textbook, you may assume that any theorem in the textbook holds; i.e., you do not have to reprove the theorems in the textbook unless the question specifically asks you to do so. If you use a theorem from the textbook in any proof, be sure to give a statement of the theorem with all of the assumptions and conclusions; e.g., say something like: "We will use the following theorem: If S is any set of strings, then $S^{**} = S^{*}$."

Problem	1	2	3	4	Total
Points					

1. **[30 points]** For each of the following, circle TRUE if the statement is always correct. Otherwise, circle FALSE

(a) TRUE FALSE — If a regular expression contains a Kleene star, then the language generated by the regular expression must be infinite.

(b) TRUE FALSE — If L is a regular language, then L' must be a nonregular lanaguge.

(c) TRUE FALSE — If $L_1, L_2, L_3, \ldots, L_n$ is a finite collection of regular languages, then $L_1 + L_2 + L_3 + \cdots + L_n$ must be a regular language.

(d) TRUE FALSE — If a language L is accepted by a nondeterministic finite automaton, then there is a (deterministic) finite automaton that accepts L.

(e) TRUE FALSE — Suppose a language L is accepted by some finite automaton with N states, and suppose that $w \in L$ with $\text{length}(w) \geq N$. Then there exist strings x, y, and z such that $w = xyz$, $\text{length}(x) + \text{length}(y) \leq N$, $y \neq \Lambda$, and $xy^k z \in L$ for all $k \geq 0$.

(f) TRUE FALSE — If L_1 and L_2 are nonregular languages, then $L_1 \cap L_2$ must be nonregular.

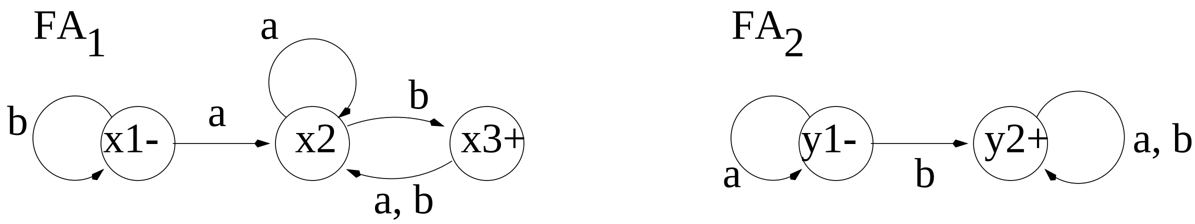
(g) TRUE FALSE — If L has a regular expression, then there is a finite automaton that accepts L.

(h) TRUE FALSE — If L is a nonregular language, then L must be infinite.

(i) TRUE FALSE — If L is a regular language, then L must be infinite.

(j) TRUE FALSE — If L_1 and L_2 are languages such that $L_1 \cap L_2' = \emptyset$, then $L_1 = L_2$.

2. **[20 points]** Suppose L_1 is accepted by finite automaton FA_1 and L_2 is accepted by finite automaton FA_2.



Give an FA that will accept exactly the language $L_1 + L_2'$.

Draw finite automaton here:

Scratch-work area

3. **[30 points]** Consider the language L over the alphabet $\Sigma = \{a, b\}$, with L consisting of exactly all words that have even length and are the same spelled forwards as backwards; i.e., $w \in L$ if and only if length(w) is even and $w =$ reverse(w).

   (a) Give a context-free grammar for L. Be sure to explicitly define your sets of terminals and nonterminals.

Scratch-work area for problem 3.

(b) Is L regular or nonregular?

REGULAR   NONREGULAR   (Circle one)

If your answer is REGULAR, give a regular expression. If your answer is NONREGULAR, give a proof. Explain your answer, and be sure to provide all details.

4. **[20 points]** Throughout this problem, assume that L is a language defined over some alphabet Σ and that L is accepted by some finite automaton with N states. Recall that Theorem 19 states L is infinite if and only if there exists some string $w \in L$ such that $N \leq \text{length}(w) < 2N$.

   (a) Using Theorem 19, develop an effective decision procedure to determine if L is infinite. Be sure to provide all of the details of your method. Also, explain why your method works and why it is an effective procedure. For example, be sure to explicitly state either the exact number of operations required by your procedure or provide an upper bound for the number of operations required. Be sure to explain why your number or bound is correct.

(b) Now consider the following claim: if there exists some string $w \in L$ such that $0 \leq \text{length}(w) < 2N$, then L is infinite. Note that the claim differs from Theorem 19 in that Theorem 19 has N as a lower bound for the length of w and the claim uses 0 as a lower bound. Is the claim always true?

YES NO (Circle one)

If your answer is YES, give a proof. If your answer is NO, give a counterexample. Explain your answer, and be sure to provide all details.