Midterm Exam I
CIS 341: Introduction to Logic and Automata — Spring 2000
Prof. Marvin K. Nakayama

Print Name (last name first): _____

Student Number: _____

I have read and understand all of the instructions below, and I will obey the Academic Honor Code.

_____
Signature and Date

- This exam has 6 pages in total, numbered 1 to 6. Make sure your exam has all the pages.
- This exam will be 1 hour and 25 minutes in length.
- This is a closed-book, closed-note exam.
- For all problems, follow these instructions:
  1. Give only your answers in the spaces provided. I will only grade what you put in the answer space, and I will take off points for any scratch work in the answer space. Use the scratch-work area to work out your answers before filling in the answer space.
  2. FA stands for finite automaton; TG stands for transition graph.
  3. For any proofs, be sure to provide a step-by-step argument, with justifications for every step.

Problem	1	2	3	4	5	Total
Points						

1. **[20 points]** For each of the following, circle TRUE if the statement is correct. Otherwise, circle FALSE

(a) TRUE FALSE — If S is any set, then $S^{****} = S^{**}$ is always true.

(b) TRUE FALSE — If S is any set, then S^{*} is always infinite.

(c) TRUE FALSE — A finite automaton may crash when processing a string.

(d) TRUE FALSE — A transition graph may have no final states.

(e) TRUE FALSE — A transition graph may have no initial states.

(f) TRUE FALSE — If a finite automaton accepts Λ, then the initial state of the finite automaton must also be a final state.

(g) TRUE FALSE — All finite automata are non-deterministic.

(h) TRUE FALSE — All transition graphs are also finite automata.

(i) TRUE FALSE — The regular expressions $(\mathbf{ab}^* + \mathbf{ba}^*)^*$ and $(\mathbf{a}^*\mathbf{b}^*)^*$ generate the same language.

(j) TRUE FALSE — If a finite automaton accepts no words, then the finite automaton must have no final states.

2. **[20 points]** For each of the following languages L over the alphabet $\Sigma = \{a, b\}$, give a regular expression for L.

   (a) L exactly consists of all words whose first and third letters are different.

   Regular Expression: _____

   (b) L exactly consists of all words that have an odd number of a's and an even number of b's.

   Regular Expression: _____

Scratch-work area

3. **[20 points]** For each of the following languages L over the alphabet $\Sigma = \{a, b\}$, give a finite automaton that accepts exactly L.

   (a) L exactly consists of all words that have contain the substring ab.

   Draw finite automaton here:

   (b) L exactly consists of all words whose first and last letters are the same.

   Draw finite automaton here:

Scratch-work area

4. **[20 points]** Let T be a transition graph, and suppose that T accepts Λ. Is it necessarily the case that an initial state of T is also a final state?

   YES NO (Circle one)

   If your answer is YES, give a proof. If your answer is NO, give a counterexample. Explain your answer.

5. **[20 points]**

   Let S be any set of strings. Prove that $S^{**} = S^{*}$.